B-meson physics with domain-wall light quarks and relativistic heavy quarks

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Time Requested: The equivalent of 3.3 million jpsi core-hours on the Fermilab clusters plus 100 Tbytes of tape storage (the equivalent of ~ 0.3 million jpsi core-hours) and 0.4 Tbytes of disk storage (the equivalent of ~ 12k jpsi core-hours) at Fermilab.

Project webpage: `http://rbc.phys.columbia.edu/USQCD/B-physics/`

Abstract

We propose to compute B-meson weak matrix elements needed to constrain the CKM unitarity triangle and test the Standard Model in the quark flavor sector using the 2+1 flavor dynamical domain-wall fermion gauge field configurations generated by the LHP, RBC, and UKQCD Collaborations and relativistic b quarks. In the upcoming year we aim to finish our calculation of the B-meson leptonic decay constants f_{B_d} and f_{B_s}, the $B^0 - \overline{B^0}$ mixing matrix elements, and their ratio, $\xi \equiv f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$. The $SU(3)$-breaking ratio ξ provides an important constraint on the apex of the CKM triangle and was therefore highlighted as a key goal in flavor physics in the USQCD Collaboration's 2007 white paper. We will also calculate the $B \to \pi \ell \nu$ form factor using the same method, which will enable a determination of the CKM Matrix element $|V_{ub}|$. Calculations of $|V_{ub}|$ are particularly critical due to the worrisome $\approx 3\sigma$ tension between inclusive and exclusive determinations. For all of these B-meson weak matrix elements we expect to obtain precise results that are competitive with other approaches, and that will place strong constraints on the CKM unitarity triangle fits. Our calculations will provide independent and valuable crosschecks of the results by HPQCD and Fermilab/MILC who both use the same set of gauge field configurations (different than those used in this proposal). We request the equivalent of 3.3 million jpsi core-hours on the Fermilab clusters plus 100 Tbytes of tape storage (the equivalent of ~ 0.3 million jpsi core-hours) and 0.4 Tbytes of disk storage (the equivalent of ~ 12k jpsi core-hours) for this project.

1 Scientific motivation

The calculation of B-meson weak matrix elements on the lattice enables precise determinations of CKM matrix elements, constraints on the CKM unitarity triangle, and tests of the Standard Model in the quark-flavor sector. The standard global unitarity-triangle fit uses lattice-QCD inputs for neutral B-meson mixing matrix elements, the $B \to D^{(*)}\ell\nu$ form factors, and the $B \to \pi\ell\nu$ form factor [1, 2]. In addition, the constraint on the unitarity triangle from $B \to \tau\nu$ decay requires a determination of the decay constant f_B. Thus lattice-QCD B-physics calculations are of great phenomenological importance.

One quantity that places a key constraint on the apex of the CKM unitarity triangle is neutral B-meson mixing. Experimentally, $B_q^0 - \overline{B_q^0}$ mixing is measured in terms of mass differences (oscillation frequencies) Δm_q, where q labels the light quark content of the B meson and is either a d- or s-quark. Within the Standard Model these oscillation frequencies are parameterized as [3]

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \tag{1}$$

where m_{B_q} is the the mass of the B_q-meson, V_{tq}^* and V_{tb} are the relevant CKM matrix elements, and the Inami-Lim function S_0 [4] and QCD coefficient η_B [3] can be calculated in perturbation theory. The hadronic matrix element $f_{B_q}^2 B_{B_q}$, where f_{B_q} is the leptonic decay constant and B_{B_q} is the B-meson bag parameter, must be computed via lattice QCD. The $SU(3)$-breaking ratio

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} \tag{2}$$

can be obtained with especially high precision because the lattice statistical and systematic uncertainties largely cancel between the numerator and denominator. This quantity allows the determination of the ratio of CKM matrix elements $|V_{ts}|^2/|V_{td}|^2$ via [5]:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2} \tag{3}$$

and currently places one of the single-tightest constraints on the apex of the CKM unitarity triangle (second only to $\sin(2\beta)$, which does not involve lattice inputs). The precision of $|V_{ts}|^2/|V_{td}|^2$ is still limited, however, by the uncertainty in lattice-QCD calculations of ξ. Recent experimental measurements of the oscillation frequencies Δm_d and Δm_s have established an accuracy of $\sim 1\%$ [6], but the $SU(3)$-breaking ratio ξ is only known to $\sim 3\%$ [7–9]. Given the phenomenological importance of ξ, this quantity was highlighted as one of three “key matrix elements” in the USQCD Collaboration’s 2007 white paper “Fundamental parameters from future lattice calculations” [10].

