# $B_s^0 \to \{D_s, K\}$ form factors from lattice QCD

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Implications of LHCb measurements and future prospects
CERN, November 09, 2017

# RBC- and UKQCD collaborations (Lattice 2017)


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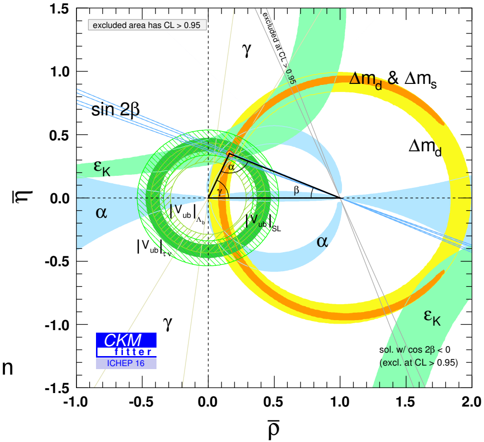
# introduction

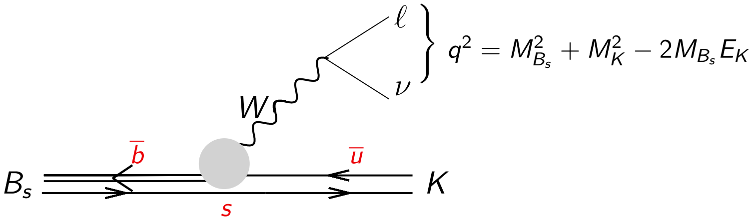
# Why $B_s$ meson decays?

- Alternative, tree-level determination of $|V_{cb}|$ and $|V_{ub}|$ from $B_s \to D_s\ell\nu$ and $B_s \to K\ell\nu$
  - → Commonly used $B \to \pi\ell\nu$ and $B \to D^{(*)}\ell\nu$
  - → Longstanding $2-3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$)
  - → $B \to \tau\nu$ has larger error
  - → Alternative, exclusive ($\Lambda_b \to p\ell\nu$) determination [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- Test of lepton flavor violation in $B_s$ decays ($R_{D_s}$, $R_K$)
- Higher precision in nonperturbative lattice calculation




[http://ckmfitter.in2p3.fr]

# $|V_{ub}|$ from exclusive semileptonic $B_s \to K\ell\nu$ decay




- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

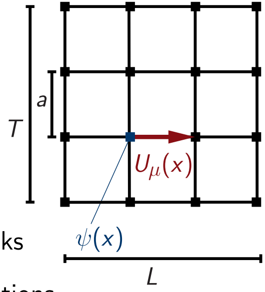
$$\frac{d\Gamma(B_s\to K\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3}\left[\left(M_{B_s}^2 + M_K^2 - q^2\right)^2 - 4M_{B_s}^2 M_K^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment known nonperturbative input CKM

# Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance
- Calculate the flavor changing currents as point-like operators using lattice QCD

# Lattice QCD




- Wick rotation of Minkowski to Euclidean time
- Discretize space-time on a 4-d hypercube with extent $L^3 \times T$ and lattice spacing $a$ [fm] $\rightsquigarrow$ $1/a$ is the cutoff [GeV]
- Quark fields $\psi(x)$ live on the lattice sites, gauge fields $U_\mu(x)$ on the links
- Numerically solve path integral using Markov chain Monte Carlo simulations with importance sampling ($\rightsquigarrow$ supercomputers)
- Different discretizations for fermion (Wilson, Staggered, DWF, ...) and gauge actions (Wilson plaquette, Iwasaki, Symanzik, ...)
- Results are expected to agree in the continuum limit where lattice artifacts are removed ($\rightsquigarrow$ see FLAG compilations)

