Beautiful physics on the lattice

RBC and UKQCD collaborations

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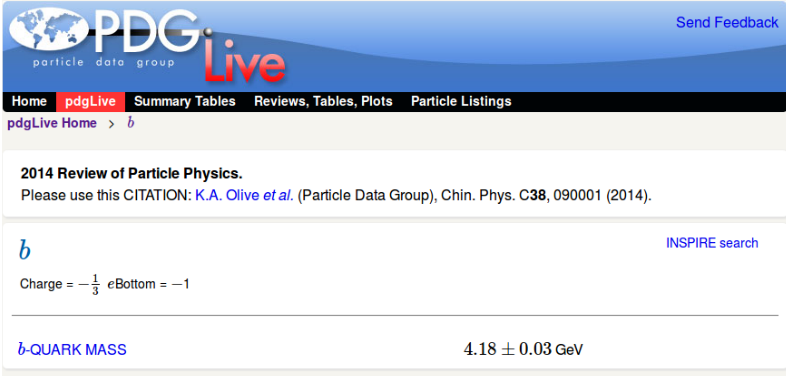
introduction

Beautiful physics . . .

- may require a controversial definition

Beautiful physics . . .

- may require a controversial definition
- or physics with a beautiful quark

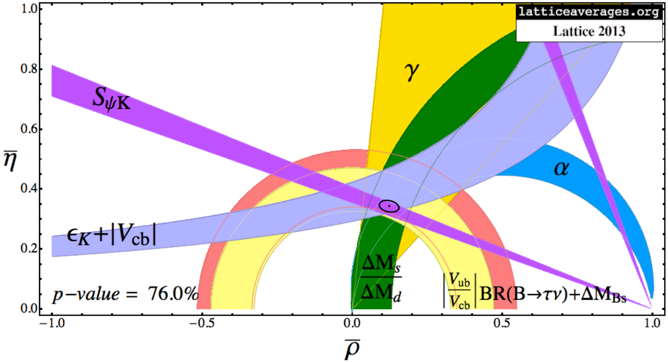


Why are b-quarks beautiful?

- Allow us to test the Standard Model
- Allow us to look for new physics
- b-quarks are heavy and hence have a lot of decay modes
- Are frequently produced in b-factories: BaBar, Belle, LHC, Belle II

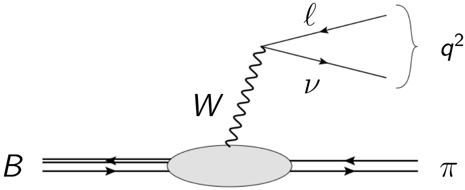
Motivation: Constraining the Standard Model

- Determination of CKM matrix elements V_{ub} and V_{cb}
- B-physics provides constraints on the apex of the CKM unitarity triangle
  - B^0–$\overline{B^0}$ mixing
  - V_{ub} and V_{cb} e.g. from $B \to \pi\ell\nu$ and $\overline{B} \to D^*\ell\nu$ form factors
- Experimental results and nonperturbative inputs are needed



http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it, http://www.latticeaverages.org

Example: V_{ub} from exclusive semileptonic decay $B \to \pi\ell\nu$



$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

- Conventionally parametrized by

$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2 + M_\pi^2 - q^2\right)^2 - 4M_B^2 M_\pi^2\right]^{3/2} \times \left|f_+(q^2)\right|^2 \times \left|V_{ub}\right|^2$$

experiment known nonperturbative input CKM

- Long standing puzzle in V_{ub} determination: $2-3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$) measurement

Motivation: New Physics in rare B-decays?

$B \to \tau\nu$ [UTfit PLB 687 (2010) 61]

- f_B is needed for the Standard-Model prediction of $BR(B \to \tau\nu)$
- Potentially sensitive to charged-Higgs exchange due to large τ mass

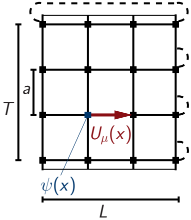
$B_s \to \mu_+\mu_-$ [Buras et al. EPJ C72 (2012) 2172], [Buras et al. JHEP07 (2013) 077]

- f_{B_s} is needed for Standard-Model prediction of $BR(B_s \to \mu_+\mu_-)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Measured by CMS and LHCb: combined analysis of 7 and 8 TeV runs yields $> 6\sigma$ significance — in agreement with SM [CMS and LHCb arXiv:1411.4413]

Both are sensitive to new physics!

lattice

Lattice QCD



- Discretize Euclidean space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- Study physics in a finite box of volume $(aL)^3$
- Compute expectation values of gauge invariant observables by

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, e^{-S(U)} \, \mathcal{O}(U), \quad \mathcal{Z} = \int \mathcal{D}U \, e^{-S(U)}$$

- Only statistical estimation possible: $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$
- Generate a sufficiently long sequence of configurations with probability distribution

$$P \propto \exp\{-S(U)\}$$

- Typically done by a Markov chain using the HMC algorithm with configurations saved e.g. every 10 MDTU

Are b-quarks on the lattice beautiful?

