Beautiful physics on the lattice

RBC and UKQCD collaborations

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Seminar

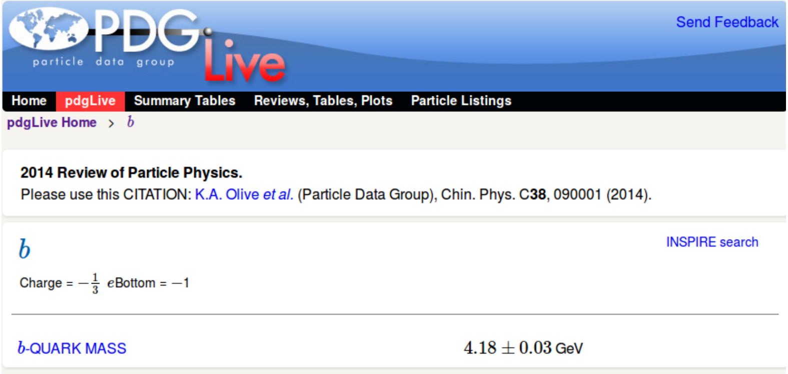
introduction

Beautiful physics . . .

- may require a controversial definition

Beautiful physics . . .

- may require a controversial definition
- or physics with a beautiful quark

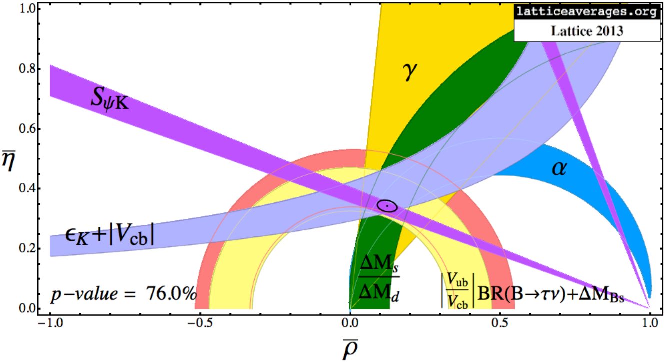


Why are b-quarks beautiful?

- Allow us to test the Standard Model
- Allow us to look for new physics
- b-quarks are heavy and hence have a lot of decay modes
- Are frequently produced in b-factories: BaBar, Belle, LHC, Belle II

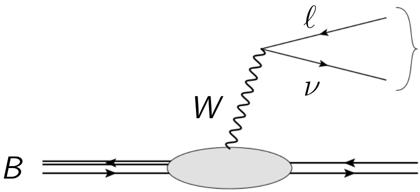
Motivation: Constraining the Standard Model

- Determination of CKM matrix elements V_{ub} and V_{cb}
- B-physics provides constraints on the apex of the CKM unitarity triangle
  - B^0–$\overline{B^0}$ mixing
  - V_{ub} and V_{cb} e.g. from $B \to \pi \ell \nu$ and $\overline{B} \to D^* \ell \nu$ form factors
- Experimental results and nonperturbative inputs are needed



[http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it, http://www.latticeaverages.org]

Example: V_{ub} from exclusive semileptonic decay $B \to \pi\ell\nu$



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parameterized by

$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2 + M_\pi^2 - q^2\right)^2 - 4M_B^2 M_\pi^2\right]^{3/2} \times \left|f_+(q^2)\right|^2 \times \left|V_{ub}\right|^2$$

experiment known nonperturbative input CKM

- Long standing puzzle in V_{ub} determination: $\sim 3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$) measurement

Motivation: New Physics in rare B-decays?

$B \to \tau\nu$ [UTfit Phys.Lett. B687 (2010) 61]

- f_B is needed for the Standard-Model prediction of $BR(B \to \tau\nu)$
- Potentially sensitive to charged-Higgs exchange due to large τ mass

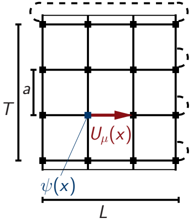
$B_s \to \mu_+\mu_-$ [Buras et al. Eur.Phys.J. C72 (2012) 2172, Buras et al. arXiv:1303.3820 [hep-ph]]

- f_{B_s} is needed for Standard-Model prediction of $BR(B_s \to \mu_+\mu_-)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Measured by LHCb with 3.5σ significance [LHCb Phys.Rev.Lett. 110 (2013) 02180], at EPS2013: combination of LHCb and CMS results gives $> 5\sigma$ significance — in agreement with SM