Another quantity that is in critical need of improvement from lattice-QCD is the $B \to \pi\ell\nu$ form factor, which enables the determination of the CKM matrix element $|V_{ub}|$ from the experimental branching fraction via

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |f_+(q^2)|^2. \tag{4}$$

Over the past several years there has been a persistent tension between the exclusive determination of $|V_{ub}|$ from $B \to \pi\ell\nu$ decay and the inclusive determination from semileptonic decays of the form $B \to X_u\ell\nu$ where X_u is any charmless hadronic final state. This tension has recently grown in magnitude to more than a 3σ discrepancy [11]. Further, the value of V_{ub} obtained from recent experimental measurements of $\mathrm{BR}(B \to \tau\nu)$ combined with lattice-QCD calculations of f_B is higher than both $|V_{ub}|_{\mathrm{excl}}$ and $|V_{ub}|_{\mathrm{incl}}$, and disagrees with their average by more than 2σ [12]. Although there is no obvious source for these discrepancies, the decay $B \to \pi\ell\nu$ is not a particularly good candidate for large new-physics contributions because it occurs at tree-level in the Standard Model. Thus one suspects that the disagreement may be due to a combination of underestimated systematic uncertainties and unkind statistical fluctuations. Lattice-QCD calculations of the $B \to \pi\ell\nu$ form factor with improved precision are key elements needed to address this puzzle. Once the situation is resolved and $|V_{ub}|$ is under better control, the constraint on the apex of the CKM unitarity triangle from $|V_{ub}|$ will strengthen tests of the Standard Model and tighten constraints on new physics in the quark-flavor sector.

Three years ago we initiated a project to compute B-meson decay constants and mixing parameters using the 2+1 flavor dynamical domain-wall ensembles generated by the LHP, RBC and UKQCD collaborations with lattice spacings $a \approx 0.11$ fm ($a^{-1} = 1.73$ GeV) and $a \approx 0.08$ fm ($a^{-1} = 2.271$ GeV) [13, 14]. Our calculation uses domain-wall fermions for the light quarks [15, 16] and the relativistic heavy quark (RHQ) action developed by Christ, Li and Lin for the heavy b quarks [17, 18]. The RHQ method extends the Fermilab approach [19] by tuning all of the parameters of the clover action nonperturbatively [20]. The RHQ action is accurate to $\mathcal{O}(a^2p^2)$, but to all orders in $(am_b)^n$; thus it allows the computation of heavy-light spectrum quantities with discretization errors of the same order as in light-light quantities.

Recently we began a computation of the $B \to \pi\ell\nu$ form factor using the same setup; this analysis is being led by visiting graduate student Taichi Kawanai from Tokyo University. A key ingredient to obtaining precise decay constants and form factors will be the use of mostly-nonperturbative renormalization of the axial-vector and vector current operators following the method of Ref. [21], which we are now implementing. Further, with non-USQCD resources, Columbia University graduate student Hao Peng is computing the D and D_s meson decay constants. The RHQ action is suitable for both bottom and charm, so only re-tuning the parameters of the action for the c-quark is needed. A comparison of f_D and f_{D_s} with experiment will allow another valuable crosscheck of the RHQ method and increase confidence in B-meson calculations using the same approach. All of these new projects are computationally inexpensive because they can re-use the domain-wall light-quark propagators that we have already generated.

Currently, the Fermilab/MILC and HPQCD collaborations are also computing B-meson weak matrix elements using the 2+1 flavor Asqtad-improved-staggered ensembles generated by the MILC Collaboration [7–9]. Our project will provide essential independent crosschecks using different light-quark and heavy-quark formulations, and we expect to obtain competitive uncertainties. With the computing time requested in this proposal we will finish our computation of B-meson decay constants and mixing parameters, compute the $B \to \pi\ell\nu$ form factor, and implement mostly-nonperturbative operator renormalization. Our results will enable precise determinations of CKM matrix elements, place stringent constraints on the CKM unitarity triangle, and allow rigorous tests of the Standard Model in the quark-flavor sector.

2 Computational method

Our computation of B-physics quantities is performed in three steps:

1. Generation and saving of domain-wall light quark propagators.
2. Nonperturbative tuning of the parameters in the RHQ b-quark action.
3. Computation of bottom-light 2-point and 3-point correlation functions to obtain matrix elements.

With the resources obtained from USQCD in the past years we have completed step 1. This was the most computationally-expensive portion of our project. We are currently writing a publication on step 2 which we expect to complete before the All Hands' meeting in May. The computing time requested in this proposal will allow us to finish step 3.

We have computed general-purpose point-source domain-wall light-quark propagators to be used for all our heavy-light physics projects. We use the same values of the domain-wall height ($M_5 = 1.8$) and extent of the fifth dimension ($L_s = 16$) as were used for the sea sector when generating the gauge field configurations. Therefore we can use the determinations of the light and strange quark masses and the residual quark mass from RBC-UKQCD's analysis of the light pseudoscalar meson masses and decay constants [13, 14]. The set of 2+1 flavor domain-wall fermion gauge field configurations used in our computation is summarized in Table 1. On each configuration we generate domain-wall light-quark propagators with several partially-quenched mass values to enable good control over the chiral extrapolation; the available propagators are listed in Table 2. For ensembles with less than 1000 independent, thermalized configurations we doubled the number of propagators by generating a second source per configuration.

L	a(fm)	$m_{\rm sea}^l$	$m_{\rm sea}^h$	$m_{\rm sea}^\pi$(MeV)	# configs.	trajectory #
32	≈ 0.08	0.004	0.030	289	628	[290:5:3425]
32	≈ 0.08	0.006	0.030	345	445	[272:8:3824]
32	≈ 0.08	0.008	0.030	394	544	[250:5:2965]
24	≈ 0.11	0.005	0.040	329	1636	[495:5:8670]
24	≈ 0.11	0.010	0.040	422	1419	[1455:5:8545]

Table 1. Analyzed RBC-UKQCD domain-wall gauge field configurations. The pion masses are taken from [13, 14]. The analyzed trajectories are specified in the last column where the number between the colons specifies the separation. On the finer ("32^3") ensembles 1 trajectory = 2 molecular dynamics time units, whereas on the coarser ("24^3") ensembles 1 trajectory = 1 molecular dynamics time unit.

