Lattice QCD

(focus on Charm and Beauty form factors, $R(D^*)$, b- & c-quark masses)

Oliver Witzel

University of Colorado Boulder

18th International Conference on B-Physics at Frontier Machines, Beauty 2019

Ljubljana, Slovenia · October 3, 2019

- Lattice QCD
- Charm and beauty form factors
  (single pseudoscalar hadronic final state)
- $R(D^*)$
  (single vector hadronic final state)
- b & c quark masses
- In addition
  $\rightarrow$ Flavor changing neutral currents (FCNC)
  $\rightarrow$ Neutral meson mixing

Lattice QCD

The Standard Model of Elementary Particle Physics

	generations I	generations II	generations III	gauge forces	Higgs boson
quarks	u	c	t	g	H
quarks	d	s	b	γ	
leptons	ν_e	ν_μ	ν_τ	Z^0	
leptons	e	μ	τ	$W^\pm$	

- Quantum Chromodynamics (QCD) describes the strong interactions of quarks and gluons (nonperturbative)
- Lattice QCD allows for first principle calculations
  - → Also in the nonperturbative regime
  - → Systematical procedures to improve uncertainties
  - → Requires large scale computing facilities

[DiRAC]

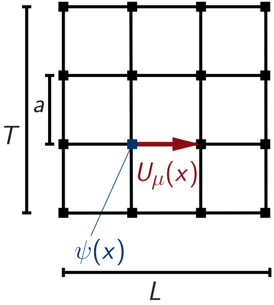
[ALCF]

Setting up a lattice calculation

- Wick-rotate to Euclidean time $t \to i\tau$
- Discretize space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- Use path integral formalism

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

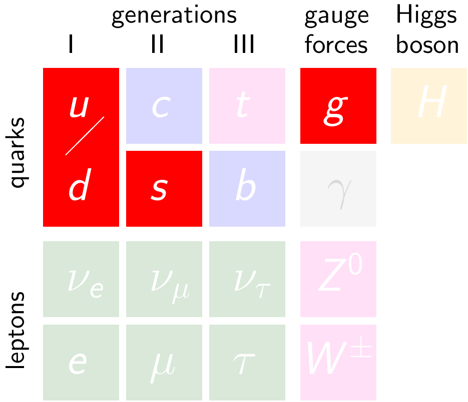
$\Rightarrow$ Large but finite dimensional path integral



- Finite lattice spacing $a \to$ UV regulator
  $\to$ Quark masses need to obey $am < 1$

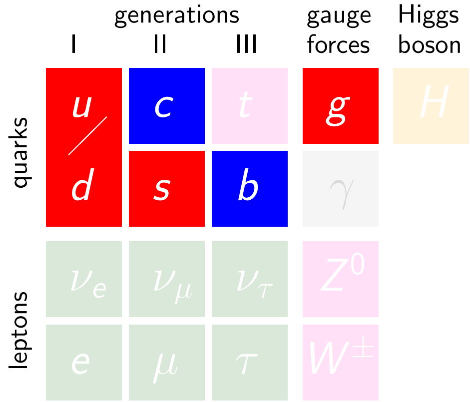
- Finite volume of length $L \to$ IR regulator
  $\to$ Study physics in a finite box of volume $(aL)^3$

The Standard Model in a typical lattice calculation



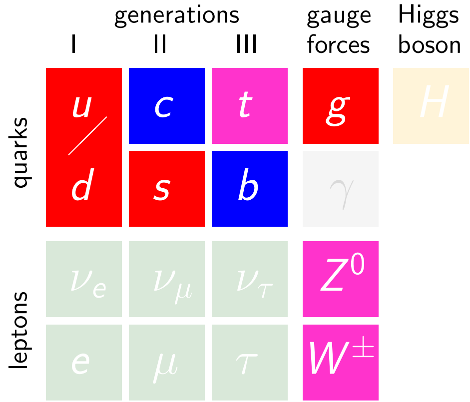
- 2+1 dynamical flavor QCD (fully relativistic action)
  → dynamical charm is possible: 2+1+1 flavors

The Standard Model in a typical lattice calculation



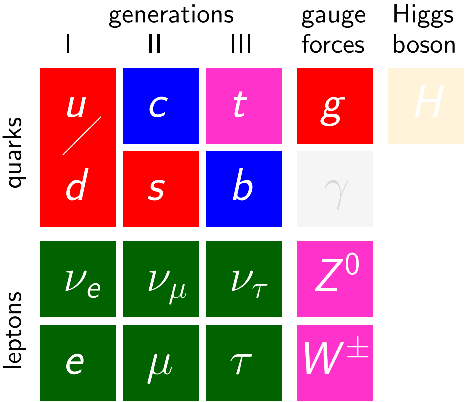
- 2+1 dynamical flavor QCD (fully relativistic action)
  → dynamical charm is possible: 2+1+1 flavors
- c- and b-quarks simulated as valence quarks

The Standard Model in a typical lattice calculation



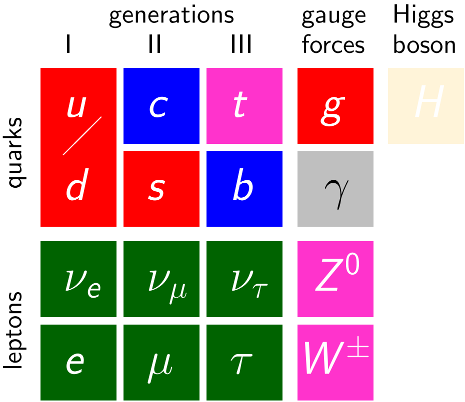
- 2+1 dynamical flavor QCD (fully relativistic action)
  → dynamical charm is possible: 2+1+1 flavors
- c- and b-quarks simulated as valence quarks
- Weak force carriers and top enter “in” point-like operators

The Standard Model in a typical lattice calculation



- 2+1 dynamical flavor QCD (fully relativistic action)
  → dynamical charm is possible: 2+1+1 flavors
- c- and b-quarks simulated as valence quarks
- Weak force carriers and top enter "in" point-like operators
- Leptons mainly in post-analysis steps

The Standard Model in a typical lattice calculation



- 2+1 dynamical flavor QCD (fully relativistic action)
  $\rightarrow$ dynamical charm is possible: 2+1+1 flavors
- c- and b-quarks simulated as valence quarks
- Weak force carriers and top enter "in" point-like operators
- Leptons mainly in post-analysis steps
- QED and iso-spin breaking required for precision $\lesssim$ 1%

Lattice QCD calculation

Simulate

- at finite lattice spacing a
- in a finite volume L^3
  $\Rightarrow$ discrete momenta $2\pi\vec{n}/L$
- lattice regularized
- bare input quark masses
  am_ℓ, am_s, am_c, am_b
  mostly: $aM_\pi \neq aM_\pi^{\text{phys}}$