# Typical workflow of a lattice calculation

1) Generate gauge field configurations containing the QCD vacuum with "light" sea-quarks and gluons
   - ⇝ Degenerate $u/d$ and $s$ quark: dynamical 2+1 flavor
   - ⇝ $s$ quarks close to physical mass
   - ⇝ $u/d$ quarks chirally extrapolated, now simulations at physical mass
   - ⇝ Need experimental inputs to set quark masses, gauge coupling, $\theta$
2) Carry out valence quark measurements on gauge field configurations
3) Combine measurements on different ensembles, extrapolate to the continuum and physical quark masses
4) Match lattice calculation to $\overline{\text{MS}}$ scheme (renormalization)
5) Account for systematic effects

# Additional challenge: *b* quark

- Masses: *b*-quark 4.18 GeV whereas *d*-quark 4.7 MeV
  - ⇒ *b*-quark $\sim$ 1000 times heavy than *d*-quark
  - ⇒ Mass of *b*-quark larger than cutoff ($a^{-1}$)
- Simulate *b*-quark with effective action
  - → Requires renormalization of mixed action
  - → Fermilab-action/RHQ, NRQCD, HQET
- Extrapolate to physical *b*-quark
  - → allows for full nonperturbative renormalization
  - → ETMC ratio method, heavy HISQ, heavy DWF
- Similar considerations for *c*-quark (1.28 GeV)

$\Longrightarrow$ systematic uncertainty to be accounted for

# Set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
    [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][arXiv:1701.02644]

- Unitary and partially quenched domain-wall up/down quarks
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Domain-wall strange quarks at/near the physical value
- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - → Simulate 3 or 2 charm-like masses then extrapolate/interpolate

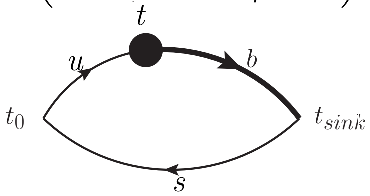
- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  - → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  - → Allows to tune the three parameters ($m_0 a$, $c_P$, $\zeta$) nonperturbatively [PRD 86 (2012) 116003]
  - → Smooth continuum limit; heavy quark treated to all orders in $(m_b a)^n$

$$B_s \to K\ell\nu$$

# $B_s \to K\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current $V^\mu$ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$




- Calculate 3-point function by
  - → Inserting a quark source for a "light" propagator at $t_0$
  - → Allow it to propagate to $t_{sink}$, turn it into a sequential source for a $b$ quark
  - → Use another "light" quark propagating from $t_0$ and contract both at $t$

# Determining $B_s \to K\ell\nu$ form factors $f_+$ and $f_0$ on the lattice

- Updating calculation [PRD 91 (2015) 074510] with new values for $a^{-1}$ and RHQ parameters
- On the lattice we prefer using the $B_s$-meson rest frame and compute

$$f_\parallel(E_K) = \langle K|V^0|B_s\rangle/\sqrt{2M_{B_s}} \quad \text{and} \quad f_\perp(E_K)p_K^i = \langle K|V^i|B_s\rangle/\sqrt{2M_{B_s}}$$

- Both are related by

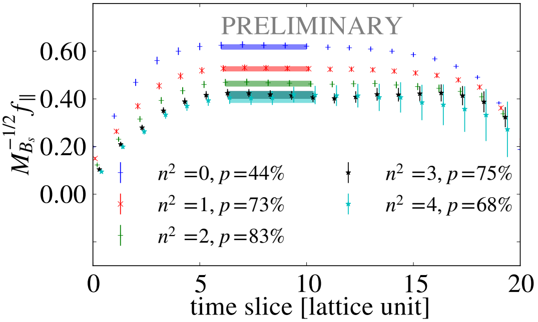
$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2}\left[(M_{B_s} - E_K)f_\parallel(E_K) + (E_K^2 - M_K^2)f_\perp(E_K)\right]$$

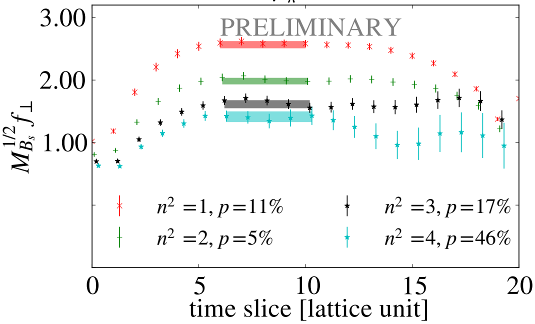
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_\parallel(E_K) + (M_{B_s} - E_K)f_\perp(E_K)\right]$$