- Not yet . . .

but they provide new challenges

Challenges for B-physics on the lattice

- The mass of the b-quark introduces another scale
  - Light quark masses: $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV
  - Mass of the b-quark: $m_b = 4.18$ GeV
- Today's lattices have an inverse lattice spacing of $a^{-1} \approx 1.7 \ldots 3 \ldots 4$ GeV
  - $am_b > 1$
  - Forced to simulate b-quarks with an effective action
    e.g. HQET (static), NRQCD, Fermilab or RHQ action
  - Requires (perturbative) mixed-action renormalization factors
- New concepts like heavy HISQ action look very promising
  [C. McNeile, et al. PRD 85 (2012) 031503]

Our project

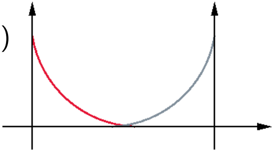
- Based on RBC-UKQCD's 2+1 flavor domain-wall Iwasaki gauge field configurations

- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - $g_{B^* B\pi}$ coupling constant (Ben Samways, Jonathan Flynn)
  - Decay constants f_B and f_{B_s} (Ruth Van de Water, OW)
  - $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors (Taichi Kawanai, Ruth Van de Water, OW)
  - B^0–$\overline{B^0}$ mixing (Taichi Kawanai, OW)

- Compute renormalization and $O(a)$-improvement factors using lattice PT (Christoph Lehner)

2+1 flavor domain-wall gauge field configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Iwasaki gauge action [Iwasaki UTHEP (1983) 118]
- Configurations generated by RBC and UKQCD collaborations [Allton et al. PRD 78 (2008) 114509], [Y. Aoki et al. PRD 83 (2011) 074508]



L	a(fm)	m_l	m_s	M_π(MeV)	# configs.	# time sources
24	≈ 0.11	0.005	0.040	329	1636	1
24	≈ 0.11	0.010	0.040	422	1419	1
32	≈ 0.08	0.004	0.030	289	628	2
32	≈ 0.08	0.006	0.030	345	889	2
32	≈ 0.08	0.008	0.030	394	544	2

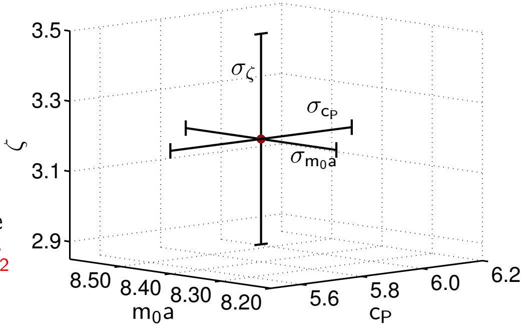
Relativistic Heavy Quark action for the *b*-quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
- Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  by tuning all parameters of the clover action non-perturbatively;
  close relation to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

Nonperturbative tuning of the RHQ action parameters

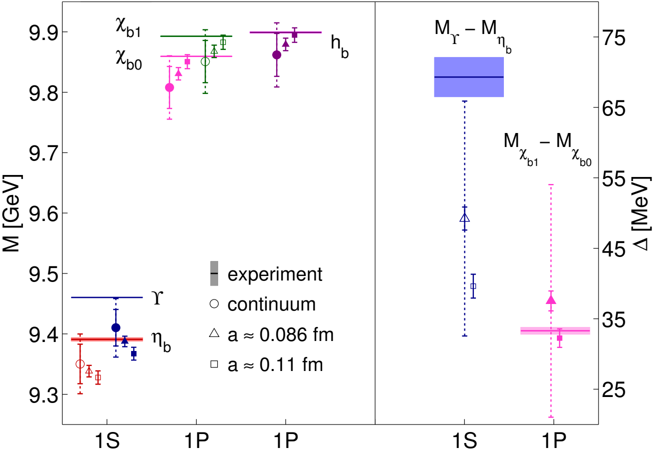
[PRD 86 (2012) 116003]

- Start from an educated guess for our three parameters m_0a, c_P, and ζ
- Probe parameter space at seven points by measuring
  spin-averaged mass: $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$
  hyperfine-splitting: $\Delta_M = M_{B_s^*} - M_{B_s}$
  ratio: $M_1/M_2 = M_{\rm rest}/M_{\rm kinetic}$
- Assume linearity to relate parameters and observables
- Obtain tuned parameters corresponding to physical b-quarks by requiring that $\overline{M}$ and Δ_M agree with experiment and that $M_1 = M_2$



Predictions for the heavy-heavy states [PRD 86 (2012) 116003]