Desired result

- take $a \to 0$ limit
- take $L \to \infty$ limit
  $\to$ continuous momenta $\vec{p}$
- match to some continuum scheme
- physical quark masses
  $m_l = m_{u/d}^{\text{phys}}$, $m_s = m_s^{\text{phys}}$, $m_c = m_c^{\text{phys}}$, $m_b = m_b^{\text{phys}}$

- Need to choose gauge and fermion action
- Need to control all limits keeping FV and discretization effects under control
  $\to$ u quarks want large volume (large L^3) such that $M_\pi \cdot L > 4$
  $\to$ b quarks want fine lattice (small a) i.e. $am_b \ll 1$

Simulating charm and bottom (schematic)

$a^{-1} > 1.5$ GeV

charm: RHQ; extrapolations of fully relativistic actions (?)

bottom: HQET, NRQCD, RHQ

$a^{-1} > 2.2$ GeV

charm: fully relativistic action

bottom: (guided) extrapolation of fully relativistic action

$a^{-1} > 4.6$ GeV

bottom: fully relativistic action

HQET: static limit, relatively noisy

NRQCD: non-relativistic QCD, no continuum limit

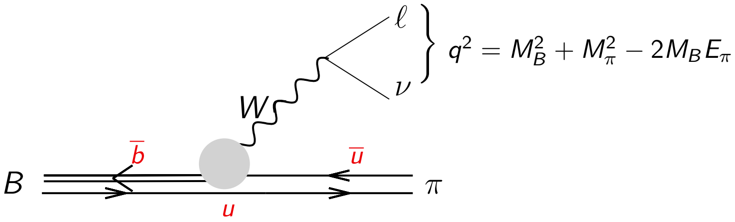
RHQ or Fermilab: relativistic heavy quark action, complicated discretization errors

fully relativistic: (heavy) HISQ, (heavy) MDWF, ...

Charm and beauty form factors

(single pseudoscalar hadronic final state)

$|V_{ub}|$ from exclusive semileptonic $B \to \pi\ell\nu$ decay



- Conventionally parametrized by (B meson at rest)

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_B^2 (E_\pi^2 - M_\pi^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 |f_0(q^2)|^2 \right],$$

nonperturbative input

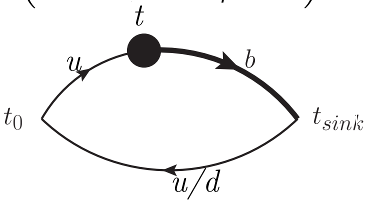
Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

$B \to \pi \ell \nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

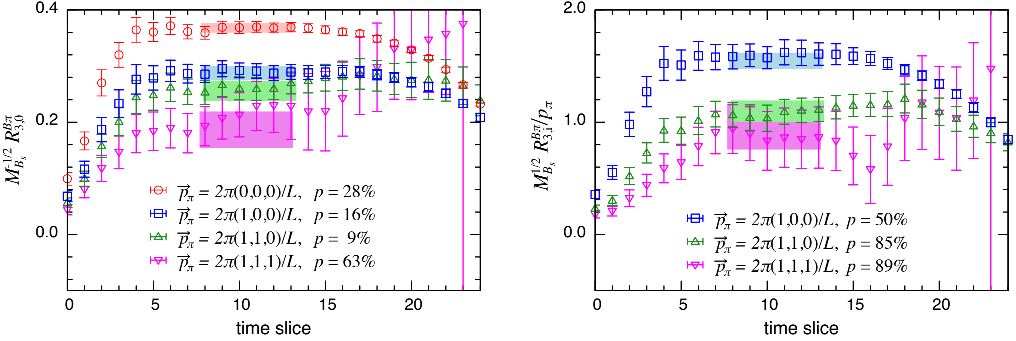


- Calculate 3-point function by
  - → Inserting a quark source for a light u/d quark propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use a "light" quark propagating from t_0 and contract both at t with $t_0 \leq t \leq t_{sink}$

Steps of the $B \to \pi\ell\nu$ calculation

1. Extract form factors on each ensemble

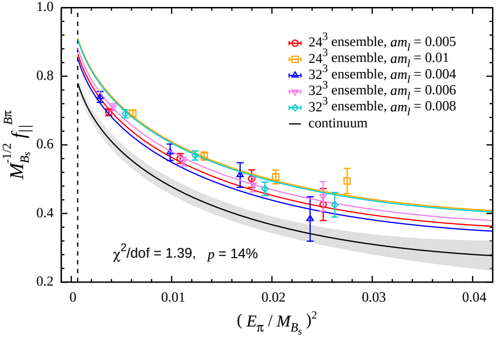
[Flynn et al. PRD 91 (2015) 074510]

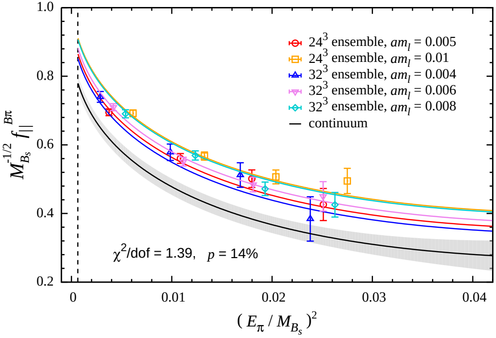


Steps of the $B \to \pi \ell \nu$ calculation

1. Extract form factors on each ensemble
2. Perform chiral-continuum extrapolation

[Flynn et al. PRD 91 (2015) 074510]



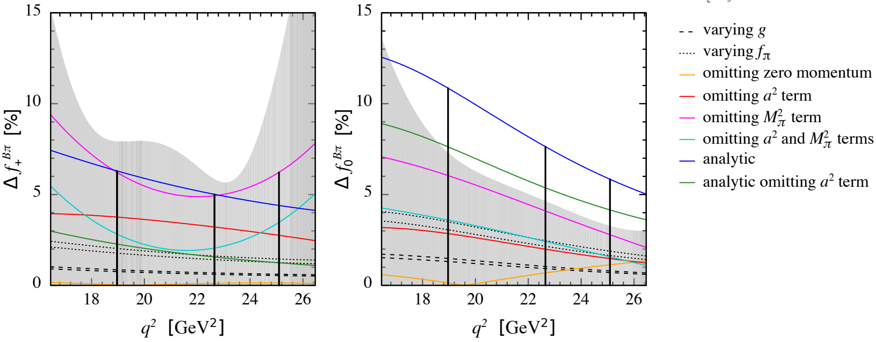


Steps of the $B \to \pi\ell\nu$ calculation

1. Extract form factors on each ensemble
2. Perform chiral-continuum extrapolation

$\to$ Explore other extrapolations, vary inputs, ...