# Lattice results for form factors $f_\parallel$ and $f_\perp$ for $B_s \to K\ell\nu$

$$R_\mu^{B_s\to K}(t, t_{\rm sink}) = \frac{C_{3,\mu}^{B_s\to K}(t, t_{\rm sink})}{C_2^K(t)C_2^{B_s}(t_{\rm sink}-t)}\sqrt{\frac{4M_{B_s}E_K}{e^{-E_k t}e^{-M_{B_s}(t_{\rm sink}-t)}}}$$

$$f_\parallel = \lim_{t,t_{\rm sink}\to\infty} R_0^{B_s\to K}(t, t_{\rm sink}) \qquad f_\perp = \lim_{t,t_{sink}\to\infty} \frac{1}{p_\pi^i} R_i^{B_s\to K}(t, t_{\rm sink})$$




⇒ Values of the form factors on one ensemble i.e. $f = f(a^{-1}, am_\ell, am_s, \ldots)$

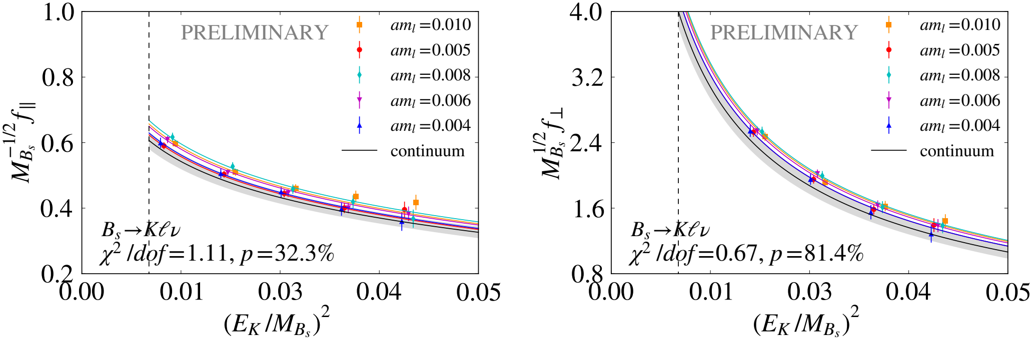
→ Kinematic range determined by largest momentum

# Chiral-continuum extrapolation using SU(2) hard-kaon $\chi$PT

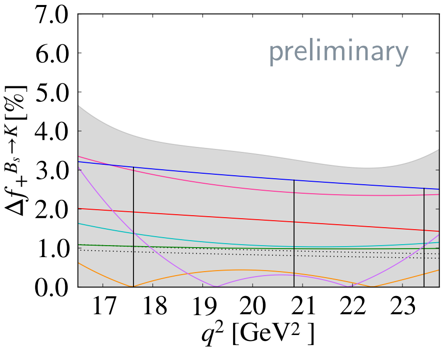
$$f_{\parallel}(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_{\parallel}^{(1)} \left[1 + \left(\frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_K}{\Lambda} + c_{\parallel}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_{\perp}(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_{\perp}^{(1)} \left[1 + \left(\frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_K}{\Lambda} + c_{\perp}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