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s-system we can predict bottomonium states and mass splittings and thereby test the method
- We find good agreement with experiment within errors



$\Upsilon = 9410(30)(38)$ MeV

$\eta_b = 9350(33)(37)$ MeV

$\chi_{b1} = 9851(35)(39)$ MeV

$\chi_{b0} = 9808(35)(39)$ MeV

$h_b = 9862(36)(39)$ MeV

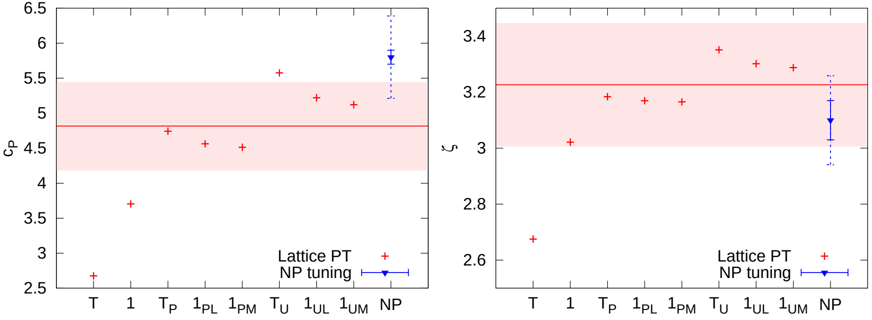
$$M_\Upsilon - M_{\eta_b} = 49(02)(17)\text{ MeV}$$

$$M_{\chi_{b1}} - M_{\chi_{b0}} = 38(01)(16)$$

Testing RHQ lattice perturbation theory [PRD 86 (2012) 116003]

(Christoph Lehner)

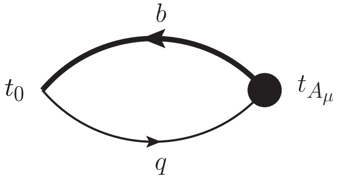
- Compute RHQ parameters in 1-loop mean field improved LPT [http://physyhcal.lhnr.de]
  - Use nonperturbative inputs for $\langle P\rangle$, $\langle R\rangle$, $\langle L\rangle$ and $m_0 a$ and predict c_P and ζ
  - Naive $\alpha_S^2 \sim 5\%$ power-counting estimate
- Agreement within errors $\Rightarrow$ MF-improved LPT can be trusted in situations for which NP matching factors are not available



decay constants

B-meson decay constant [PRD 91 (2015) 054502]

(Ruth Van de Water, OW)



- Use point-source light quark and generate Gaussian smeared-source heavy quark
- On the lattice we compute Φ_{B_q}

$$f_B = \Phi^{\text{ren}}_{B_q} \cdot a_{32}^{-3/2} / \sqrt{M_{B_q}}$$

- Improve axial current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)

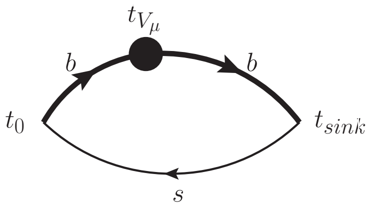
Mostly nonperturbative renormalization

For f_B, f_{B_s} and $B \to \pi$ we compute mostly non-perturbative renormalization factors á la [El-Khadra et al. PRD 64 (2001) 014502]

$$Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}$$

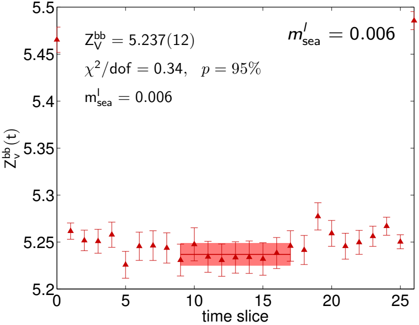
- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative series of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections (Christoph Lehner)
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. PRD 83 (2011) 074508] and compute Z_v^{bb} ourselves

Determination of Z_V^{bb} [PRD 91 (2015) 054502]



$$Z_V^{bb} \times \langle B | V^{bb,0} | B \rangle = 2 m_B$$

$$\frac{C_2^B(T)}{C_3^{B\to B}(T,t)} \lim_{T,t\to\infty} Z_V^{bb}$$



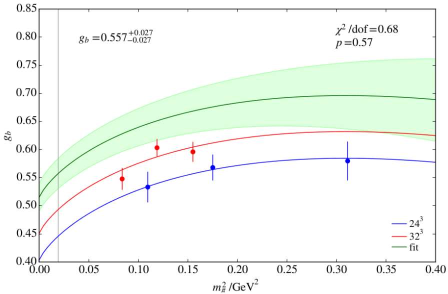
$a_{24}\, m^l_{sea}$	Z_V^{bb}	$a_{32}\, m^l_{sea}$	Z_V^{bb}
0.005	10.037(34)	0.004	5.270(13)
0.010	10.042(37)	0.006	5.237(12)
		0.008	5.267(15)
Avg.$^{(24)}$	10.093(25)	Avg.$^{(32)}$	5.2560(76)
$\text{PT}^{(24)}_{\text{1-loop}}$	10.72(16)(0)	$\text{PT}^{(32)}_{\text{1-loop}}$	5.725(74)(1)