Both are sensitive to new physics!

lattice

Lattice QCD



- Discretize Euclidean space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- Study physics in a finite box of volume $(aL)^3$
- Compute expectation values of gauge invariant observables by

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, e^{-S(U)} \, \mathcal{O}(U), \quad \mathcal{Z} = \int \mathcal{D}U \, e^{-S(U)}$$

- Only statistical estimation possible: $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$
- Generate a sufficiently long sequence of configurations with probability distribution

$$P \propto \exp\{-S(U)\}$$

- Typically done by a Markov chain using the HMC algorithm with configurations saved e.g. every 10 MDTU

Are b-quarks on the lattice beautiful?

- Not yet . . .

but they provide new challenges

Challenges for B-physics on the lattice

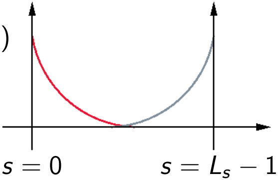
- The mass of the b-quark introduces another scale
  - Light quark masses: $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV
  - Mass of the b-quark: $m_b = 4.18$ GeV
- Today's lattices have an inverse lattice spacing of $a^{-1} \approx 1.7 \ldots 3 \ldots 4$ GeV
  - $am_b > 1$
  - Forced to simulate b-quarks with an effective action
    e.g. HQET (static), NRQCD, Fermilab or RHQ action
  - Requires (perturbative) mixed-action renormalization factors
- New concepts like heavy HISQ action look very promising
  [C. McNeile, et al. Phys.Rev. D85 (2012) 031503]

Our project

- Based on RBC-UKQCD's 2+1 flavor domain-wall Iwasaki gauge field configurations
- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - B^0–$\overline{B^0}$ mixing
  - Decay constants f_B and f_{B_s}
  - $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors [Taichi Kawanai and Ruth Van de Water]
  - $g_{B^*B\pi}$ coupling constant [Ben Samways and Jonathan Flynn]
- Provides important cross-check of other $N_f = 2+1$ determinations using the MILC staggered ensembles

2+1 flavor domain-wall gauge field configurations

- Domain-wall fermions for the light quarks (u, d, s)
  [Kaplan Phys.Lett. B288 (1992) 342]
  [Shamir Nucl.Phys. B406 (1993) 90]
- Iwasaki gauge action [Iwasaki UTHEP-118(1983)]
- Configurations generated by RBC and UKQCD collaborations [C. Allton et al. Phys.Rev. D78 (2008) 114509, Y. Aoki et al. Phys.Rev. D83 (2011) 074508]



L	a(fm)	m_l	m_s	M_π(MeV)	approx. # configs.	# time sources
24	$\approx$ 0.11	0.005	0.040	329	1636	1
24	$\approx$ 0.11	0.010	0.040	422	1419	1
32	$\approx$ 0.08	0.004	0.030	289	628	2
32	$\approx$ 0.08	0.006	0.030	345	889	2
32	$\approx$ 0.08	0.008	0.030	394	544	2

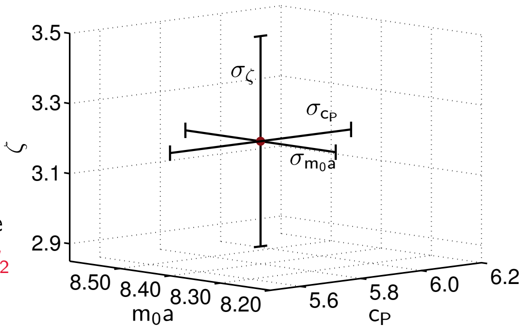
Relativistic Heavy Quark action for the *b*-quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin
  [Christ et al. Phys.Rev. D76 (2007) 074505; Lin and Christ Phys.Rev. D76 (2007) 074506]
- Builds upon Fermilab approach [El-Khadra et al. Phys.Rev. D55 (1997) 3933] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation
  [S. Aoki et al. Prog.Theor.Phys. 109 (2003) 383]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

Nonperturbative tuning of the RHQ action parameters

[Phys.Rev. D86 (2012) 116003]