with $\delta f$ non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$



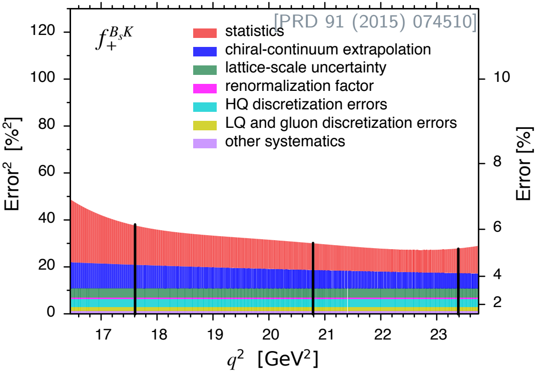
# Estimate systematic errors due to

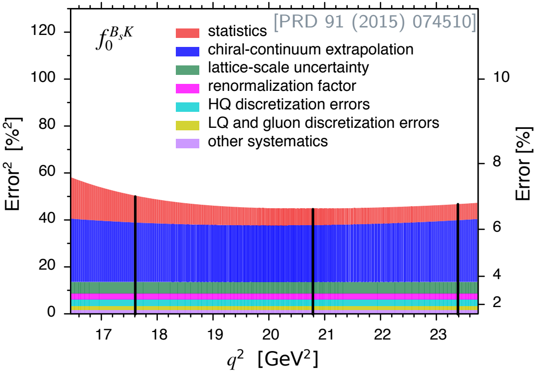



- Chiral-continuum extrapolation
  - → Use alternative fit functions
  - → Impose different cuts on the data
- Uncertainties of the lattice spacing ($a^{-1}$)
  - → Repeat the fit varying $a^{-1}$ by its uncertainty
- Uncertainty of the renormalization factors
  - → Estimate effect of higher loop corrections
- Discretization errors and uncertainties of light and heavy quarks
  - → Vary by uncertainty
  - → Carry out additional simulations to test effects on form factors
- Finite volume, iso-spin breaking, . . .

$\Longrightarrow$ full error budget

# Graphical error budget (plots from previous analysis!)



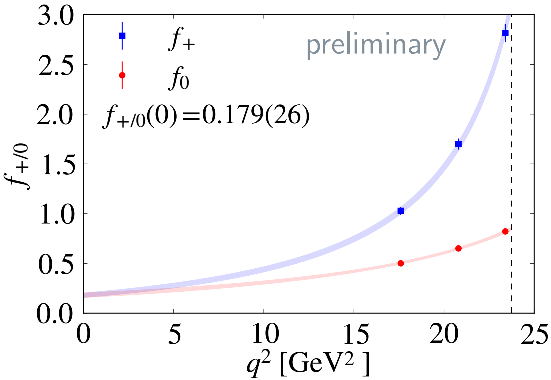




- Read off values for "synthetic" data points
  - → Use values in the chiral-continuum limit with uncertainties representing the full error budget
  - → Chiral-continuum extrapolation performed over range of our data
  - → Avoids parametrizing lattice artifacts in kinematic expansion

# Kinematical extrapolation ($z$-expansion)

- Map $q^2$ to $z$ with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with} \quad \begin{array}{l} t_\pm = (M_B \pm M_\pi)^2 \\ t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2 \end{array}$$




[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Express $f_+$ as convergent power series
- $f_0$ is analytic, except for $B^*$ pole
- Exploit kinematic constraint $f_+ = f_0$ at $q^2 = 0$
- Use HQ power counting to constrain size of $f_+$ coefficients

# Kinematical extrapolation ($z$-expansion)

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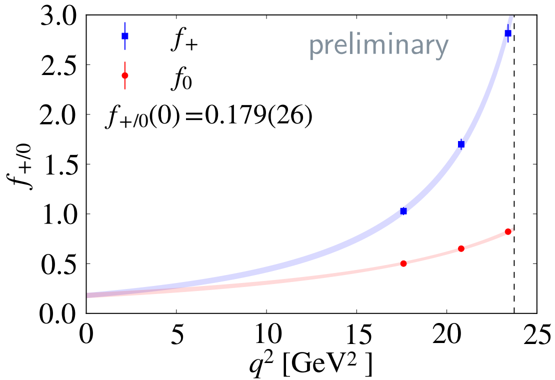
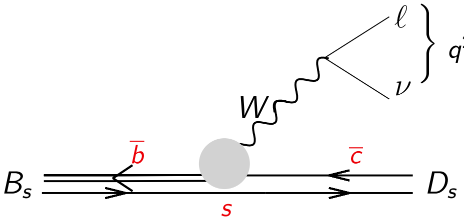



