B-physics with domain-wall light quarks and nonperturbatively tuned relativistic $b$-quarks

RBC and UKQCD collaborations

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Motivation: Constraining the Standard Model

- Determination of CKM matrix elements $V_{ub}$ and $V_{cb}$
- $B$-physics provides constraints on the apex of the CKM unitarity triangle
  - $B^0 - \overline{B}^0$ mixing
  - $V_{ub}$ and $V_{cb}$ e.g. from $B \to \pi \ell \nu$ and $\overline{B} \to D^* \ell \nu$ form factors
- Experimental results and nonperturbative inputs are needed

Example: $V_{ub}$ from exclusive semileptonic decay $B \rightarrow \pi \ell \nu$

\[
q^2 = M_B^2 + M_{\pi}^2 - 2M_B E_\pi
\]

Conventionally parameterized by

\[
\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3} \left[ \left( M_B^2 + M_{\pi}^2 - q^2 \right)^2 - 4M_B^2 M_{\pi}^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2
\]
Motivation: New Physics in rare $B$-decays?

$B \to \tau\nu$ [UTfit Phys.Lett. B687 (2010) 61]

- $f_B$ is needed for the Standard-Model prediction of $BR(B \to \tau\nu)$
- Potentially sensitive to charged-Higgs exchange due to large $\tau$ mass


- $f_{B_s}$ is needed for Standard-Model prediction of $BR(B_s \to \mu^+\mu^-)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Measured by LHCb with $3.5\sigma$ significance [LHCb Phys.Rev.Lett. 110 (2013) 02180], at EPS2013: combination of LHCb and CMS results gives $>5\sigma$ significance — in agreement with SM

Both are sensitive to new physics!
Challenges for $B$-physics on the lattice

- The mass of the $b$-quark introduces another scale
  - Light quark masses: $m_u = 2.3 \text{ MeV}, m_d = 4.8 \text{ MeV}, m_s = 95 \text{ MeV}$
  - Mass of the $b$-quark: $m_b = 4.18 \text{ GeV}$

- Today’s lattices have an inverse lattice spacing of $a^{-1} \approx 1.7 \ldots 3 \text{ GeV}$
  - $am_b > 1$
  - Forced to simulate $b$-quarks with an effective action
    e.g. HQET (static), NRQCD, Fermilab or RHQ action

- Relativistic Heavy Quark (RHQ) action
  - Anisotropic action to simulate dynamical $b$-quarks
  - Similar size discretization errors in heavy-light quantities as in light-light
  - Requires (perturbative) mixed-action renormalization factors
Our project

- Based on RBC-UKQCD’s 2+1 flavor domain-wall Iwasaki gauge field configurations

- Use domain-wall light quarks and nonperturbatively tuned relativistic $b$-quarks to compute at few-percent precision
  - $B^0 - \bar{B}^0$ mixing
  - Decay constants $f_B$ and $f_{B_s}$
  - $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$ form factors [Taichi Kawanai]
  - $g_{B^*B\pi}$ coupling constant [Ben Samways]

- Provides important cross-check of other $N_f = 2 + 1$ determinations using the MILC staggered ensembles
**B-meson decay constant** [arXiv:1404.4670 [hep-lat]]

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- On the lattice we compute $\Phi_{Bq}$
  \[
  f_B = \Phi^{\text{ren}}_{Bq} \cdot a^{-3/2} / \sqrt{M_{Bq}}
  \]
- Improve axial current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)
  \[
  Z_{V}^{bl} = \varrho^{bl} \cdot \sqrt{Z_{V}^{bb} Z_{V}^{ll}}
  \]
$B$-meson decay constant [arXiv:1404.4670 [hep-lat]]

- Perform analysis in terms of dimensionless ratios over $M_{B_s}$

\[ a_{B_q} = 2.3252(13) \]
\[ \chi^2/\text{dof} = 0.87, \quad p = 56\% \]
\[ a_{m_q} = 0.006 \]

\[ \Phi_{\text{eff}}^{B_q}/M_{B_s}^{3/2} = 0.03954(31) \]
\[ \chi^2/\text{dof} = 0.50, \quad p = 89\% \]

\[ a_{m_q} = 0.006 \]
Chiral-continuum extrapolation of $f_B$ [arXiv:1404.4670 [hep-lat]]

- NLO SU(2) HM\(\chi\)PT
- Only data points with filled symbols included in the fit
  \((M_\pi < 420 \text{ MeV})\)
- Statistical errors only
- \(g_{B^*B\pi} = 0.57(8)\)
- \(f_\pi = 130.4 \text{ MeV} \) [PDG]
- \(\Lambda_\chi = 1 \text{ GeV}\)
Continuum extrapolation of $f_{B_s}$ [arXiv:1404.4670 [hep-lat]]

- Data for $\Phi_{B_s}$ show no sea-quark mass dependence
- Average data at same lattice spacing
- Assume $a^2$ scaling to remove LQ and gluon discretization errors
- Statistical errors only
## Error budget [arXiv:1404.4670 [hep-lat]]