L	$m_{\rm sea}^l$	$m_{\rm val}$	# time sources per config	# propagators
32	0.004	0.004, 0.006, 0.008, 0.025, 0.0272, 0.030	2	1256
32	0.006	0.004, 0.006, 0.008, 0.025, 0.0272, 0.030	2	1778
32	0.008	0.004, 0.006, 0.008, 0.025, 0.0272, 0.030	2	1088
24	0.005	0.005, 0.010, 0.020, 0.030, 0.0343, 0.040	1	1636
24	0.010	0.005, 0.010, 0.020, 0.030, 0.0343, 0.040	1	1419

Table 2. Generated domain-wall valence-quark propagators. To compensate for the lower number of gauge field configurations on the 32^3 ensembles we generate additional time source(s) per configuration. The propagators with masses $m_{\rm val} = 0.0272$ on the 32^3 ensembles and $m_{\rm val} = 0.0343$ on the 24^3 ensembles correspond to the physical strange quark [14]).

We have completed the tuning of the parameters of the RHQ action for b quarks, as shown in the section on "Recent Progress," and can now proceed to computing weak matrix elements involving b quarks, as we now describe.

Decay constants and B-mixing matrix elements

We determine the $B_q^0 - \overline{B_q^0}$ mixing matrix elements, where q denotes the light-quark mass, from the 3-point correlation function shown in Fig. 1 (along with the B_q meson 2-point correlator). We keep the location of the effective four-quark operator $t_{\mathcal{O}^{\Delta B=2}}$ fixed and vary the locations of the B_q^0 and $\overline{B_q^0}$ mesons, t_1 and t_2, over all possible time slices. This setup requires one point-source light quark and one point source b-quark propagator originating from $t_{\mathcal{O}^{\Delta B=2}}$. These propagators can be used for both the B_q^0 as well as the $\overline{B_q^0}$ mesons, thereby reducing the overall computational cost. For the heavy b quarks we project out the zero momentum component using a gauge-invariant Gaussian smeared sink. The Gaussian smearing has been optimized to minimize excited-state contamination for B_q meson correlators.

We determine the decay constants f_{B_q} from the 2-point correlation function shown in Fig. 2. For this calculation we combine a Gaussian-smeared source b-quark propagator originating at time t_0 with the existing point-source light-quark propagators. Note that by identifying $t_{\mathcal{O}^{\Delta B=2}} \equiv t_0$, the same light-quark propagators can be used in both computations.

We calculate the B_q^0-mixing matrix elements and decay constants f_{B_q} at several values of the light-quark mass to aid in the chiral extrapolation to the physical d-quark mass. We will obtain our final results for the $B_d(B_s)$-meson decay constants and mixing parameters by fitting data on the coarse "24^3" and fine "32^3" lattices together and performing a simultaneous extrapolation (interpolation) to the physical light-

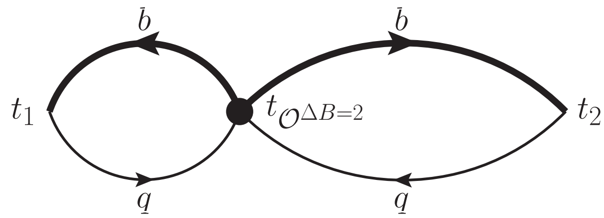

Figure 1. Three-point correlation function for computing $B_q^0 - \overline{B_q^0}$ mixing on the lattice.

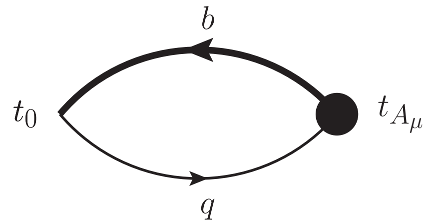

Figure 2. Two-point correlation function for computing the decay constant f_{B_q} on the lattice.

quark mass and the continuum using partially-quenched heavy-meson χPT [22] supplemented by analytic terms $\propto a^2$ to parameterize light-quark discretization effects. The use of the RHQ action leads to residual discretization errors that are more complicated functions of the bare-quark mass $m_0 a$. We estimate the size of these contributions using heavy-quark power-counting suitable for bottom-light systems where the typical B-meson momentum is of $\mathcal{O}(\Lambda_{\rm QCD})$ [23].