- Allows to compare shape of form factors
  - → Obtained by other lattice calculations [HPQCD PRD90 (2014) 054506]
  - → Predicted by QCD sum rules and alike

- Combination with experiment leads to the overall normalization: $|V_{ub}|$

$$B_s \to D_s \ell \nu$$

# $|V_{cb}|$ from exclusive semileptonic $B_s \to D_s\ell\nu$ decay




$$q^2 = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}E_{D_s}$$

- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

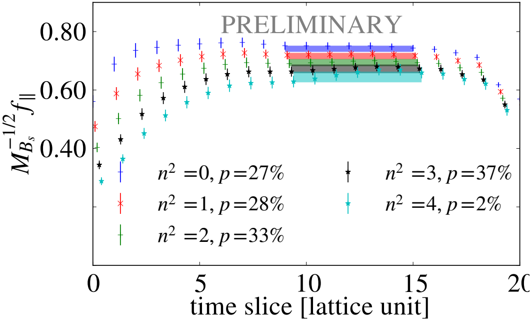
$$\frac{d\Gamma(B_s \to D_s\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3}\left[\left(M_{B_s}^2 + M_{D_s}^2 - q^2\right)^2 - 4M_{B_s}^2 M_{D_s}^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{cb}|^2$$

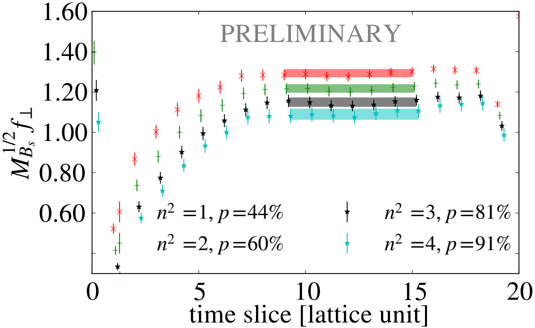
experiment known nonperturbative input CKM

# Lattice results for form factors $f_\parallel$ and $f_\perp$ for $B_s \to D_s\ell\nu$

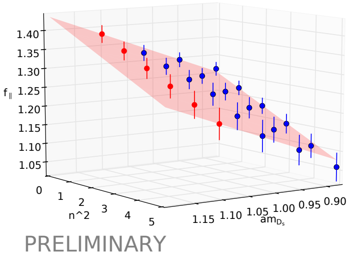
$$R_\mu^{B_s\to D_s}(t, t_{\rm sink}) = \frac{C_{3,\mu}^{B_s\to D_s}(t, t_{\rm sink})}{C_2^{D_s}(t)\, C_2^{B_s}(t_{\rm sink}-t)} \sqrt{\frac{4 M_{B_s} E_{D_s}}{e^{-E_{D_s} t} e^{-M_{B_s}(t_{\rm sink}-t)}}}$$

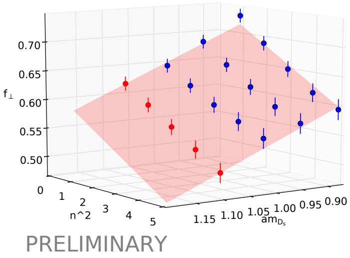
$$f_\parallel = \lim_{t, t_{\rm sink}\to\infty} R_0^{B_s\to D_s}(t, t_{\rm sink}) \qquad f_\perp = \lim_{t, t_{sink}\to\infty} \frac{1}{p_\pi^i} R_i^{B_s\to D_s}(t, t_{\rm sink})$$