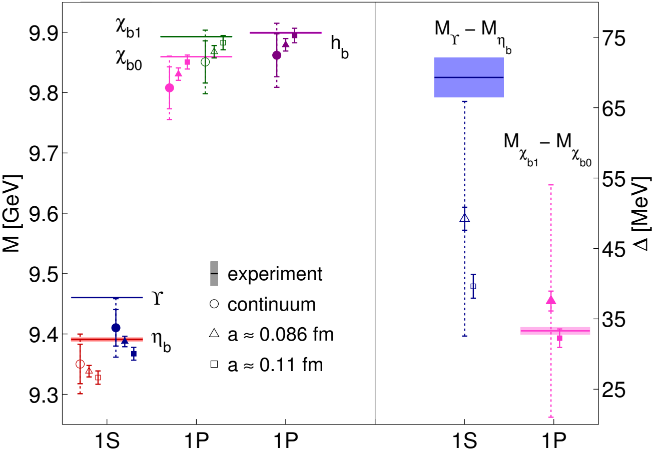
- Start from an educated guess for our three parameters $m_0 a$, c_P, and ζ
- Probe parameter space at seven points by measuring
  spin-averaged mass: $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$
  hyperfine-splitting: $\Delta_M = M_{B_s^*} - M_{B_s}$
  ratio: $M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}}$
- Assume linearity to relate parameters and observables
- Obtain tuned parameters corresponding to physical b-quarks by requiring that $\overline{M}$ and Δ_M agree with experiment and that $M_1 = M_2$



Predictions for the heavy-heavy states

[Phys.Rev. D86 (2012) 116003]

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s-system we can predict bottomonium states and mass splittings and thereby test the method
- We find good agreement with experiment within errors



$\Upsilon = 9410(30)(38)$ MeV

$\eta_b = 9350(33)(37)$ MeV

$\chi_{b1} = 9851(35)(39)$ MeV

$\chi_{b0} = 9808(35)(39)$ MeV

$h_b = 9862(36)(39)$ MeV

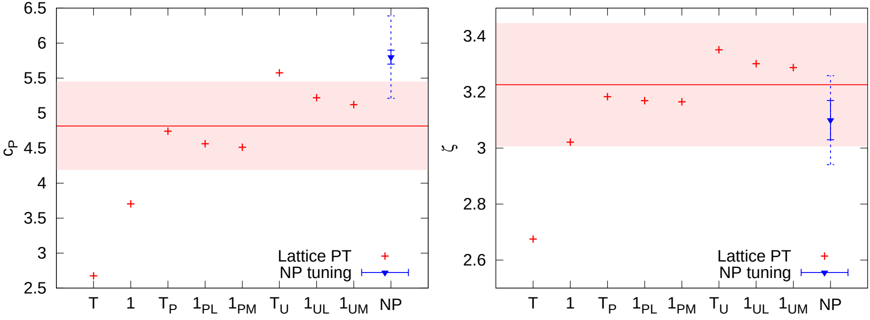
$M_\Upsilon - M_{\eta_b}$
$= 49(02)(17)$ MeV

$M_{\chi_{b1}} - M_{\chi_{b0}}$
$= 38(01)(16)$

Testing RHQ lattice perturbation theory

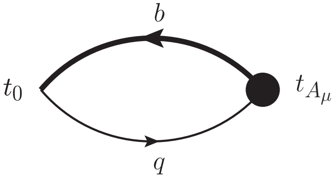
[Phys.Rev. D86 (2012) 116003]

- Compute RHQ parameters in 1-loop mean field improved LPT [http://physyhcal.lhnr.de]
  - Use nonperturbative inputs for $\langle P\rangle$, $\langle R\rangle$, $\langle L\rangle$ and m_0a and predict c_P and ζ
  - Naive $\alpha_S^2 \sim 5\%$ power-counting estimate
- Agreement within errors ⇒ MF-improved LPT can be trusted in situations for which NP matching factors are not available



decay constants

B-meson decay constant [arXiv:1404.4670 [hep-lat]]



- Use point-source light quark and generate Gaussian smeared-source heavy quark
- On the lattice we compute Φ_{B_q}

$$f_B = \Phi^{\text{ren}}_{B_q} \cdot a_{32}^{-3/2} / \sqrt{M_{B_q}}$$

- Improve axial current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)