The lattice and continuum axial-current operator are related via a matching factor Z_A^{hl} and a series of improvement coefficients $c_A^{(i)}$:

$$\mathcal{A}_\mu^{\rm cont.} \doteq Z_A^{bl}\left(A_\mu^{{\rm lat},(0)} + c_A^{(ap),i} A_\mu^{{\rm lat},(ap),i} + \dots\right), \tag{5}$$

where the "$\doteq$" denotes equality of matrix elements and the "..." denote higher-order terms in the expansion parameter ap. A similar expression holds for the $\Delta B = 2$ four-fermion operator and the $b \to u$ vector current. The improvement coefficients adjust for short-distance mismatches between the lattice and continuum operators and are functions of the parameters in the RHQ action, while the renormalization factor Z_A^{hl} matches to the continuum regularization and renormalization scheme. In order to reduce discretization errors in the current and four-quark operators, we implement $O(a)$ operator improvement. Only one additional matrix element is needed to improve the decay constant at $O(ap)$ to all orders in α_s because no momentum leaves the axial-current operator in Fig. 2. We are currently computing the needed improvement coefficient $c_A^{(ap)}$ at 1-loop in tadpole improved lattice perturbation theory; this will improve the decay constants through $O(\alpha_s ap)$, such that truncation errors are of $O(\alpha_s^2 ap, a^2p^2)$. The Standard Model B_q-$\overline{B}_q$ mixing four-quark operator requires several additional matrix elements, and we are currently working on obtaining the minimal set needed for improvement through $O(\alpha_s ap)$. We are also computing the bottom-light current renormalization factor Z_A^{bl} using 1-loop tadpole-improved lattice perturbation theory [24], leading to truncation errors of $O(\alpha_s^2 ap, a^2p^2)$. For the most important quantity ξ we expect much of the uncertainty due to the truncation of perturbation theory to cancel in the ratio.

Mostly nonperturbative operator renormalization

Ultimately, to obtain precise determinations of the decay constants, matrix elements, and form factors, we need more precise heavy-light renormalization factors than can be obtained from 1-loop lattice perturbation theory alone. We therefore are implementing the "mostly-nonperturbative" renormalization method introduced by El Khadra *et al.* for the calculation of the $B(D) \to \pi\ell\nu$ form factors in Ref. [21]. This approach takes advantage of rewriting the heavy-light axial-vector (or vector) current renormalization factor as the following product:

$$Z_{A(V)}^{bl} = \varrho_{A(V)}^{bl}\sqrt{Z_V^{bb} Z_V^{ll}}. \tag{6}$$

Because the flavor-conserving renormalization factors Z_V^{bb} and Z_V^{ll} can be obtained nonperturbatively from standard heavy-light and light-light meson charge normalization conditions, only the residual correction $\varrho_{A(V)}^{bl}$ needs to be computed perturbatively. The flavor-conserving factors Z_V^{bb} and Z_V^{ll} account for most of the operator renormalization, while $\varrho_{A(V)}^{bl}$ is expected to be close to unity because most of the radiative

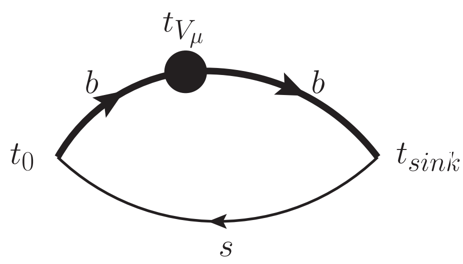

Figure 3. Three-point correlation function for computing Z_V^{bb} on the lattice.

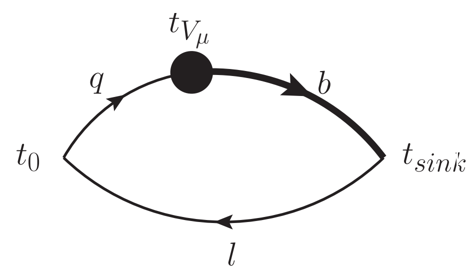

Figure 4. Three-point correlation function for computing the $B \to \pi\ell\nu$ form factor. The spectator quark is labeled l and the daughter quark is labeled q.

corrections, including contributions from tadpole graphs, cancel in the ratio $Z^{bl}_{A(V)}/\sqrt{Z^{bb}_V Z^{ll}_V}$ [25]. Therefore $\varrho^{bl}_{A(V)}$ has a more convergent series expansion in α_s than $Z^{bl}_{A(V)}$ and can be computed at 1-loop in tadpole-improve lattice perturbation theory to few-percent precision.

In practice, Z^{ll}_V has already been obtained by the RBC/UKQCD Collaborations (see Ref. [14]), where we use the fact that $Z_A = Z_V$ for domain-wall fermions up to corrections of $\mathcal{O}(m_{\rm res})$. We therefore need only calculate Z^{bb}_V ourselves. Figure 3 shows the 3-point correlation function needed to compute Z^{bb}_V. We compute the matrix element of the $b \to b$ vector current between two B_q mesons, where q is the domain-wall light quark. In practice, Z^{bb}_V is independent of the light "spectator" quark mass, so we fix the the spectator mass to be $m_q = m_s$ in order to reduce the statistical errors. As we show in the "Recent Progess" section, with this method we can obtain Z^{bb}_V with sub-percent statistical uncertainty.

The $B \to \pi\ell\nu$ form factor

We are also calculating the $B \to \pi\ell\nu$ form factor using the same general setup. This requires computing the matrix element of the $b \to u$ vector current between a B meson and a pion. The relevant 3-point correlation function is shown in Fig. 4. We fix the location of the pion at time t_0 and the location of the B meson at a time separation $T = (t_{\rm sink} - t_0)$ away. We then vary the location of the current operator t_{V_μ} over all time slices in between. We improve the operator at $\mathcal{O}(a)$, and are computing the improvement coefficients at 1-loop in lattice perturbation theory, such that the residual errors are of $\mathcal{O}(\alpha_s^2 ap, a^2p^2)$. The B meson is at rest, so we can use a Gaussian-smeared sequential b quark to reduce excited-state contamination, and we inject momentum on the pion side to obtain the form factor's momentum dependence.