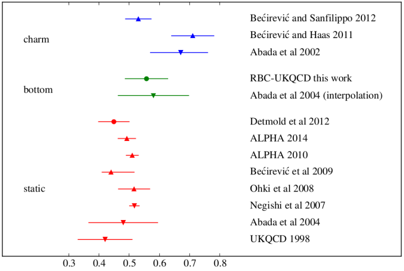
PT values: [http://physyhcal.lhnr.de]

Coupling constant $g_{B^*B\pi}$ [PoS(Lattice2013)408]

(Ben Samways, Jonathan Flynn)

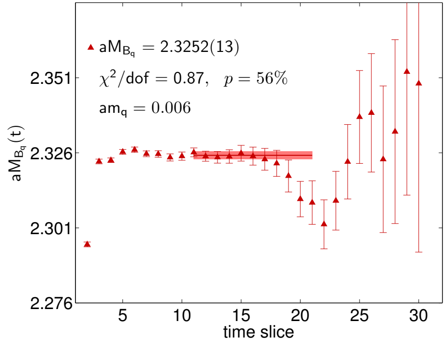
- Strong coupling $g_{B^*B\pi}$ parametrizes $\langle B\pi|B^*\rangle$
- Related to leading order LEC $g_b = g_{B^*B\pi} \cdot f_\pi/(2M_B)$ of HMχPT
- g_b important for chiral extrapolations of f_B, B_B, ξ, $f_+^{B\pi}$, $f_0^{B\pi}$, ...
- First determination at physical b-quark mass
- Not accessible experimentally

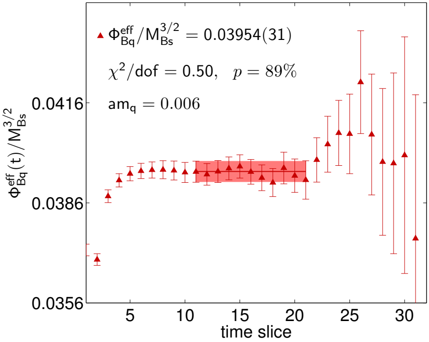




B-meson decay constant [PRD 91 (2015) 054502]

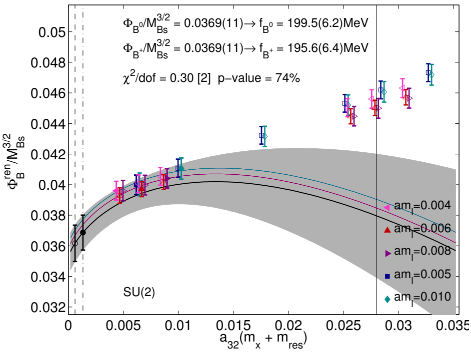
- Perform analysis in terms of dimensionless ratios over M_{B_s}

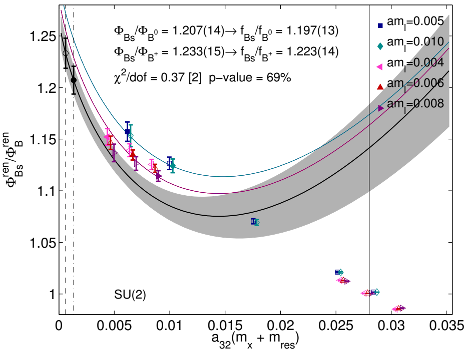




Chiral-continuum extrapolation of f_B and f_{B_s}/f_B

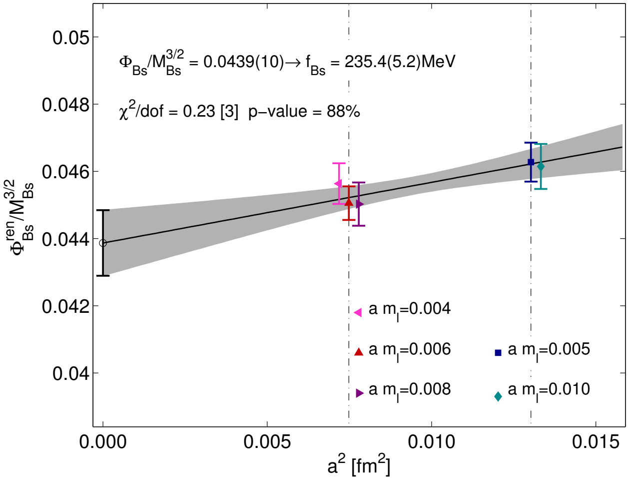
[PRD 91 (2015) 054502]