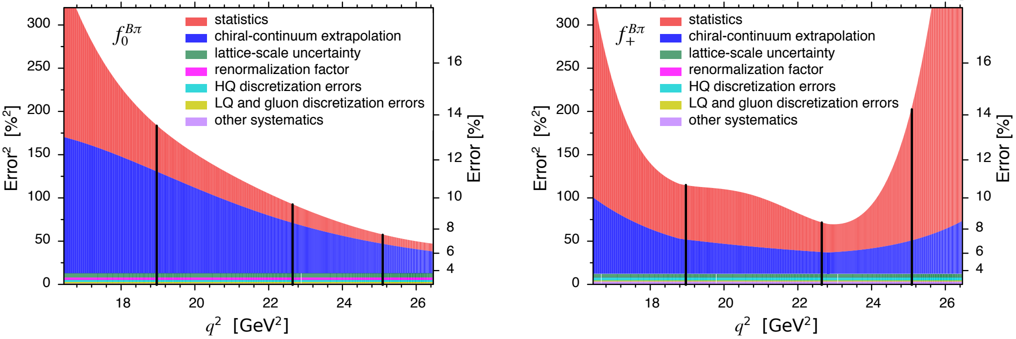
[Flynn et al. PRD 91 (2015) 074510]



Steps of the $B \to \pi\ell\nu$ calculation

1. Extract form factors on each ensemble
2. Perform chiral-continuum extrapolation
   $\to$ Explore other extrapolations, vary inputs, ...
3. Estimate further systematic uncertainties

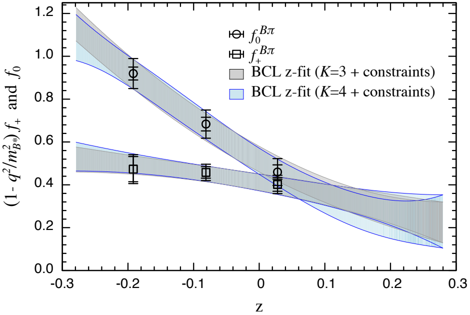
[Flynn et al. PRD 91 (2015) 074510]

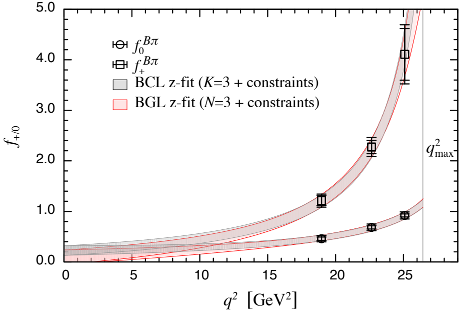


Steps of the $B \to \pi\ell\nu$ calculation

1. Extract form factors on each ensemble
2. Perform chiral-continuum extrapolation
   → Explore other extrapolations, vary inputs, ...
3. Estimate further systematic uncertainties
4. Kinematical extrapolation
   → z-expansion (BGL, BCL, CLN (if applicable))
   → Compare to other calculations, QCD sum rules
   → Combine with experimental data, extract $|V_{ub}|$

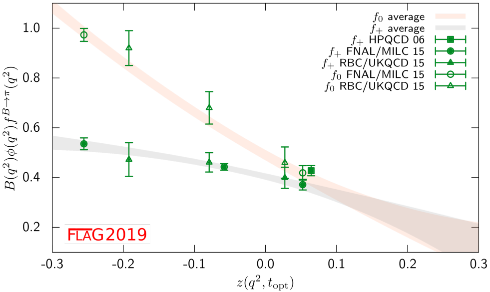
[Flynn et al. PRD 91 (2015) 074510]





FLAG: $B \to \pi\ell\nu$ [FLAG 2019]

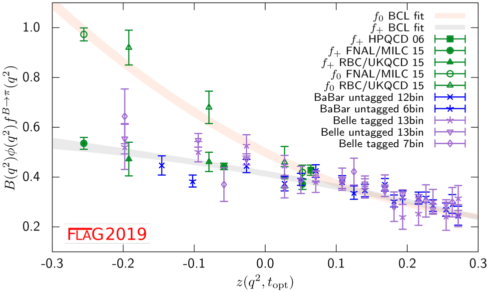
Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	z-parameterization	$\Delta\zeta^{B\pi}$
FNAL/MILC 15	[575]	2+1	A	★	○	★	○	✓	BCL	n/a
RBC/UKQCD 15	[576]	2+1	A	○	○	○	○	✓	BCL	1.77(34)
HPQCD 06	[573]	2+1	A	○	○	○	○	✓	n/a	2.07(41)(39)

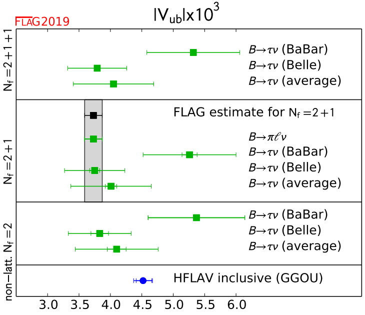


- Summary table indicating quality of key features
- Combined analysis accounting for correlations (if needed)

- Please cite calculations feeding into FLAG averages
  [Bailey et al. PRD92(2015)014024] [Flynn et al. PRD91(2015)074510]
  [Dalgic et al. PRD73(2006)074502]

FLAG: $B \to \pi\ell\nu$ [FLAG 2019]



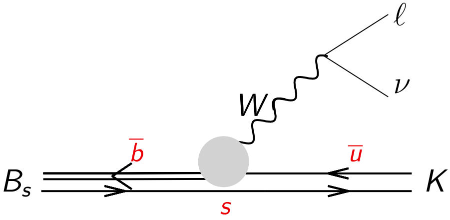


- Combination of lattice average with experimental results
- Extraction of $|V_{ub}|$: $3.73(14) \cdot 10^{-3}$

- Please cite calculations feeding into FLAG averages
  [Bailey et al. PRD92(2015)014024] [Flynn et al. PRD91(2015)074510]
  [Dalgic et al. PRD73(2006)074502]

- Many groups are working on updates:
  - → Fermilab/MILC [Z. Gelzer Lattice X IF 2019]
  - → HPQCD [C. Bouchard Lattice 2019]
  - → JLQCD [J. Koponen Lattice 2019]
  - → RBC-UKQCD [R. Hill Lattice X IF 2019]

$|V_{ub}|$ from exclusive semileptonic $B_s \to K\ell\nu$ decay

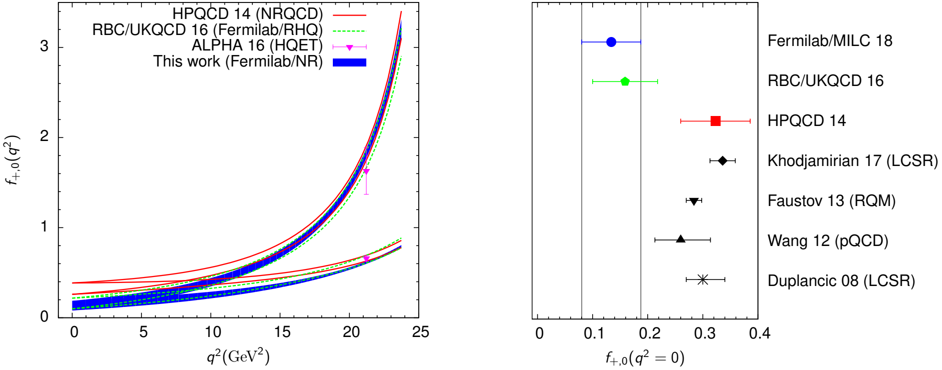


- Compared to $B \to \pi\ell\nu$ only spectator quark differs

- Lattice QCD prefers s quark over u quark: statistically more precise, computationally cheaper
- B factories run mostly at $\Upsilon(4s)$ threshold $\Rightarrow$ B mesons
- LHC collisions create many B and B_s mesons which decay $\Rightarrow$ LHCb
  $\rightarrow$ LHCb prefers the ratio $(B_s \to D_s\ell\nu)/(B_s \to K\ell\nu) \Rightarrow |V_{cb}/V_{ub}|$