Because the 3-point correlation function in Fig. 4 requires a new b-quark inversion for every light spectator-quark mass, we compute the form factor only with a unitary spectator mass (*i.e.* equal to the lighter quark mass in the sea sector). We compute the 3-point correlation function, however, for all available partially-quenched daughter-quark masses. These additional pion masses will help us to better resolve both the quark-mass dependence and the pion-energy dependence. As in the case of the B-mixing parameters and decay constants, we will extrapolate our data to the physical light-quark mass and continuum using next-to-leading order partially-quenched heavy meson χPT supplemented by higher-order analytic terms [26, 27]. We will estimate the residual discretization errors from the RHQ action using heavy-quark power-counting.

3 Recent progress

Over the past year we have made progress in several fronts needed for our B-physics program, which we describe here.

Nonperturbative tuning

A significant challenge associated with the RHQ approach is the nonperturbative tuning of the three parameters in the action m_0a, c_P and ζ. These are the bare-quark mass, clover coefficient, and anisotropy parameter,

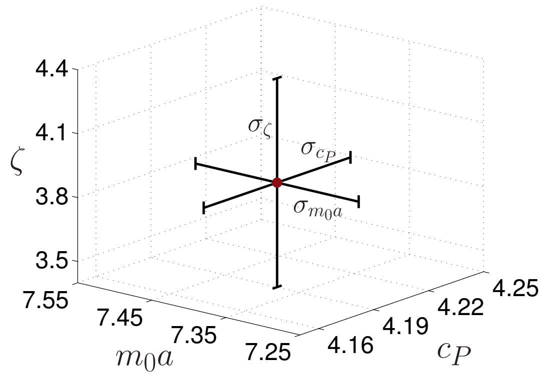

Figure 5. Visualization of the seven sets of parameters used to obtain the tuned values of $\{m_0a, c_P, \zeta\}$.

	m_0a	c_P	ζ
$am_l = 0.005$	8.4(1)	5.7(2)	3.1(1)
$am_l = 0.01$	8.5(1)	5.8(3)	3.1(2)
average	8.45(7)	5.73(17)	3.10(9)

(a) Preliminary results on the 24^3 ensembles.

	m_0a	c_P	ζ
$am_l = 0.004$	3.99(7)	3.6(1)	1.97(9)
$am_l = 0.006$	3.97(6)	3.5(1)	1.93(8)
$am_l = 0.008$	3.96(8)	3.6(1)	2.0(1)
average	3.97(4)	3.57(6)	1.96(5)

(b) Preliminary results on the 32^3 ensembles.

Table 3. Preliminary results for the nonperturbatively-tuned values of the RHQ parameters at both lattice spacings $a \approx 0.11$ fm (24^3) and $a \approx 0.08$ fm (32^3) [28].

respectively. We tune $\{m_0a, c_P, \zeta\}$ to describe b quarks by requiring that calculations of specified physical on-shell quantities correctly reproduce the experimentally-measured results. In particular, we use the bottom-strange system for tuning because both discretization errors and chiral extrapolation errors are expected to be small. We need three experimental inputs to fix $\{m_0a, c_P, \zeta\}$. We match to the experimental values of the spin-averaged B_s meson mass, $\overline{M}_{B_s} = \frac{1}{4}\left(M_{B_s} + 3M_{B_s^*}\right)$, and hyperfine splitting, $\Delta M_{B_s} = M_{B_s^*} - M_{B_s}$. We also require that the B_s meson rest and kinetic masses are equal, *i.e.* $M_1^{B_s}/M_2^{B_s} = 1$, so that the B_s meson satisfies the continuum energy-momentum dispersion relation.

We tune $\{m_0a, c_P, \zeta\}$ using an iterative procedure in which we compute the quantities $\{\overline{M}_{B_s}, \Delta M_{B_s}, M_1^{B_s}/M_2^{B_s}\}$ at seven sets of parameters in the RHQ action. These seven sets are obtained by varying one of the three parameters $\{m_0a, c_P, \zeta\}$ by a chosen uncertainty $\pm\sigma_{\{m_0a, c_P, \zeta\}}$ while holding the other two fixed (see Fig. 5). We must work in a region sufficiently close to the true parameters that the output quantities $\{\overline{M}_{B_s}, \Delta M_{B_s}, M_1^{B_s}/M_2^{B_s}\}$ depend linearly on the input parameters $\{m_0a, c_P, \zeta\}$. We can then interpolate using our "box" of seven input parameter sets to the values of $\{m_0a, c_P, \zeta\}$ corresponding to the physical b quark. We have finished tuning the RHQ parameters on all five sea-quark ensembles and are currently writing up the calculation for publication, which we expect to complete before the All Hands' meeting. The preliminary values for the RHQ parameters on the 24^3 and 32^3 ensembles are given in Table 3.