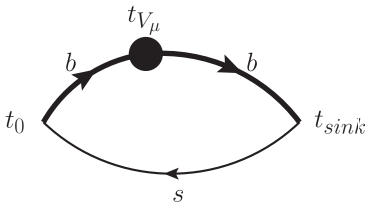
Mostly nonperturbative renormalization

For f_B, f_{B_s} and $B \to \pi$ we compute mostly non-perturbative renormalization factors á la [El-Khadra et al. Phys.Rev. D64 (2001) 014502]

$$Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}$$

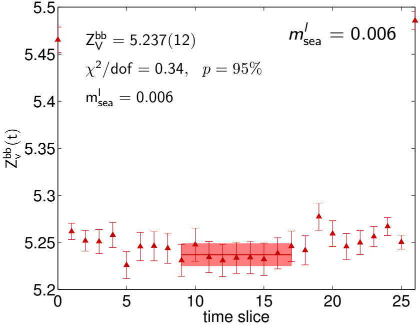
- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative series of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. Phys.Rev. D83 (2011) 074508] and compute Z_V^{bb} ourselves

Determination of Z_v^{bb} [arXiv:1404.4670 [hep-lat]]



$$Z_V^{bb} \times \langle B|V^{bb,0}|B\rangle = 2m_B$$

$$\frac{C_2^B(T)}{C_3^{B\to B}(T,t)} \lim_{T,t\to\infty} Z_V^{bb}$$

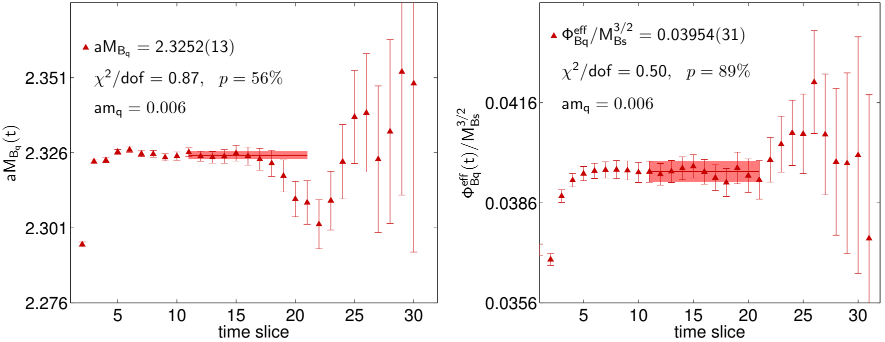


$a_{24} m^l_{sea}$	Z_v^{bb}	$a_{32} m^l_{sea}$	Z_v^{bb}
0.005	10.037(34)	0.004	5.270(13)
0.010	10.042(37)	0.006	5.237(12)
		0.008	5.267(15)
Avg.$^{(24)}$	10.093(25)	Avg.$^{(32)}$	5.2560(76)
$\text{PT}^{(24)}_{\text{1-loop}}$	10.72(16)(0)	$\text{PT}^{(32)}_{\text{1-loop}}$	5.725(74)(1)

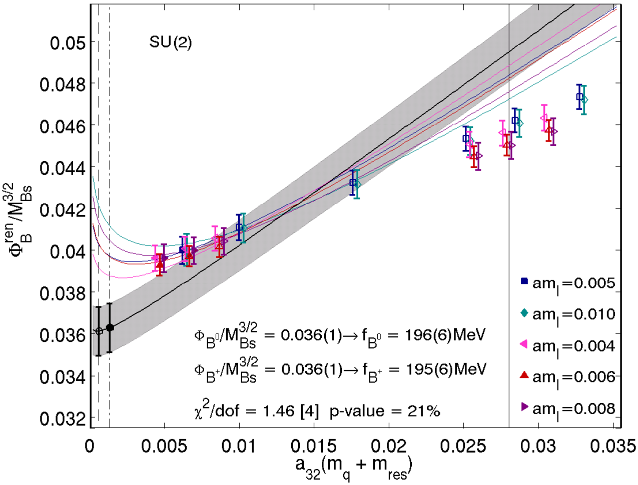
PT values: http://physyhcal.lhnr.de

B-meson decay constant [arXiv:1404.4670 [hep-lat]]