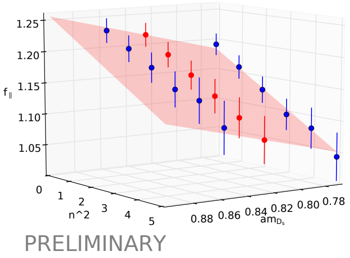


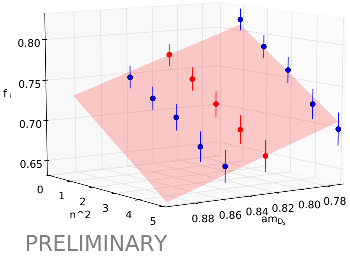



# Charm extra-/interpolation for $B_s \to D_s\ell\nu$



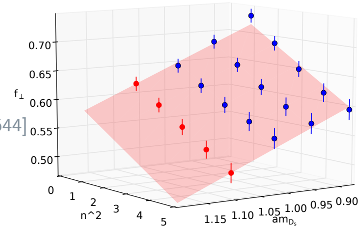



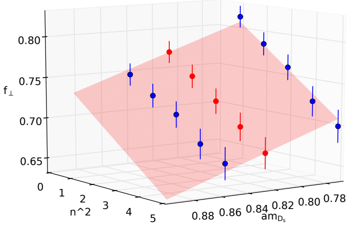



# Charm extra-/interpolation for $B_s \to D_s\ell\nu$

- Simulate charm quarks using DWF
  - → Similar action as for $u$, $d$, $s$ quarks
  - → "Fully" relativistic setup simplifies renormalization
  - → Established by calculating $f_{D_{(s)}}$ [Boyle et al. arXiv:1701.02644]
- Coarse ensembles
  - → Linearly extrapolate three charm-like masses
- Medium and fine ensembles
  - → Interpolate between two charm-like masses
- Analysis of data at third, finer lattice spacing will help to better estimate uncertainty




PRELIMINARY



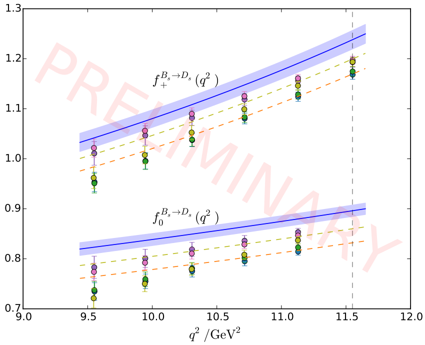

PRELIMINARY

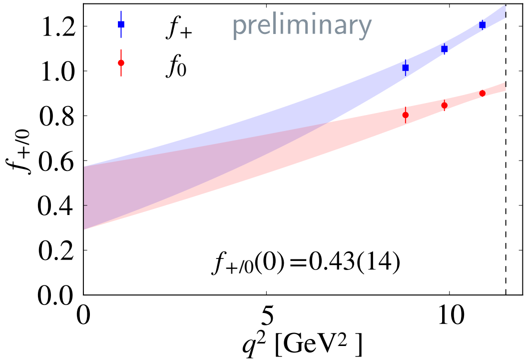
# Chiral-continuum extrapolation and first $z$-expansion

- No light valence quarks, no need for $\chi$PT

$$f(q,a) = \frac{c_0 + c_1(\Lambda_{\rm QCD}\, a)^2}{1 + c_2(q/M_{B_c})^2}$$

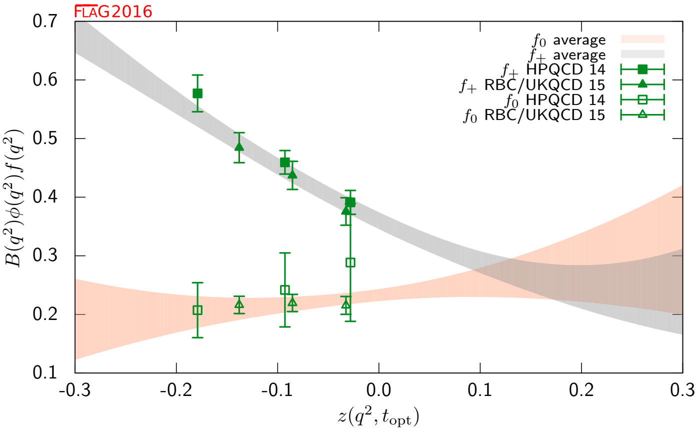
- Based on incomplete/preliminary error budget
- Expect improvement from additional data