Given the values of the parameters $\{m_0a, c_P, \zeta\}$ tuned for b quarks, we can make predictions for the masses and mass-splittings of bottomonium states. This provides a simple test of the RHQ method. Figure 6 shows our preliminary determinations of the η_b and Υ masses as well as the fine splittings $M_\Upsilon - M_{\eta_b}$ and $M_{\chi_{b1}} - M_{\chi_{b0}}$. For all quantities studied, our results agree with experiment within estimates of systematic uncertainties, confirming the validity of the RHQ approach and bolstering confidence in our computations of heavy-light weak matrix elements with the RHQ action. The systematic errors in the bottomonium masses and mass-splittings shown in Fig. 6 are dominated by heavy-quark discretization errors, which we estimate via power-counting. We expect the heavy-quark discretization errors in bottom-light systems to be much smaller than in bottomonium, however, because the typical b-quark momentum is smaller: in $b\bar{q}$ states it is of $\mathcal{O}(\Lambda_{\rm QCD})$ whereas in $b\bar{b}$ states it is of $\mathcal{O}(\alpha_s m_b)$.

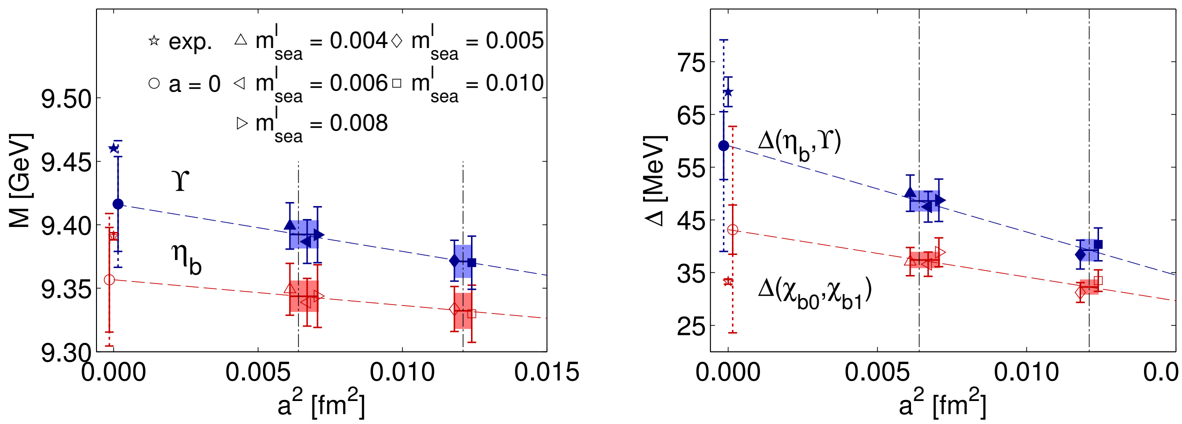

Figure 6. Continuum extrapolation of η_b and Υ meson masses (left plot) and mass-splittings (right plot) [28]. For the continuum values, the solid error bars show the statistical error, while the dotted error bars show the total error (which comes from adding the systematic error in quadrature).

Lattice perturbation theory and operator improvement

Even though we will use a mostly nonperturbative framework for the operator renormalization, we need to employ lattice perturbation theory (LPT) to obtain improvement coefficients for the heavy-light (axial-) vector currents and four-quark operators. Although for the Tsukuba formulation of the RHQ action one-loop results for the matching of (axial-)vector currents are known [29], these results do not translate trivially to the Columbia formulation of RHQ [18] which we use. We have implemented an automated framework to perform the necessary LPT calculations taking into account the complications introduced by non-trivial field rotations in the Columbia formulation. As a first test of the framework we extended the tadpole-improved one-loop predictions of Ref. [30] for the RHQ parameter tuning to the Columbia formulation. A publication containing these results is in preparation. Currently we are finalizing the matching of the currents and preparing the matching of the four-quark operators. Our automated LPT framework will allow for rapid progress for similar calculations in the future.

Mostly-nonperturbative operator renormalization (Z_V^{bb})

We compute the flavor-conserving renormalization factor Z_V^{bb} nonperturbatively from the 3-point correlation function shown in Fig. 3. We have written code to compute the vector-current 3-point function and have verified it in two ways: we implemented two independent versions ourselves, and we also cross-checked against the code used by the Fermilab/MILC Collaboration. We have computed Z_V^{bb} on the 24^3 ensembles with $m^l_{\rm sea} = 0.005$ using two different spectator-quark masses: the unitary mass $m_{\rm val} = 0.005$ and the strange-quark mass $m_{\rm val} = m_s = 0.0343$. The comparison is shown in Fig. 7. As we expect, Z_V^{bb} is independent of spectator mass within the statistical errors, and the use of the heavier spectator quark significantly reduces the statistical uncertainty, allowing us to obtain Z_V^{bb} with sub-percent accuracy. We will also study the light sea-quark mass dependence of Z_V^{bb}, but we again expect it to be negligible within statistical errors. We are currently computing Z_V^{bb} with different source-sink separations in order to determine the separation that leads to the best plateau quality. The separation of $T = 20$ shown in Fig. 3 already looks quite promising.