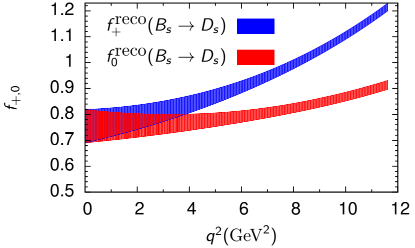
$B_s \to K\ell\nu$

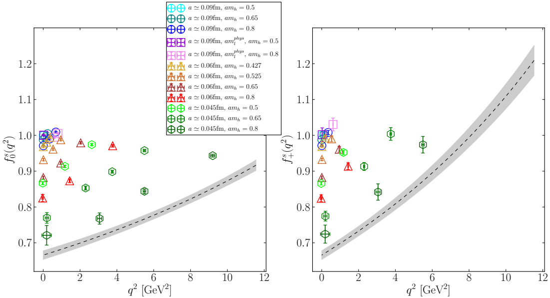
- HPQCD, RBC-UKQCD, ALPHA
  [Bouchard et al. PRD90(2014)054506] [Flynn et al. PRD91(2015)074510] [Bahr et al. PLB757(2016)473]
- New 2019: Fermilab/MILC [Bazavov et al. PRD100(2019)034501]

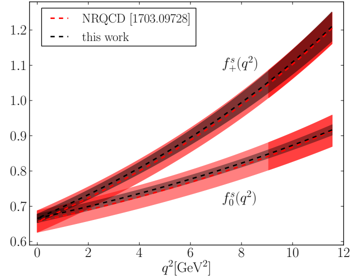


$B_s \to D_s \ell \nu$

- HPQCD, Fermilab/MILC
  [Monahan et al. PRD95(2017)114506] [Bazavov et al. PRD100(2019)034501 (reco)]
- New 2019: HPQCD [McLean et al. arXiv:1906.00701]
  - → Using heavy HISQ and twisted BC to cover full q^2 range
  - → Simulate an array of "lighter" b-quarks and then extrapolate

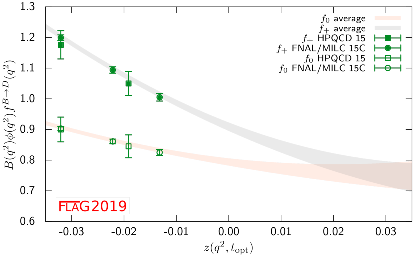


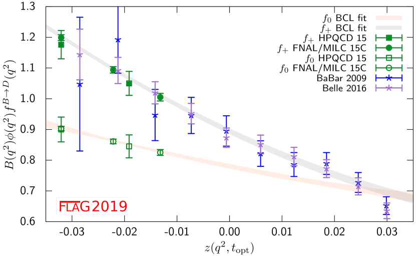


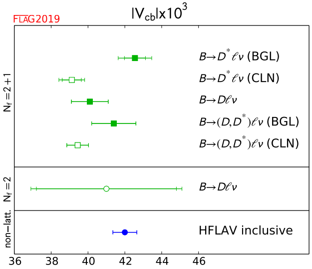


- Updates: RBC-UKQCD [OW Lattice X IF 2019]

$B \to D\ell\nu$







- Please cite calculations feeding into FLAG averages [FLAG 2019]

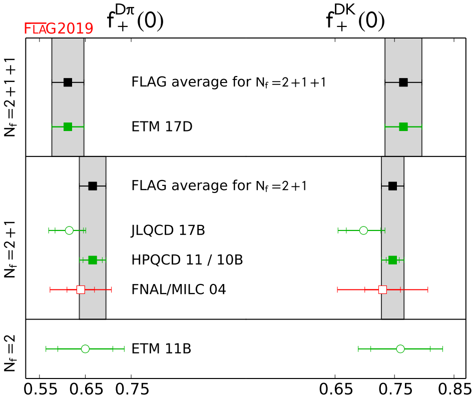
  [Na et al. PRD92(2015)054510] [Bailey et al. PRD92(2015)034506]

$D \to \pi\ell\nu$ and $D \to K\ell\nu$

- Only very few published calculations
  [Lubizc et al. PRD96(2017)054514]
  [Na et al. PRD84(2011)114505] [PRD82(2010)114506]

- Full q^2 range can be covered using twisted boundary conditions

- Updates:
  → HPQCD [B. Chakraborty Lattice 2019]
  → Fermilab/MILC [Li et al. PoS Lattice2018(2019)269]
  → JLQCD [Kaneko et al. EPJ WebConf.175(2018)13007]



[FLAG 2019]

$R(D^*)$

(single vector hadronic final state)

$B \to D^* \ell \nu$ form factors

- Vector final state with narrow width approximation

$$\langle D^*(k,\lambda)|\bar{c}\gamma^\mu b|B(p)\rangle = f_V \frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_B + M_D^*}$$

$$\begin{aligned}\langle D^*(k,\lambda)|\bar{c}\gamma^\mu\gamma_5 b|B(p)\rangle =& f_{A_0}(q^2)\frac{2M_{D^*}\varepsilon^*\cdot q}{q^2}q^\mu \\ &+ f_{A_1}(q^2)(M_B + M_{D^*})\left[\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ &- f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_B + M_{D^*}}\left[k^\mu + p^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2}q^\mu\right]\end{aligned}$$

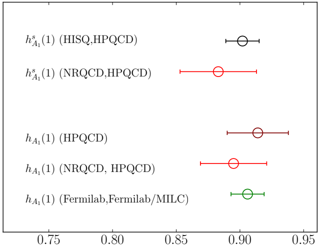
- Commonly the HQET variable $w = v \cdot v' > 1$ is used with $v = p/M_B$ and $v' = k/M_{D^*}$

$B \to D^{(*)}_{(s)} \ell \nu$

- Still only results at zero recoil [FLAG 2019]

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$w=1$ form factor / ratio	
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B\to D}(1)$	1.035(40)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$\mathcal{G}^{B\to D}(1)$	1.054(4)(8)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B\to D}(1)$	1.033(95)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B_s\to D_s}(1)$	1.068(40)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B_s\to D_s}(1)$	1.052(46)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.895(10)(24)
FNAL/MILC 14	[612]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.906(4)(12)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B_s\to D_s^*}(1)$	0.883(12)(28)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$R(D)$	0.300(8)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$R(D)$	0.299(11)