- Perform analysis in terms of dimensionless ratios over M_{B_s}

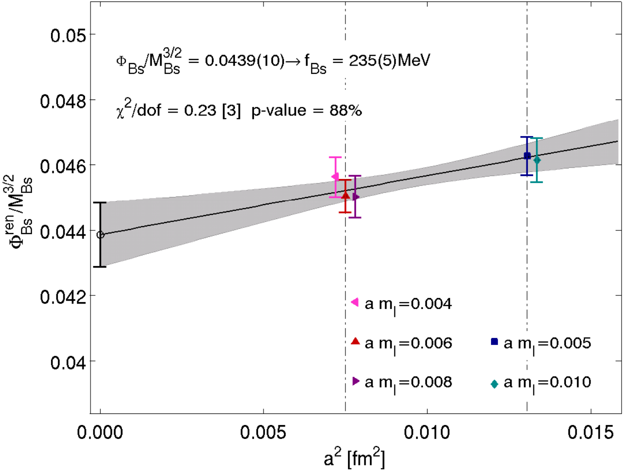


Chiral-continuum extrapolation of f_B [arXiv:1404.4670 [hep-lat]]



- NLO SU(2) HMχPT
- Only data points with filled symbols included in the fit ($M_\pi < 420$ MeV)
- Statistical errors only
- $g_{B^*B\pi} = 0.57(8)$ [PoS(Lattice 2013)408]
- $f_\pi = 130.4$ MeV [PDG]
- $\Lambda_\chi = 1$ GeV

Continuum extrapolation of f_{B_s} [arXiv:1404.4670 [hep-lat]]

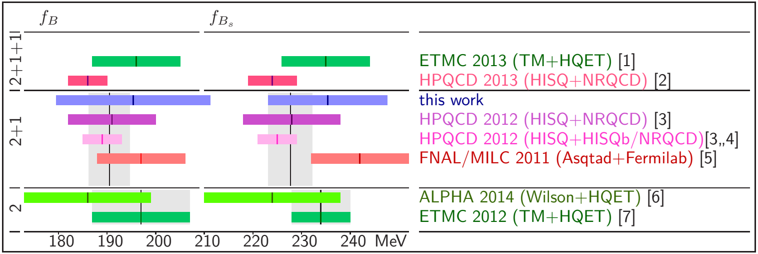


- Data for Φ_{B_s} show no sea-quark mass dependence
- Average data at same lattice spacing
- Assume a^2 scaling to remove LQ and gluon discretization errors
- Statistical errors only

Error budget [arXiv:1404.4670 [hep-lat]]

	f_{B^0}(%)	f_{B^+}(%)	f_{B_s}(%)
statistics	3.2	3.2	2.2
chiral-continuum extrapolation	6.5	6.6	3.1
lattice-scale uncertainty	1.5	1.5	1.5
light- and strange-quark mass uncertainty	0.1	0.2	0.9
RHQ parameter tuning	1.2	1.2	1.2
HQ discretization errors	1.7	1.7	1.7
LQ and gluon disc. errors	1.1	1.1	1.2
renormalization factor	1.7	1.7	1.7
finite volume	0.4	0.4	0.0
isospin-breaking and EM	0.7	0.7	0.7
total	8.0	8.1	5.2

Comparison with other results [arXiv:1404.4670 [hep-lat]]

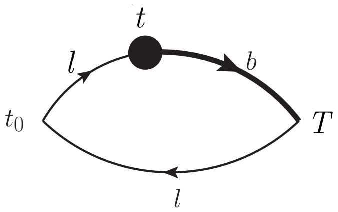


- Good agreement with other results

FLAG: S. Aoki et al. arXiv:1310.8555 [hep-lat]
[1] Carrasco et al. arXiv:1311.2837 [hep-lat]
[2] Dowdall et al. Phys.Rev.Lett. 110 (2012) 222003
[3] Na et al. Phys.Rev.D86 (2012) 034506
[4] McNeile et al. Phys.Rev.D85 (2012) 031503
[5] Bazavov et al. Phys.Rev.D85 (2012) 114506
[6] Bernardoni et al. arXiv:1404.3590 [hep-lat]
[7] Carrasco et al. PoS LATTICE2012 (2012) 104

form factors

$B \to \pi \ell \nu$ form factors



- Parameterize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0 \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

- Re-use point-source light quark propagators and generate Gaussian smeared-source sequential heavy quark propagators

- Improve vector current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)