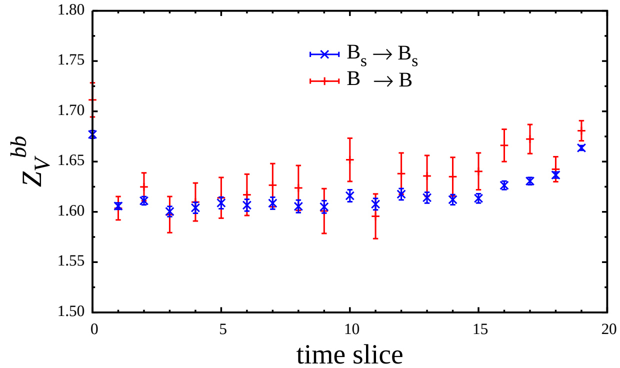

Figure 7. The flavor-conserving heavy-heavy vector current Z_V^{bb} for two different light spectator-quark masses, $m_{\rm val} = 0.005$ and $m_{\rm val} = m_s = 0.0343$, on the 24^3 ensemble with $m^l_{\rm sea} = 0.005$. The source-sink separation used is T = 20.

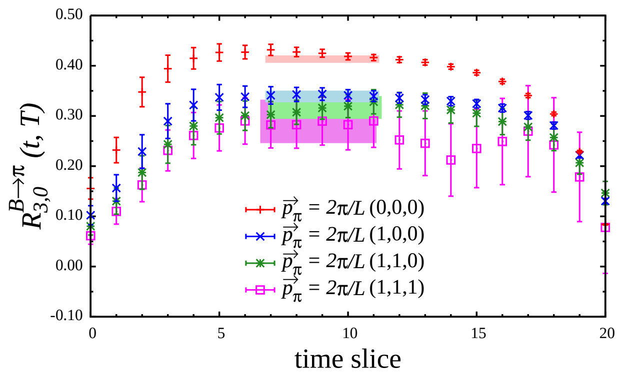

Figure 8. Temporal component of the $B \to \pi\ell\nu$ form factor on the 24^3 ensemble with $m^l_{\rm sea} = 0.005$ using a source-sink separation $T = 20$.

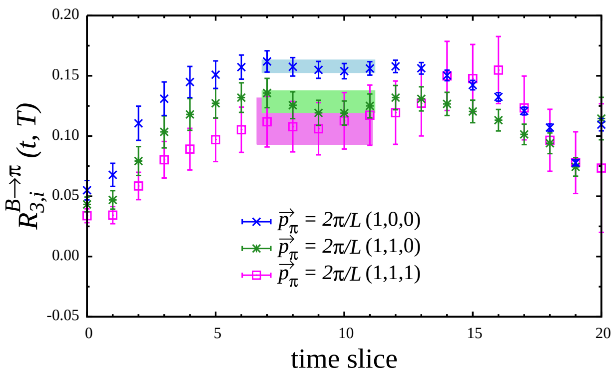

Figure 9. Spatial component of the $B \to \pi\ell\nu$ form factor on the 24^3 ensemble with $m^l_{\rm sea} = 0.005$ using a source-sink separation $T = 20$.

The $B \to \pi\ell\nu$ form factor

The computation of the $B \to \pi\ell\nu$ form factor is performed using the same 3-point code as for Z_V^{bb}, but with momentum inserted on the pion side. We extract the $B \to \pi\ell\nu$ form factor from the ratio of the 3-point function shown in Fig. 4 over the pion and B-meson 2-point functions:

$$R_{3,\mu}^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{\sqrt{C_2^\pi(t)C_2^B(T-t)}}\sqrt{\frac{2E_0^\pi}{\exp(-E_0^\pi t)\exp(-m_0^B t)}}\,, \tag{7}$$

where the phenomenologically-relevant form factor f_+ is a linear combination of the temporal form factor $f_\parallel \propto R_{3,0}^{B\to\pi}(t,T)$ and the spatial form factor $f_\perp \propto R_{3,i}^{B\to\pi}(t,T)$. We have computed the ratios $R_{3,0}^{B\to\pi}(t,T)$ and $R_{3,i}^{B\to\pi}(t,T)$ for several values of the source-sink separation in order to optimize its value before beginning production running. We have found $T = 20$ to be quite good; as shown in Figs. 8 and 9, the plateaus are of sufficient length and quality that we can reliably extract the temporal and spatial components of the form factor up to pion momentum $\vec{p}_\pi = 2\pi(1,1,1)/L$. These preliminary results indicate that our statistical errors should be in the range of 1.5%-3% from lowest to highest momentum. At the moment the coding of the $\mathcal{O}(a)$ improvement operators for the vector current is in progress.

4 Run Plan and Resource Allocation

Because we have already generated all of the necessary domain-wall light-quark propagators, the computing time in this proposal is only needed for b-quark inversions and contractions to obtain 2-point and 3-point correlation functions. All of our computations are done in `Chroma`: we use the default Clover inverter and implement the various measurements as inline functions. The bulk of the computational cost is due to the b-quark inversions. For each configuration and time source we compute the decay constants and other weak matrix elements with the same "box" of RHQ parameters used in the tuning procedure. This allows us to cleanly propagate the statistical uncertainties of the RHQ parameters via single-elimination jackknife, but increases the number of b-quark inversions by seven. Of course, this is still quite inexpensive compared to the initial cost of generating the domain-wall light-quark propagators. The remaining computational cost goes towards computing 2-point and 3-point correlation functions, of which there are a significant number due to the many partially-quenched points.