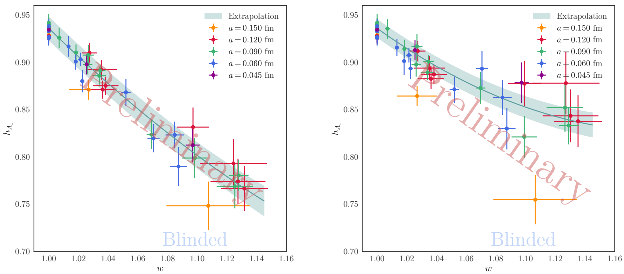
- New 2019: HPQCD $B_s \to D_s^* \ell \nu$ at zero recoil [McLean et al. PRD99(2019)114512]



[Atoui et al. EPJC74(2014)2861] [Bailey et al. PRD89(2014)114504] [Bailey et al. PRD92(2015)034506] [Na et al PRD92(2015)054510] [Monahan et al. PRD95(2017)114506] [Harrison et al. PRD97(2018)054502]

Update Fermilab/MILC: $B \to D^* \ell \nu$ [A. Vaquero Lattice X IF 2019]

Results: Chiral-continuum fits



- **Left** Old fit, **Right** New fit. Preliminary blinded results.
- Both plots differ on the accounting of discretization effects, which seem to be large at large recoil

- Old double ratio

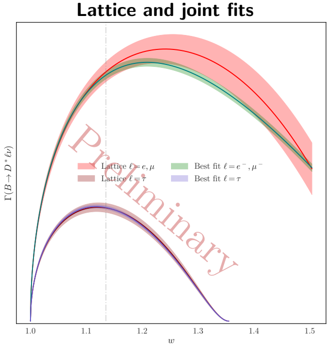
$$\frac{C^{3pt,A_j}_{B\to D^*}(p_\perp,t,T)\, C^{3pt,A_j}_{D^*\to B}(p_\perp,t,T)}{C^{3pt,V^4}_{D^*\to D^*}(0,t,T)\, C^{3pt,V^4}_{B\to B}(0,t,T)} = \frac{M_{D^*}}{E_{D^*}(p_\perp)} \frac{Z^2_{D^*}(p_\perp)}{Z^2_{D^*}(0)} e^{-(E_{D^*}(p_\perp)-M_{D^*})T} \left(\frac{1+w}{2} h_{A_1}(w)\right)^2$$

- New ratio

$$\frac{C^{3pt,A_1}_{B\to D^*}(p_\perp,t,T)}{C^{3pt,A_1}_{B\to D^*}(0,t,T)} \to \frac{C^{3pt,A_1}_{B\to D^*}(p_\perp,t,T)}{C^{3pt,A_1}_{B\to D^*}(0,t,T)} \times \sqrt{\frac{C^{2pt}_{D^*}(0,t)}{C^{2pt}_{D^*}(p_\perp,t)}}$$

Update Fermilab/MILC: $B \to D^* \ell\nu$ [A. Vaquero Lattice X IF 2019]

Results: $R(D^*)$

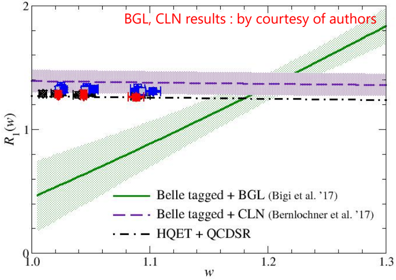


Update JLQCD: $B \to D^* \ell \nu$ [T. Kaneko Lattice X IF 2019]

- Ratio method: $\frac{\langle D^* | V_\mu^{\rm lat} | B \rangle}{\langle D^* | A_\mu^{\rm lat} | B \rangle} \to \frac{h_V(w)}{h_{A_1}(w)}$ $\Rightarrow$ renormalization factors Z_A, Z_V cancel [Hashimoto et al. PRD61(1999)014502]

LQCD vs BGL vs CLN vs HQET

$R_1 = h_V / h_{A1}$

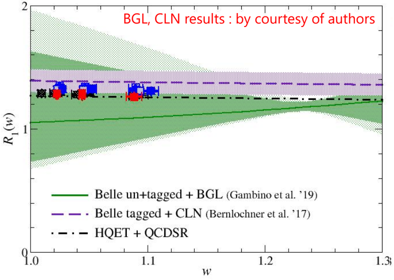


w/ Belle tagged '17
BGL vs CLN, HQET
Bernlochner et al.
Bigi et al.
Grinstein-Kobach

our data favor the CLN results

LQCD vs BGL vs CLN vs HQET

$R_1 = h_V / h_{A1}$



w/ Belle tagged '17
BGL vs CLN, HQET
Bernlochner et al.
Bigi et al.
Grinstein-Kobach

our data favor the CLN results

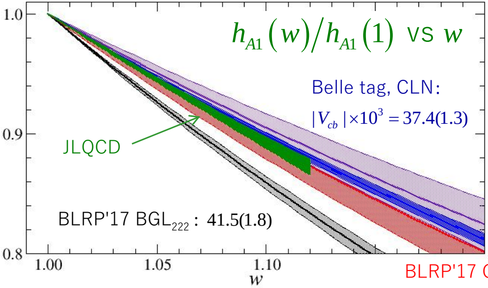
Gambino et al '19 tagged+untagged

consistency among LQCD, BGL, CLN, HQET

Update JLQCD: $B \to D^* \ell\nu$ [T. Kaneko Lattice X IF 2019]

LQCD vs BGL vs CLN

shape of h_{A1}



reduced stat.+sys. error in the ratio

Belle untag, CLN: 37.4(0.9)

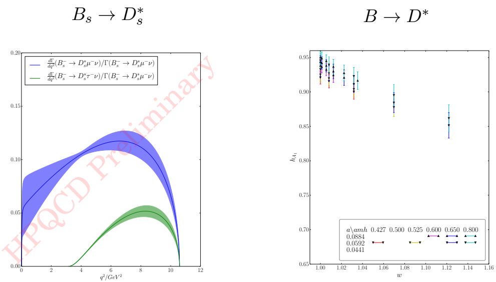
BLRP'17 CLN: 38.2(1.5)

$$h_{A1}(w)/h_{A1}(1) = 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 + (231\rho^2 - 91)z^3$$

Further updates: $B \to D^* \ell\nu$

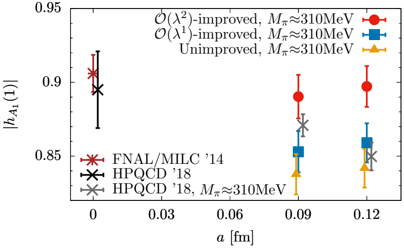
- HPQCD form factors for $B_{(s)} \to D^*_{(s)} \ell\nu$
  [Plenary talk A. Lytle Lattice 2019]