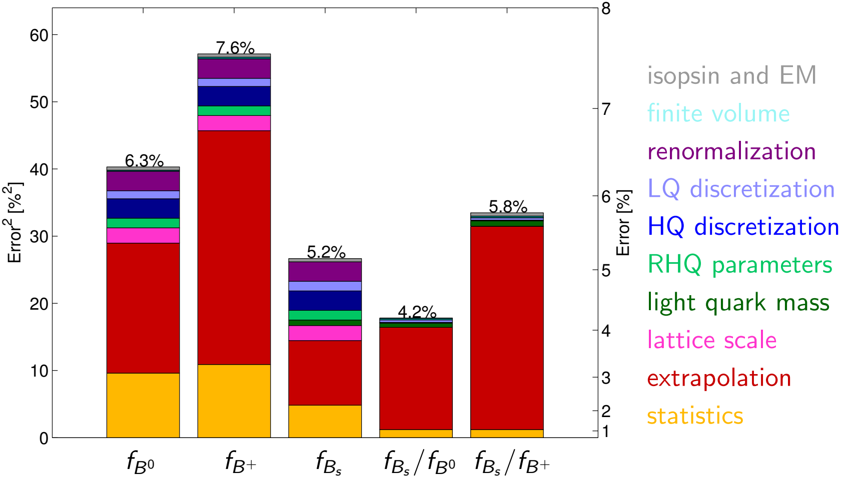
- NLO SU(2) HMχPT to data with unitary M_π
- Only data points with filled symbols included in the fit ($M_\pi \lesssim$ 425 MeV)
- $g_{B^*B\pi} = 0.57(8)$ [PoS(Lattice2013)408] ▸ $f_\pi = 130.4$ MeV [PDG] ▸ $\Lambda_\chi = 1$ GeV
- Statistical errors only

Continuum extrapolation of f_{B_s} [PRD 91 (2015) 054502]

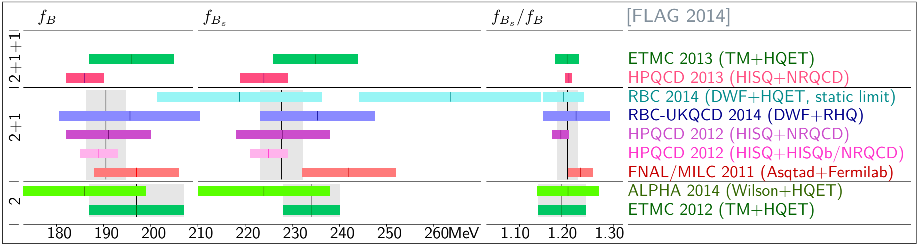


- Data for Φ_{B_s} show no sea-quark mass dependence
- Average data at same lattice spacing
- Assume a^2 scaling to remove LQ and gluon discretization errors
- Statistical errors only

Graphical error budget [PRD 91 (2015) 054502]



Comparison with other results



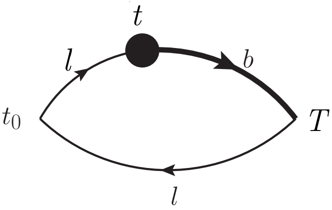
- Good agreement with other results

- $f_{B^0} = 199.5(6.2)(12.6)$ MeV
- $f_{B^+} = 195.6(6.4)(14.9)$ MeV
- $f_{B_s} = 235.4(5.2)(11.1)$ MeV
- $f_{B_s}/f_{B^0} = 1.197(13)(49)$
- $f_{B_s}/f_{B^+} = 1.223(14)(70)$

form factors

$B \to \pi \ell \nu$ form factors [PRD 91 (2015) 074510]

(Taichi Kawanai)



- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

- Re-use point-source light quark propagators and generate Gaussian smeared-source sequential heavy quark propagators

- Improve vector current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient (Christoph Lehner))

Relating form factors f_+ and f_0 to $f_\parallel$ and $f_\perp$

- On the lattice we prefer using the B-meson rest frame and compute

$$f_\parallel(E_\pi) = \langle\pi|V^0|B\rangle/\sqrt{2M_B} \quad \text{and} \quad f_\perp(E_\pi)p_\pi^i = \langle\pi|V^i|B\rangle/\sqrt{2M_B}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2-M_\pi^2}\left[(M_B - E_\pi)f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2)f_\perp(E_\pi)\right]$$

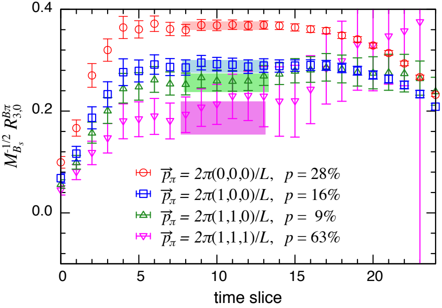
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(E_\pi) + (M_B - E_\pi)f_\perp(E_\pi)\right]$$