We have divided our running into three components with minimal computational overlap: decay constants and mixing matrix elements, the heavy-heavy renormalization factor Z_V^{bb}, and $B \to \pi\ell\nu$. The timing estimates for these computations on both the 24^3 and 32^3 ensembles are given in Table 4. The individual times in the table include the loop over all partially-quenched light-quark masses and all seven sets of b-quark parameters. Table 5 shows the total time required to analyze all of the configurations listed in Table 2.

We would like to continue running on the Fermilab clusters. Both jpsi and ds are well-suited for the computation of b quarks and 2-point and 3-point correlation functions. Several of the proposal authors are experienced in running on the Fermilab clusters and our additions to the Chroma code are compiled for both jpsi and ds. Further, all of our domain-wall propagators are saved on tape at Fermilab and can easily be transferred to the lustre file system as needed for production running.

In addition to computing time, we need temporary space to store our propagators on the lustre file system at Fermilab while running, as well as permanent storage for our propagators on tape and for our data in the project area. We anticipate a typical use of approximately 30 TB of lustre space to hold domain-wall propagators while running. We are currently using approximately 90 TB of tape to store our domain-wall propagators, and anticipate increasing this amount by about 10 TB before the end of this year's allocation. We are also currently using 0.2 TB of backed-up disc storage in the project area for our data, and expect to increase this by about a factor of two in the coming year to 0.4 TB.

5 Summary

By the end of the allocation period 2012/2013 we expect to have a precise determination of the B-meson decay constants f_{B_d} and f_{B_s}, the $B^0 - \overline{B^0}$ mixing matrix elements and their ratio ξ, and the $B \to \pi\ell\nu$ form factor. Our calculation of the $SU(3)$-breaking ratio ξ, in particular, will fulfill one of the key goals in flavor physics as stated in the 2002 strategic plan and the 2007 white paper "Fundamental parameters from future lattice calculations" of the USQCD collaboration [10]. Our final results will be based on computations at two lattice spacings with multiple quark masses, allowing us good control over the systematic errors asscoiated with both chiral and continuum extrapolations. We expect to obtain the decay constants and mixing matrix elements with few-percent errors, and to obtain the $B \to \pi\ell\nu$ form factor at high q^2 with an error below 10%. This will provide valuable independent and competitive crosschecks of the results of Fermilab/MILC and HPQCD. When used in the unitarity triangle analysis, our results will place an important constraint on physics beyond the Standard Model.

In the following year we may request further time to add data on additional sea-quark ensembles. This year the RBC and UKQCD collaborations will be generating an additional ensemble with $a^{-1} \approx 1.73$ GeV, a spatial volume of 48^3, and a pion mass of approximately 135 GeV. The addition of this ensemble would reduce our chiral extrapolation error, which we expect to be one of our largest sources of uncertainty.

We encourage other members of the lattice QCD community to make use of the domain-wall propagators we generate as part of this project in order to compute other physics quantities. Within this project we intend to compute the $B^0 - \overline{B^0}$ mixing matrix elements and their ratio ξ , the decay constants f_{B_d}, f_{B_s} and their ratio f_{B_s}/f_{B_d}, and the $B \to \pi\ell\nu$ form factor. We would also like to retain exclusive rights to calculate $D-$ and D_s-meson decay constants and beyond the Standard Model contributions to B- and D-meson mixing

quantity	a(fm)	L	nodes (jpsi)	time (hours)	jpsi core-hours
f_B, $B-\overline{B}$	≈ 0.08	32	16	3.0	384
Z_V^{bb}	≈ 0.08	32	8	2.5	160
$B \to \pi l \nu$	≈ 0.08	32	8	2.0	128
f_B, $B-\overline{B}$	≈ 0.11	24	8	1.5	96
Z_V^{bb}	≈ 0.11	24	4	1.5	48
$B \to \pi l \nu$	≈ 0.11	24	4	1.0	32

Table 4. Time to compute each quantity for all 7 "box" parameters of the RHQ action and for all partially-quenched light-quark masses using Chroma on the Fermilab "jpsi" cluster. These times are for a single source location and include both the heavy-quark inversions and contractions.

32^3 $a \approx 0.08$ fm	f_B, $B-\overline{B}$	1.583 $\times 10^6$ jpsi core-hours
32^3 $a \approx 0.08$ fm	Z_V^{bb}	0.660 $\times 10^6$ jpsi core-hours
32^3 $a \approx 0.08$ fm	$B \to \pi l \nu$	0.528 $\times 10^6$ jpsi core-hours
24^3 $a \approx 0.11$ fm	f_B, $B-\overline{B}$	0.293 $\times 10^6$ jpsi core-hours
24^3 $a \approx 0.11$ fm	Z_V^{bb}	0.147 $\times 10^6$ jpsi core-hours
24^3 $a \approx 0.11$ fm	$B \to \pi l \nu$	0.098 $\times 10^6$ jpsi core-hours
Total		3.309 $\times 10^6$ jpsi core-hours

Table 5. Total computer time needed to determine the B^0-$\overline{B^0}$ mixing matrix elements, B-meson decay constants, and $B \to \pi \ell \nu$ form factor using the sea quark ensembles, valence quark masses, and numbers of propagators listed in Table 2.

as well as the coupling constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$ using these propagators in the future. All generated propagators will be stored at Fermilab and will be made available immediately for non-competing analyses. Researchers who wish to use them should contact us to arrange access.

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