Relating form factors f_+ and f_0 to $f_\parallel$ and $f_\perp$

- On the lattice we prefer using the B-meson rest frame and compute

$$f_\parallel(E_\pi) = \langle \pi | V^0 | B \rangle / \sqrt{2M_B} \quad \text{and} \quad f_\perp(E_\pi) p_\pi^i = \langle \pi | V^i | B \rangle / \sqrt{2M_B}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 - M_\pi^2} \left[(M_B - E_\pi) f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2) f_\perp(E_\pi) \right]$$

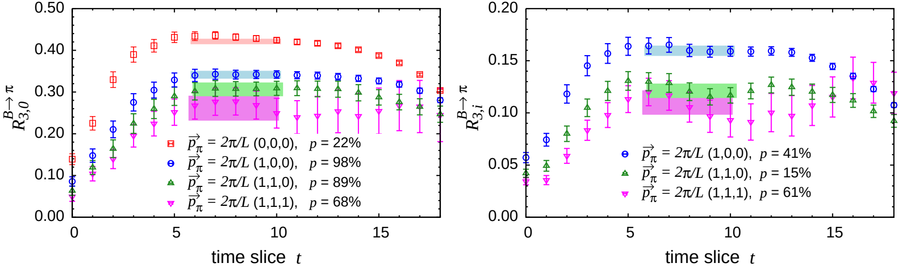
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}} \left[f_\parallel(E_\pi) + (M_B - E_\pi) f_\perp(E_\pi) \right]$$

Lattice results for form factors $f_\parallel$ and $f_\perp$

$$f_\parallel = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_\perp = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

$$R_\mu^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{C_2^\pi(t)C_2^B(T-t)}\sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T-t)}}}$$



[plots by Taichi Kawanai]

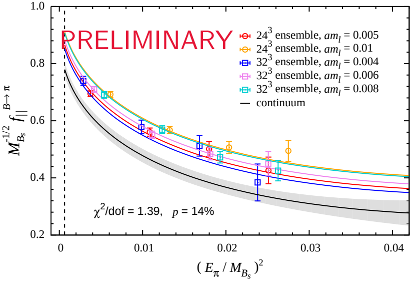
Chiral-continuum extrapolation using SU(2) hard-pion χPT

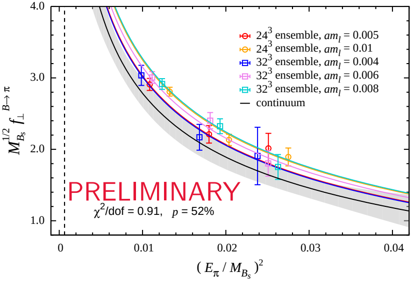
$$f_{\parallel}(M_\pi, E_\pi, a^2) = c_{\parallel}^{(1)} \left(1 + (\delta f_{\parallel})^{\text{hard-pion}} + c_{\parallel}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_\pi}{\Lambda} + c_{\parallel}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2} a_{32}^4\right)$$

$$f_{\perp}(M_\pi, E_\pi, a^2) = \frac{1}{E_\pi + M_B^* - M_B} c_{\perp}^{(1)} \left(1 + (\delta f_{\perp})^{\text{hard-pion}} + c_{\perp}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_\pi}{\Lambda} + c_{\perp}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2} a_{32}^4\right)$$

with δF non-analytic logs of the pion mass and hard-pion limit is taken by $\frac{M_\pi}{E_\pi} \to 0$

- Again we perform the analysis in terms of dimensionless ratios over M_{B_s}

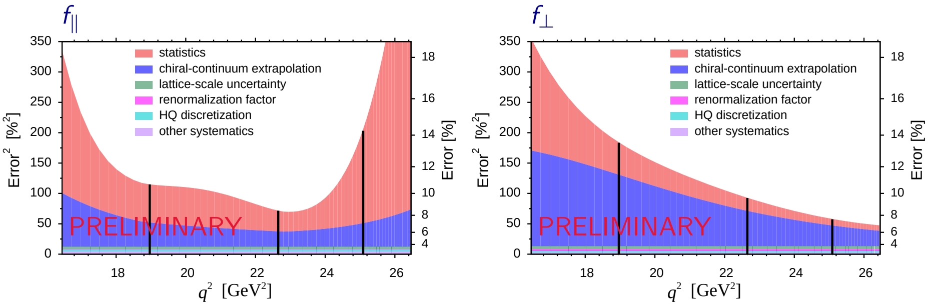




[plots by Taichi Kawanai]