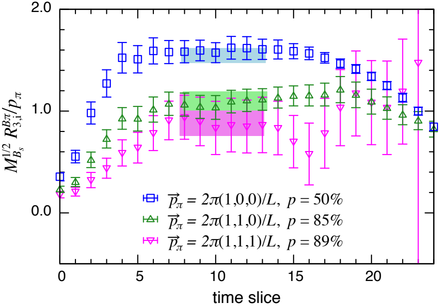
Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ [PRD 91 (2015) 074510]

$$f_{\parallel} = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_{\perp} = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

$$R_\mu^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{C_2^{\pi}(t)C_2^{B}(T-t)}\sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T-t)}}}$$





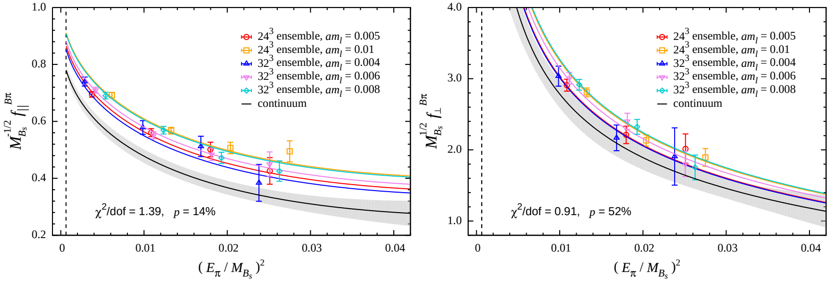
Chiral-continuum extrapolation using SU(2) hard-pion χPT

$$f_{\|}(M_\pi, E_\pi, a^2) = c_{\|}^{(1)}\left[1 + \left(\frac{\delta f_{\|}}{(4\pi f)^2} + c_{\|}^{(2)}\frac{M_\pi^2}{\Lambda^2} + c_{\|}^{(3)}\frac{E_\pi}{\Lambda} + c_{\|}^{(4)}\frac{E_\pi^2}{\Lambda^2} + c_{\|}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_{\perp}(M_\pi, E_\pi, a^2) = \frac{1}{E_\pi+\Delta}c_{\perp}^{(1)}\left[1 + \left(\frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)}\frac{M_\pi^2}{\Lambda^2} + c_{\perp}^{(3)}\frac{E_\pi}{\Lambda} + c_{\perp}^{(4)}\frac{E_\pi^2}{\Lambda^2} + c_{\perp}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

with δf non-analytic logs of the pion mass and hard-pion limit is taken by $\frac{M_\pi}{E_\pi} \to 0$

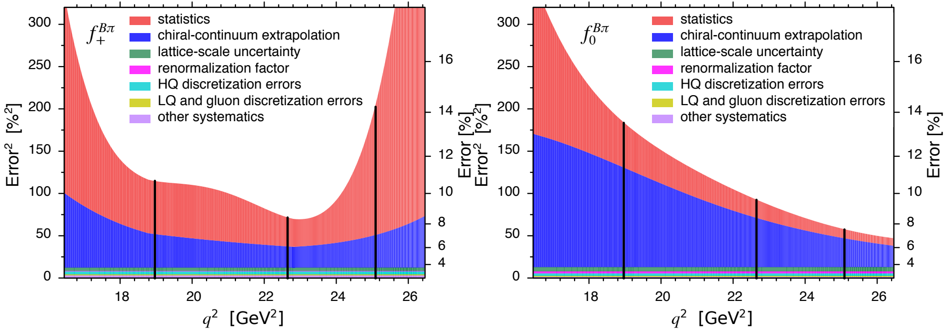
- Again we perform the analysis in terms of dimensionless ratios over M_{B_s}



[PRD 91 (2015) 074510]

Obtaining form factors f_+ and f_0 [PRD 91 (2015) 074510]

- Extract $f_\parallel$ and $f_\perp$ for three different q^2 values (synthetic data points)
- Estimate all systematic errors and them add in quadrature
- Convert results to f_+ and f_0



z-expansion [PRD 91 (2015) 074510]

- Use the model-independent z-expansion fit to extrapolate lattice results to the full kinematic range [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