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{B^0}$ (%)</th>
<th>$f_{B^+}$ (%)</th>
<th>$f_{B_s}$ (%)</th>
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<td>isospin-breaking and EM</td>
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<td>0.7</td>
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<td><strong>total</strong></td>
<td><strong>8.0</strong></td>
<td><strong>8.1</strong></td>
<td><strong>5.2</strong></td>
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</table>
### Comparison with other results

| $f_B$ | $f_{B_s}$ | 
|---|---|---|
| 2+1+1 |  nutshell | ETMC 2013 (TM+HQET) [1] |
| 2+1 |  nutshell | HPQCD 2013 (HISQ+NRQCD) [2] |
| 2 |  nutshell | this work |
| |  nutshell | HPQCD 2012 (HISQ+NRQCD) [3] |
| |  nutshell | HPQCD 2012 (HISQ+HISQb/NRQCD) [3,4] |
| |  nutshell | FNAL/MILC 2011 (Asqtad+Fermilab) [5] |
| 1 |  nutshell | ALPHA 2014 (Wilson+HQET) [6] |
| |  nutshell | ETMC 2012 (TM+HQET) [7] |

► **Good agreement with other results**

**FLAG:** S. Aoki et al. arXiv:1310.8555 [hep-lat]


$B \rightarrow \pi \ell \nu$: computation of lattice form factors $f_\parallel$ and $f_\perp$

(which are linearly related to $f_+$ and $f_0$)

\begin{align*}
  f_\parallel &= \lim_{t,T \to \infty} R_{0}^{B \rightarrow \pi}(t, T) \\
  f_\perp &= \lim_{t,T \to \infty} \frac{1}{p_i} R_{i}^{B \rightarrow \pi}(t, T)
\end{align*}

\begin{align*}
  R_{\mu}^{B \rightarrow \pi}(t, T) &= \frac{C_{3,\mu}^{B \rightarrow \pi}(t, T)}{C_{2}^{\pi}(t)C_{2}^{B}(T - t)} \sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T - t)}}}
\end{align*}

[plots by Taichi Kawanai]
Chiral-continuum extrapolation using SU(2) hard-pion $\chi$PT

$\chi^2$/d.o.f. = 1.39, p-value = 14%

$B \to \pi l \nu$

- Extract $f_\parallel$ and $f_\perp$ for three different $q^2$ values
- Estimate all systematic errors and add in quadrature
- Convert results to $f_+$ and $f_0$

[plots by Taichi Kawanai]
Combine with experimental data to determine $|V_{ub}|$

$|V_{ub}| \approx 3.6(3) \cdot 10^{-3}$ (preliminary)

in good agreement with existing results
Conclusion

- Results for decay constants $f_B$ and $f_{B_s}$ are posted and submitted to PRD
  - Good agreement with results in the literature
  - Errors dominated by chiral-continuum extrapolation
- Already improving our errors by simulating with physical pion mass
- Another ensemble with finer lattice spacing will be added

- Finalizing analyses for $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$ form factors
  - Promising looking results

- First determination of the $g_{B^*B\pi}$ coupling with relativistic $b$-quarks
  (publication in preparation)

- Measurements for $B^0 - \overline{B^0}$ mixing are in progress
Appendix
2+1 flavor domain-wall gauge field configurations

▶ Domain-wall fermions for the light quarks (u, d, s)

▶ Iwasaki gauge action [Iwasaki UTHEP-118(1983)]

▶ Configurations generated by RBC and UKQCD collaborations

<table>
<thead>
<tr>
<th>L</th>
<th>a(fm)</th>
<th>m_l</th>
<th>m_s</th>
<th>M_π(MeV)</th>
<th>approx. # configs.</th>
<th># time sources</th>
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<tr>
<td>32</td>
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<td>0.008</td>
<td>0.030</td>
<td>394</td>
<td>544</td>
<td>2</td>
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</tbody>
</table>
Relativistic heavy quark action for the $b$-quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin


- Heavy quark mass is treated to all orders in $(m_b a)^n$
  - Applies for all values of the quark mass

- Expand in powers of the spatial momentum through $O(\vec{p} a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities

- Tune parameters to physical $b$-quarks i.e. require:
  - Spin-averaged mass and hyperfine splitting of the $B_s$-meson agree with experiment
  - $B_s$-meson rest and kinetic masses are equal

- Validated by computing $b\bar{b}$ masses and splittings [Phys.Rev. D86 (2012) 116003]
$B \to \pi \ell \nu$: relating form factors $f_+$ and $f_0$ to $f_\parallel$ and $f_\perp$

- Parametrize the hadronic matrix element for the flavor changing vector current $V^\mu$ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$
\langle \pi | V^\mu | B \rangle = f_+(q^2) \left( p_B^\mu + p_\pi^\mu - \frac{M_B^2-M_\pi^2}{q^2} q^\mu \right) + f_0 \frac{M_B^2-M_\pi^2}{q^2} q^\mu
$$

- Using the $B$-meson rest frame we compute on the lattice

$$
f_\parallel(E_\pi) = \frac{\langle \pi | V^0 | B \rangle}{\sqrt{2M_B}} \quad \text{and} \quad f_\perp(E_\pi) p_\pi^i = \frac{\langle \pi | V^i | B \rangle}{\sqrt{2M_B}}
$$

- Both are related by

$$
f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2-M_\pi^2} \left[ (M_B - E_\pi) f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2) f_\perp(E_\pi) \right]
$$

$$
f_+(q^2) = \frac{1}{\sqrt{2M_B}} \left[ f_\parallel(E_\pi) + (M_B - E_\pi) f_\perp(E_\pi) \right]
$$