Obtaining form factors f_+ and f_0

- Extract $f_\parallel$ and $f_\perp$ for three different q^2 values (synthetic data points)
- Estimate all systematic errors and them add in quadrature



- Convert results to f_+ and f_0

[plots by Taichi Kawanai]

z-expansion

- Use the model-independent z-expansion fit to extrapolate lattice results to the full kinematic range [Boyd, Grinstein, Lebed, Phys.Rev.Lett. 74 (1995) 4603] [Bourrely, Caprini, Lellouch, Phys.Rev. D79 (2009) 013008]

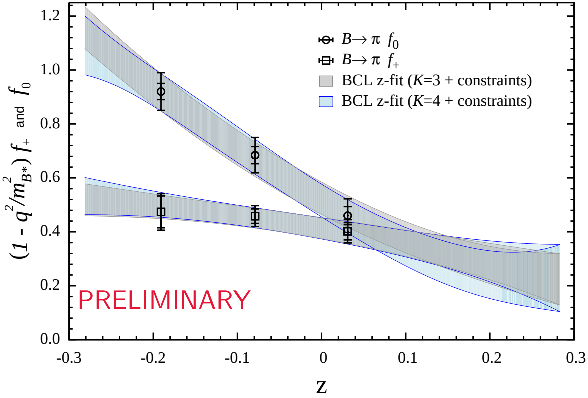
$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$$

  with $t_\pm = (M_B \pm M_\pi)^2$ and $t_0 \equiv t_{\rm opt} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$
- Minimizes the magnitude of z in the semi-leptonic region: $|z| \leq 0.279$
- The form factor $f(q^2)$ is analytic in the semi-leptonic region except at the B^*
- $f_+(q^2)$ can be expressed as convergent power series

$$f_+(q^2) = \frac{1}{1-q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^k \right]$$

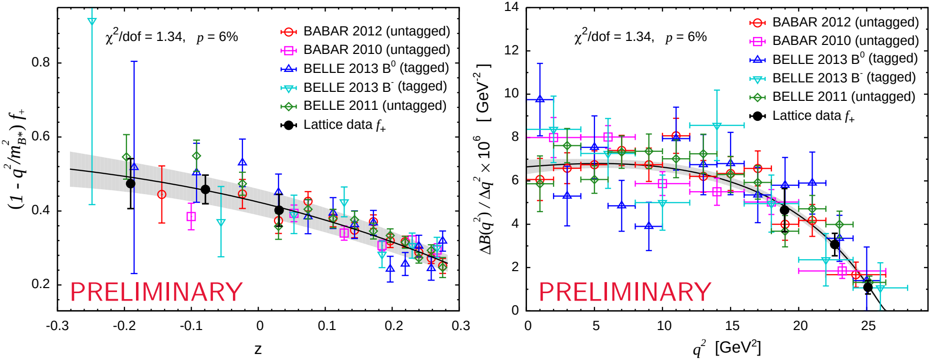
  and use for $f_0(q^2)$ the functional form $f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$
- Exploit the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$ and use HQ power counting to constrain the size of the f_+ coefficients

z-expansion fit



[plot by Taichi Kawanai]

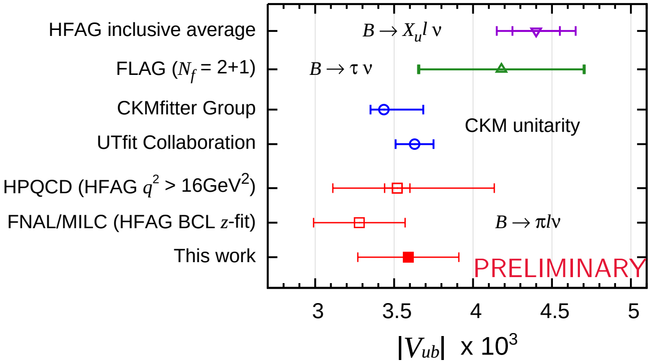
Combine with experimental data to determine $|V_{ub}|$



- Preliminary result: $|V_{ub}| = 3.59(32) \cdot 10^{-3}$

[plot by Taichi Kawanai]

Comparison with other determinations

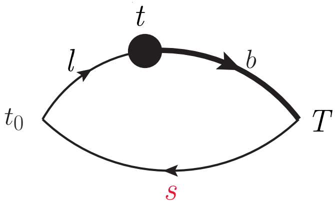


- In good agreement with existing results
- Result agrees with value obtained CKM unitarity
- Exhibits 2σ tension to inclusive results