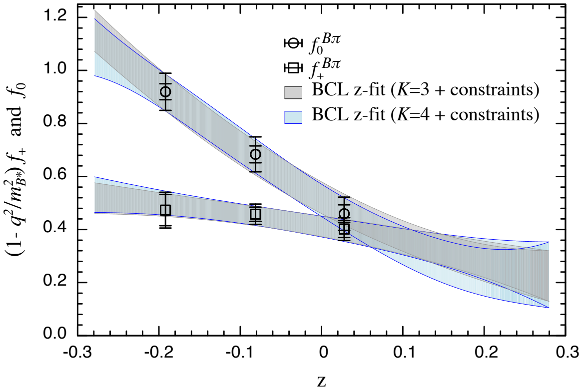
$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+}-\sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+}+\sqrt{1-t_0/t_+}}$$

  with $t_\pm = (M_B \pm M_\pi)^2$ and $t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$
- Minimizes the magnitude of z in the semi-leptonic region: $|z| \leq 0.279$
- $f(q^2)$ is analytic in the semi-leptonic region except at the B^* pole
- $f_+(q^2)$ can be expressed as convergent power series

$$f_+(q^2) = \frac{1}{1-q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^k\right]$$

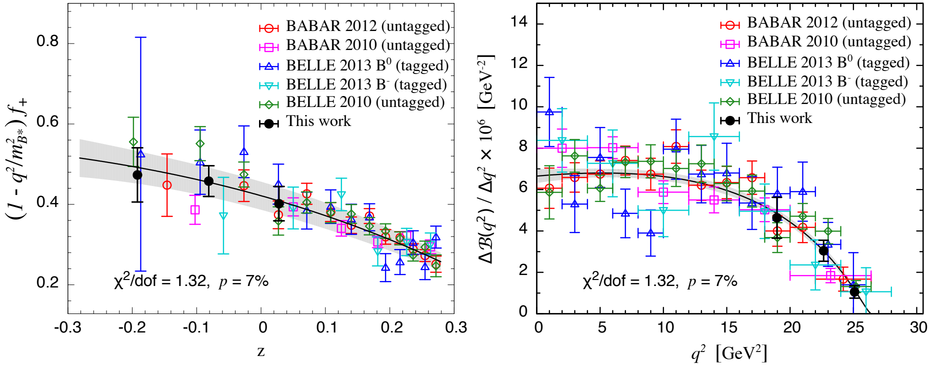
  and use for $f_0(q^2)$ the functional form $f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$
- Exploit the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$ and use HQ power counting to constrain the size of the f_+ coefficients

z-expansion fit [PRD 91 (2015) 074510]



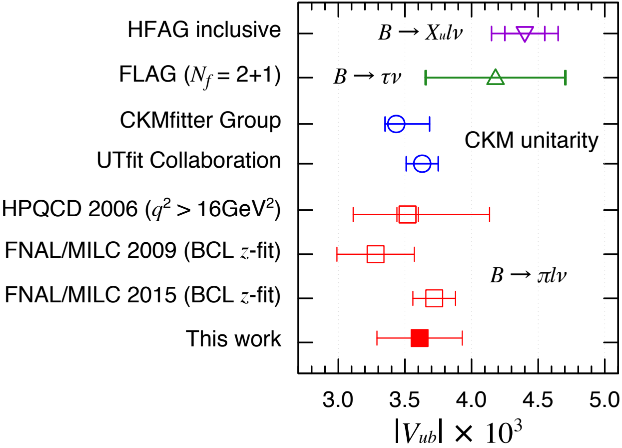
Combine with experimental data to determine $|V_{ub}|$

[PRD 91 (2015) 074510]



- Result: $|V_{ub}| = 3.61(32) \cdot 10^{-3}$

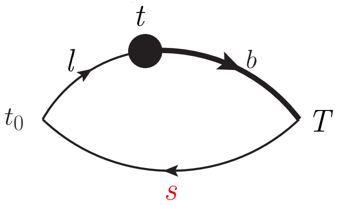
Comparison with other determinations



- In good agreement with existing and new FNAL/MILC result
- Result agrees with value obtained CKM unitarity
- Exhibits 2σ tension to inclusive results

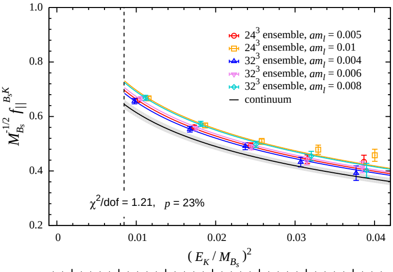
$B_s \to K\ell\nu$ [PRD 91 (2015) 074510]
(Taichi Kawanai)

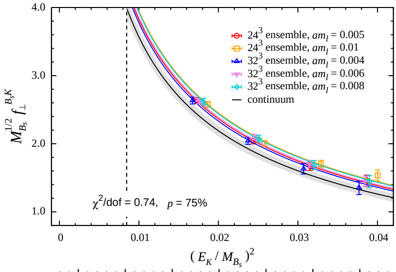
- Lattice calculation: replace light spectator quark with s-quark