HPQCD $B_{(s)} \to D^*_{(s)}$



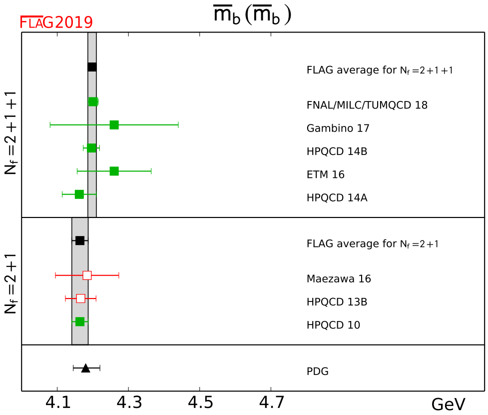
Figs. courtesy Judd Harrison

- LANL/SWME form factors at zero recoil
  [PoS Lattice2018 283]



b & c quark masses

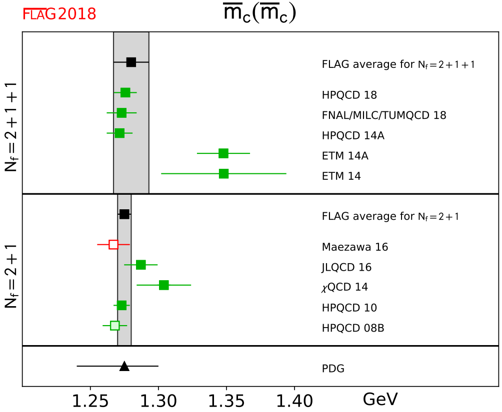
b quark mass



- Determinations
  - → Experimental values for M_Υ, M_{η_b}
  - → Euclidean time moments of ps correlator for heavy-quark current followed by OPE
  - → Ratio of moments of heavy current-current correlator functions
  - → New FNAL/MILC/TUMQCD determination based on sophisticated fit strategy using HQET, HMrASχPT, Symanzik effective theory [Bazavov et al. PRD98(2018)054517]

[FLAG 2019] 2+1: $\overline{m}_b(\overline{m}_b) = 4.164(23)$ GeV   2+1+1: $\overline{m}_b(\overline{m}_b) = 4.198(12)$ GeV

c quark mass



- Determinations
  - → Experimental values for $M_{D_{(s)}}$, M_{η_c}
  - → Euclidean time moments of ps correlator for heavy-quark current followed by OPE
  - → New FNAL/MILC/TUMQCD determination based on sophisticated fit strategy using HQET, HMrASχPT, Symanzik effective theory [Bazavov et al. PRD98(2018)054517]
- Tension with ETMC 2+1+1 determinations

[FLAG 2019] 2+1: $\overline{m}_c(\overline{m}_c) = 1.275(5)$ GeV   2+1+1: $\overline{m}_c(\overline{m}_c) = 1.280(13)$ GeV

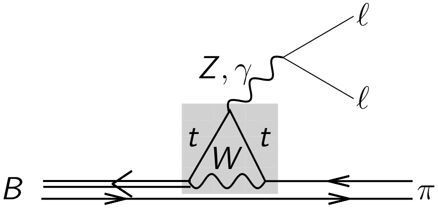
summary

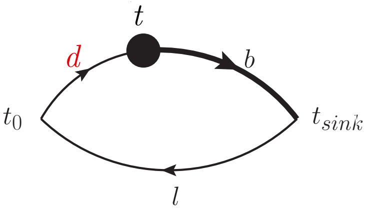
Summary

- Many calculation for exclusive decays are in progress; calculations are hard, tedious, and take time
- New ideas to compute inclusive decays using lattice techniques
  [Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993] [Bailas Lattice 2019]
- Interesting new ideas for radiative decays
  [Kane et al. arXiv:1907.00279] [Martinelli Lattice 2019] [Sachrajda Lattice 2019]
- Not covered: B_c decays, $R(J/\psi)$, etc.
  [see Poster by J. Harrison]
- Not covered: exclusive baryonic decays
  - $\rightarrow \Lambda_b \rightarrow \Lambda_c \ell\nu$ and $\Lambda_b \rightarrow p\ell\nu \Rightarrow |V_{cb}|/|V_{ub}|$ [Detmold, Lehner, Meinel, PRD92(2015)034503]
  - $\rightarrow \Lambda_b \rightarrow \Lambda_c \tau\nu$ [Datta et al. JHEP08(2017)131]
  - $\rightarrow \Lambda_c \rightarrow \Lambda\ell\nu$ [Meinel PRL118(2017)082001]

Flavor Changing Neutral Currents

$B \to \pi\ell\ell$ form factor





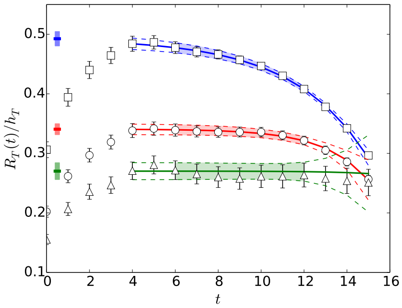
- If the daughter quark is a d-quark, we have a FCNC decay at loop-level
  - $\to$ Need to implement additional operators
- Dominant contributions at short distance: f_0, f_+, and f_T

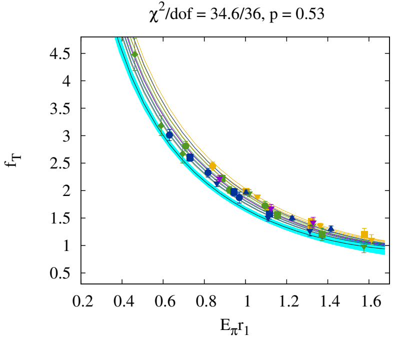
$$\langle\pi(k)|i\bar{d}\sigma^{\mu\nu}b(p)|B\rangle = 2\frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2)$$

$B \to \pi\ell\ell$ form factor

- Fermilab/MILC

[Bailey et al. PRD92(2015)014024]

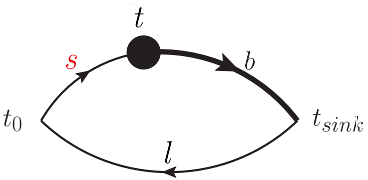




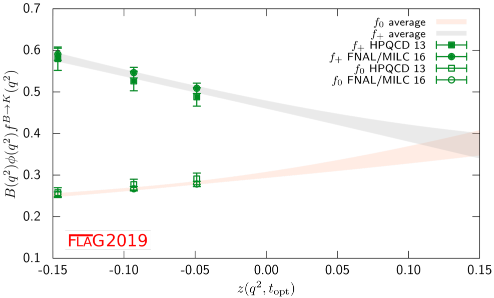
- Extract form factor on each ensemble
- Perform chiral-continuum extrapolation

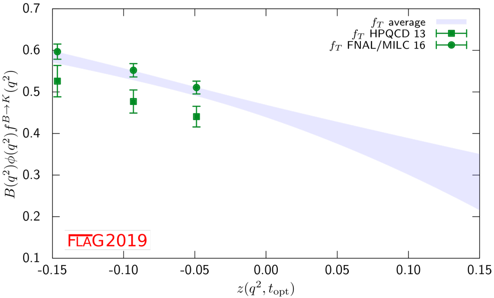
- Updates: Fermilab/MILC [Z. Gelzer Lattice X IF 2019], HPQCD [C. Bouchard Lattice 2019]