[plot by Taichi Kawanai]

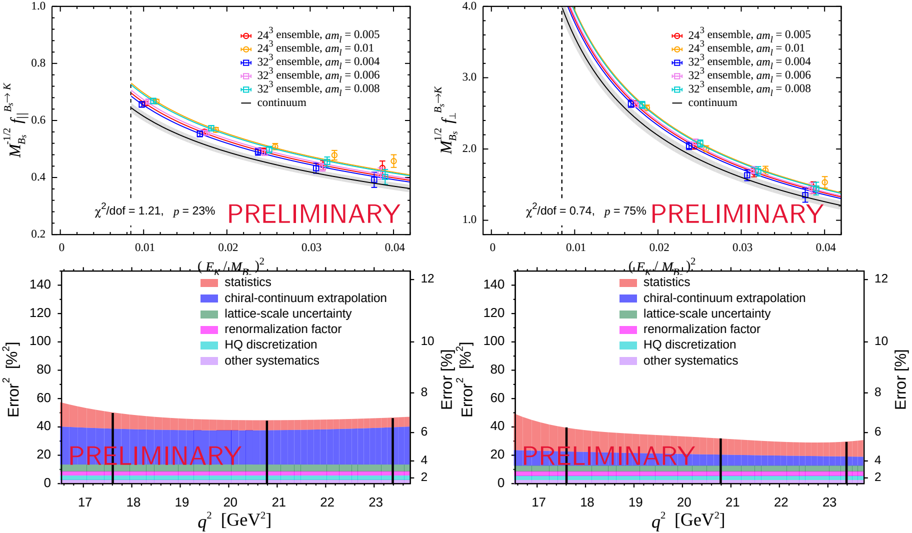
$B_s \to K\ell\nu$

- Lattice calculation: replace light spectator quark with s-quark



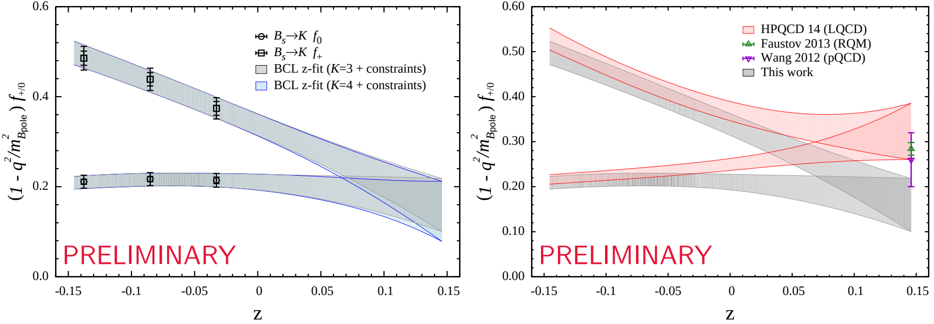
- Chiral-continuum extrapolation is similar but pole masses change
- Smaller statistical and extrapolation errors
- Perform z-expansion
- Experimental results for $B_s \to K\ell\nu$ not (yet) available
- Can make phenomenological predictions to be compared with future measurements

Chiral-continuum extrapolation for $B_s \to K\ell\nu$



[plots by Taichi Kawanai]

z-expansion fit for $B_s \to K\ell\nu$ and comparison to others



- Bouchard et al., arXiv:1406.2279 [hep-lat]
- Faustov and V. Galkin, Phys.Rev. D87 (2013) 094028
- Wang and Xiao, Phys.Rev. D86 (2012) 114025

[plots by Taichi Kawanai]

Conclusion & outlook

Conclusion & outlook

- Results for decay constants f_B and f_{B_s} are posted and submitted to PRD
  - Good agreement with results in the literature
  - Errors dominated by chiral-continuum extrapolation
- Already improving our errors by simulating with physical pion mass
- Another ensemble with finer lattice spacing will be added

- Finalizing analyses for $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors
  - $B \to \pi\ell\nu$: independent determination of V_{ub} with competitive errors
    - Agreement with CKM unitarity results
    - 2σ tension to inclusive determination
  - $B_s \to K\ell\nu$: phenomenological predictions
    - Allows with future measurements an alternative determination of V_{ub}