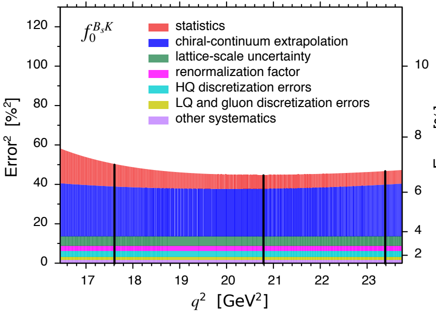


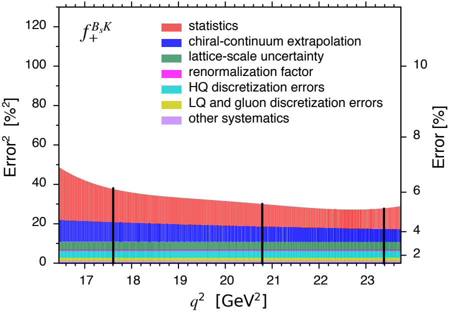
- Chiral-continuum extrapolation is similar but pole masses change
- Smaller statistical and extrapolation errors
- Perform z-expansion
- Experimental results for $B_s \to K\ell\nu$ not (yet) available
- Can make phenomenological predictions to be compared with future measurements

Chiral-continuum extrapolation for $B_s \to K\ell\nu$ [PRD 91 (2015) 074510]



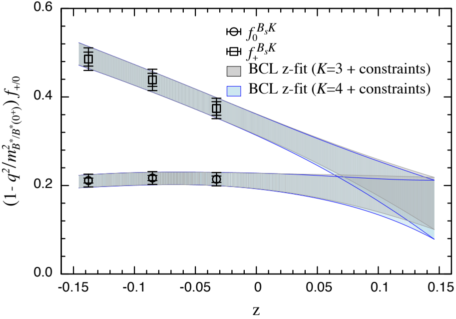


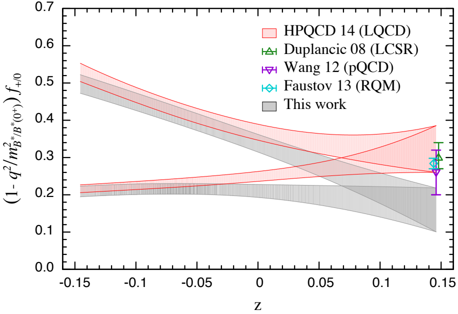




z-expansion fit for $B_s \to K\ell\nu$ and comparisons

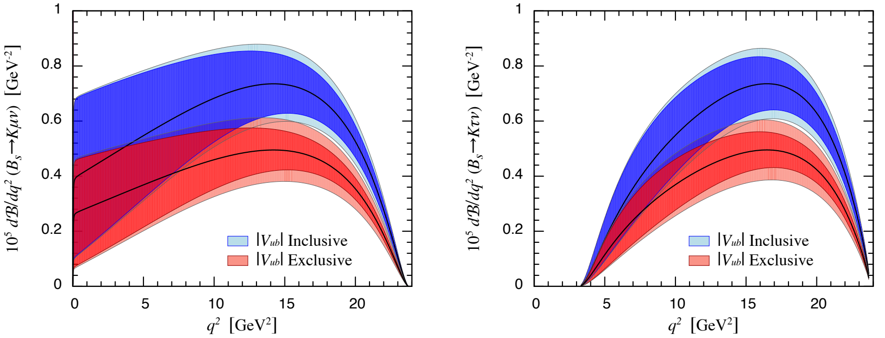
[PRD 91 (2015) 074510]





- [Bouchard et al., PRD 90 (2014) 054506]
- [Duplancic and Melic, PRD 78 (2008) 054015]
- [Faustov and Galkin, PRD 87 (2013) 094028]
- [Wang and Xiao, PRD 86 (2012) 114025]

Phenomenological prediction



- Using our value for $|V_{ub}|$ we can make predictions for the $B_s \to K\ell\nu$ differential branching fraction for $\ell = \mu, \tau$
- Given an experimental measurement of branching fractions at $q^2 \gtrsim 13$ GeV one may distinguish between curves corresponding to $|V_{ub}|_{\rm excl}$ and $|V_{ub}|_{\rm incl}$

Conclusion & outlook

Conclusion

- Published results
  - B-meson decay constants f_B, f_{B_s}, and f_{B_s}/f_B
  - $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$ form factors
- Finalizing first determination of the $g_{B^*B\pi}$ coupling with relativistic b-quarks
- Errors dominated by chiral-continuum extrapolation

Outlook

- Improving upon our errors
  - Add ensemble with physical pions
    $48^3 \times 96$, $a^{-1} = 1.73$ GeV, $M_\pi = 139$ MeV
  - New DWF-Iwasaki ensemble with finer lattice spacing in production
    $48^3 \times 96$, $a^{-1} \approx 2.8$ GeV, $M_\pi \approx 200$ MeV
- Finally working on $B^0 - \overline{B^0}$ mixing