$B \to K\ell\ell$



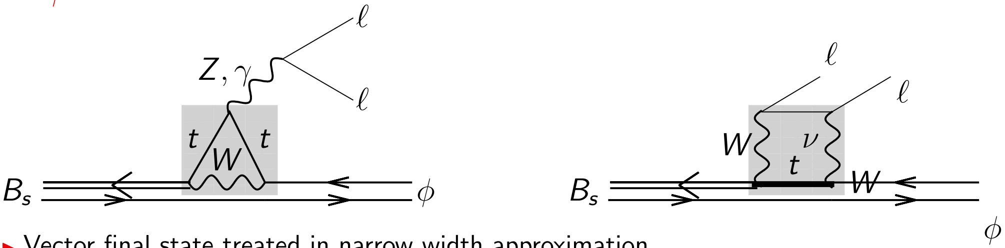
- Similar to $B \to \pi\ell\ell$ but use a strange daughter quark
- Fermilab/MILC, HPQCD





- Please cite calculations feeding into FLAG averages [FLAG 2019]
  [Bailey et al. PRD93(2016)025026] [Bouchard et al. PRD88(2013)054509]
- Updates: Fermilab/MILC [Z. Gelzer Lattice X IF 2019]

$B_s \to \phi\ell^+\ell^-$ form factors



- Vector final state treated in narrow width approximation
- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(\prime)}$$

- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2}\bar{s}\sigma^{\mu\nu}P_{R(L)}bF_{\mu\nu} \qquad O_9^{(\prime)} = \frac{e^2}{16\pi^2}\bar{s}\gamma^\mu P_{L(R)}b\bar{\ell}\gamma_\mu\ell \qquad O_{10}^{(\prime)} = \frac{e^2}{16\pi^2}\bar{s}\gamma^\mu P_{L(R)}b\bar{\ell}\gamma_\mu\gamma^5\ell$$

- $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ [Horgan et al. PRD89(2014)090501] [PoS Lattice2014 372]
  - $\to$ Angular analysis [PRL112(2014)212003]

The beauty of seven form factors

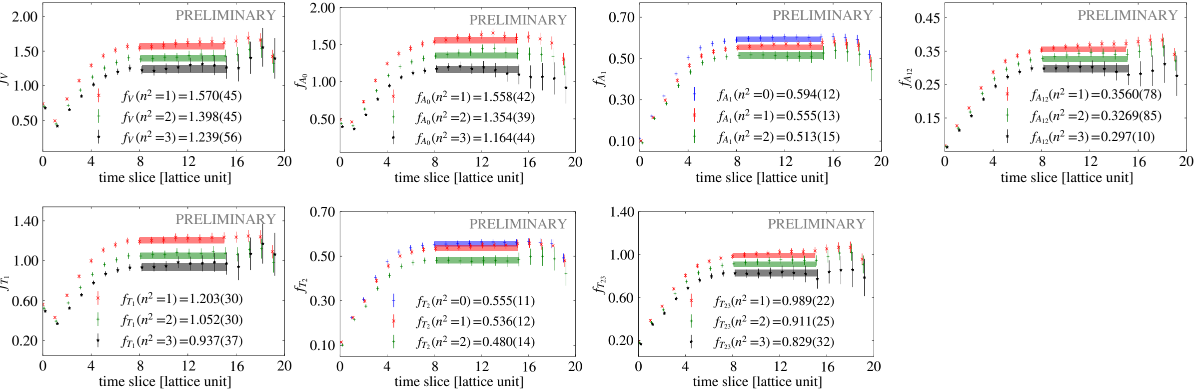
$$\langle\phi(k,\lambda)|\bar{s}\gamma^\mu b|B_s(p)\rangle = f_V(q^2)\frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_{B_s}+M_\phi}$$

$$\langle\phi(k,\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B_s(p)\rangle = f_{A_0}(q^2)\frac{2M_\phi\varepsilon^*\cdot q}{q^2}q^\mu + f_{A_1}(q^2)(M_{B_s}+M_\phi)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] - f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_\phi}\left[k^\mu+p^\mu-\frac{M_{B_s}^2-M_\phi^2}{q^2}q^\mu\right]$$

$$q_\nu\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}b|B_s(p)\rangle = 2f_{T_1}(q^2)\epsilon^{\mu\rho\tau\sigma}\varepsilon^*_\rho k_\tau p_\sigma\ ,$$

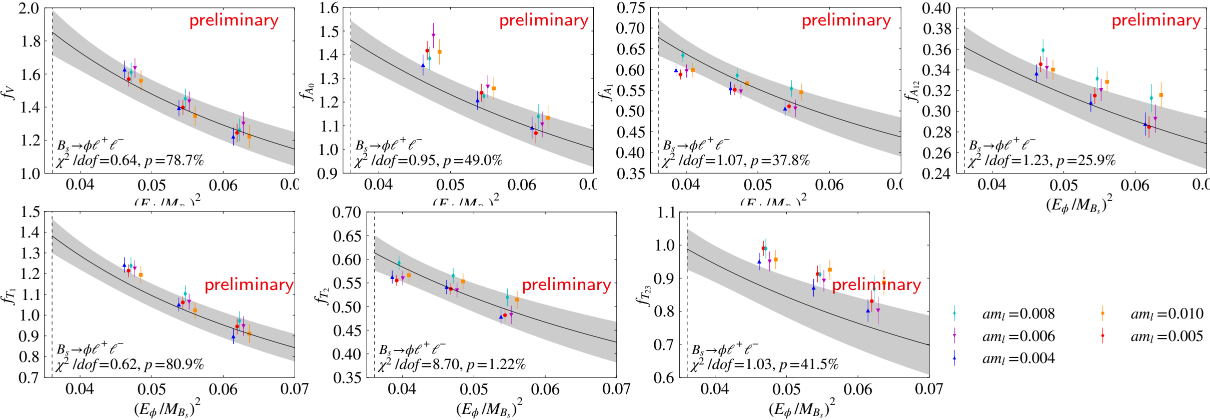
$$q_\nu\langle\phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}\gamma^5 b|B_s(p)\rangle = if_{T_2}(q^2)\left[\varepsilon^{*\mu}(M_{B_s}^2-M_\phi^2)-(\varepsilon^*\cdot q)(p+k)^\mu\right] + if_{T_3}(q^2)(\varepsilon^*\cdot q)\left[q^\mu-\frac{q^2}{M_{B_s}^2-M_\phi^2}(p+k)^\mu\right]$$

$B_s \to \phi\ell\ell$: seven form factors ($a^{-1} = 1.784$ GeV, $am_l^{\rm sea} = 0.005$, $am_s = 0.03224$)



[Flynn et al. PoS Lattice2016(2016)296]