# Flavor Lattice Averaging Group

# Lattice determinations of $B_s \to K\ell\nu$ form factors

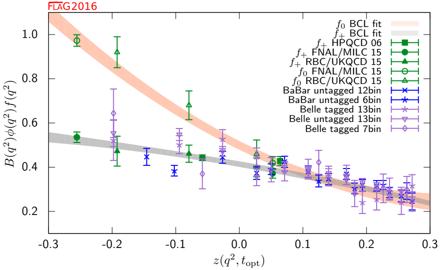


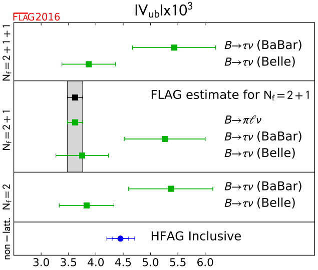

- Two independent calculations
  - → Different actions
  - → Different gauge field ensembles
  - ⇒ Entirely uncorrelated
- Combined analysis by FLAG

[FLAG2016]

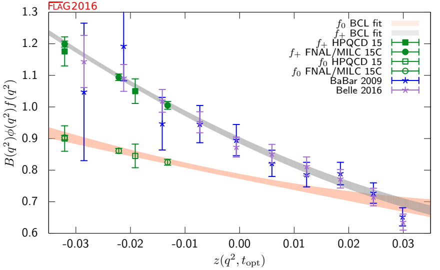
# Lattice determinations of $|V_{ub}|$ and $|V_{cb}|$

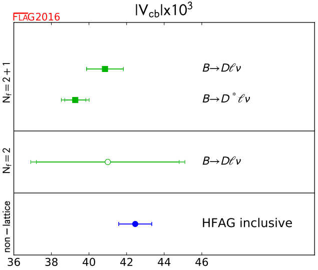
$B \to \pi\ell\nu$




$B \to D\ell\nu$




[FLAG2016]

# conclusion

# Conclusion

- In the final stages to complete $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ form factor calculation
  - → As usual, carefully estimating all systematic uncertainties is tedious
  - → Even requires additional simulations (currently running)
  - → Additional data at finer lattice spacing ready to be included

- Our lattice calculation also includes
  - → $B \to \pi\ell\nu$, $B \to \pi\ell^+\ell^-$
  - → $B \to K^*\ell^+\ell^-$
  - → $B \to D^{(*)}\ell\nu$
  - → $B_s \to K^*\ell^+\ell^-$
  - → $B_s \to D_s^*\ell\nu$
  - → $B_s \to \phi\ell^+\ell^-$
  - → ...

# Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)
RBC qcdcl (RIKEN) and cuth (Columbia U)
UK: ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)

# appendix

# 2+1 Flavor Domain-Wall Iwasaki ensembles

| L | $a^{-1}$(GeV) | $am_l$ | $am_s$ | $M_\pi$(MeV) | # configs. | #sources | |
|---|---|---|---|---|---|---|---|
| 24 | 1.784 | 0.005 | 0.040 | 338 | 1636 | 1 | [PRD 78 (2008) 114509] |
| 24 | 1.784 | 0.010 | 0.040 | 434 | 1419 | 1 | [PRD 78 (2008) 114509] |
| 32 | 2.383 | 0.004 | 0.030 | 301 | 628 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.006 | 0.030 | 362 | 889 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.008 | 0.030 | 411 | 544 | 2 | [PRD 83 (2011) 074508] |
| 48 | 1.730 | 0.00078 | 0.0362 | 139 | 40 | 81/1* | [PRD 93 (2016) 074505] |
| 64 | 2.359 | 0.000678 | 0.02661 | 139 | — | — | [PRD 93 (2016) 074505] |
| 48 | 2.774 | 0.002144 | 0.02144 | 234 | 70 | 24 | [arXiv:1701.02644] |

* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- $a$: $\sim 0.11$ fm, $\sim 0.08$ fm, $\sim 0.07$ fm