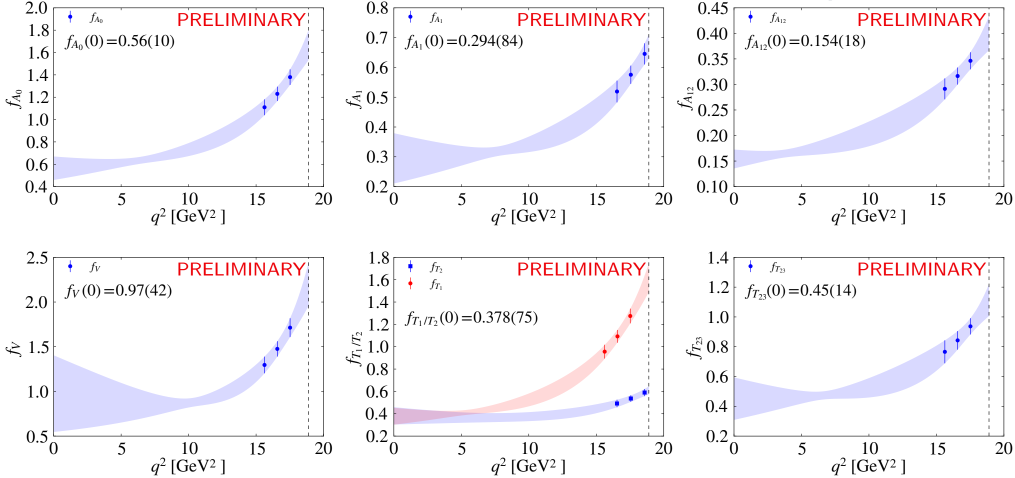
$B_s \to \phi\ell^+\ell^-$: seven form factors vs. q^2



[Flynn et al. PoS Lattice2016(2016)296]

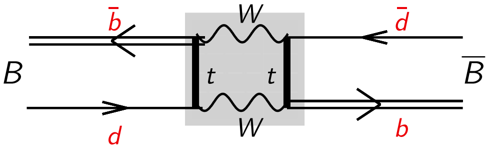
$B_s \to \phi \ell^+ \ell^-$: first attempt to use z-parametrization

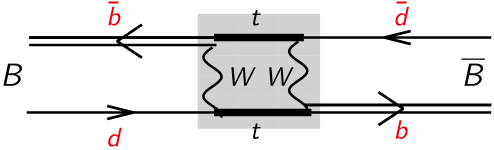
[Flynn et al. PoS Lattice2016(2016)296]



neutral meson mixing

Neutral B meson mixing





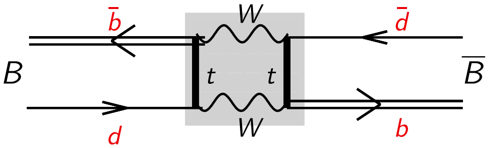
$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \qquad q = d, s$$

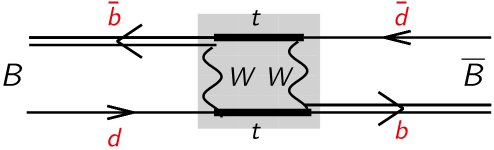
- Experiments measure oscillation frequencies Δm_q extremely precise
- Process is short distance dominated (top quark)
- Theory provides decay constant f_{B_q} and bag parameter B_{B_q}

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p^\mu_{D_q}$$

$$\langle \bar{B}_q^0|[\bar{b}\gamma^\mu(1-\gamma_5)q][\bar{b}\gamma_\mu(1-\gamma_5)q]|B_q^0\rangle = \frac{3}{8} f_{B_q}^2 M_{B_q}^2 B_{B_q}$$

Neutral B meson mixing





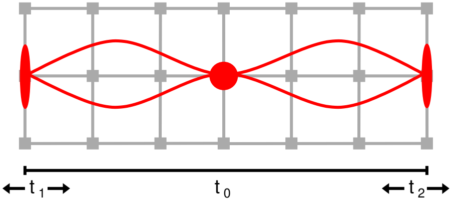
$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2, \qquad q = d, s$$

- Experiments measure oscillation frequencies Δm_q extremely precise
- Process is short distance dominated (top quark)
- Theory provides decay constant f_{B_q} and bag parameter B_{B_q}
- Advantageous to consider
  [Bernard, Blum, Soni PRD58(1998)014501]

$$\frac{\Delta m_s}{\Delta m_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}, \qquad \xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

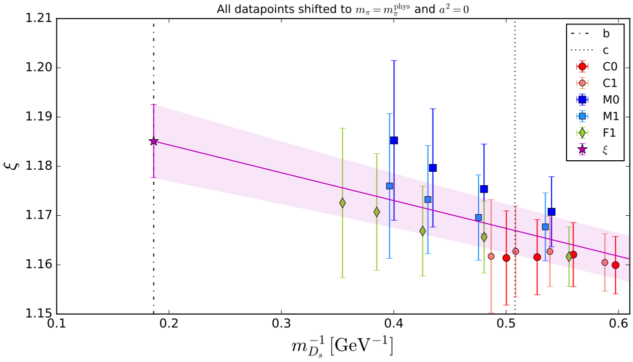
Basics of the lattice calculation

- Schematic set-up



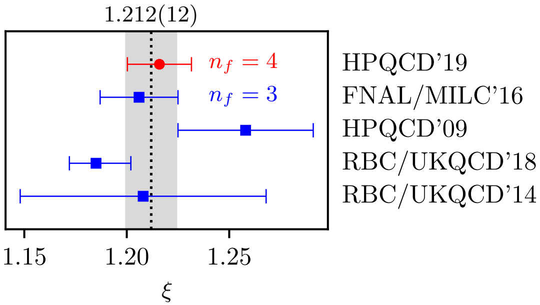
- Only zero momentum ground-states (bag parameters and decay constants)
- Fit symmetrized plateau between source and operator location

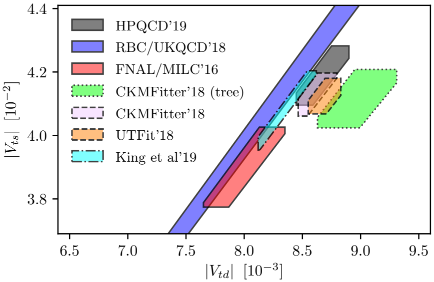
- Ratio ξ ideal to explore simulating with "heavier than charm" and extrapolating to bottom e.g. [Boyle et al. arXiv:1812.08791]



ξ and $|V_{td}|$ vs. $|V_{ts}|$

- Two latest calculation feature simulations at physical pion masses





[Dowdall et al. arXiv:1907.01025] [Boyle et al. arXiv:1812.08791] [Bazavov et al. PRD93(2016)113016]
[Aoki et al. PRD91(2015)114505] [Gamiz et al. PRD80(2009)014503]

- Today on the arXiv: dimension 7 operators of $B_s^0 - \overline{B_s^0}$ mixing
[Davies et al. arXiv:1910.00970]