# Semi-leptonic $B$ decays

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# RBC- and UKQCD collaborations [Lattice 2016]


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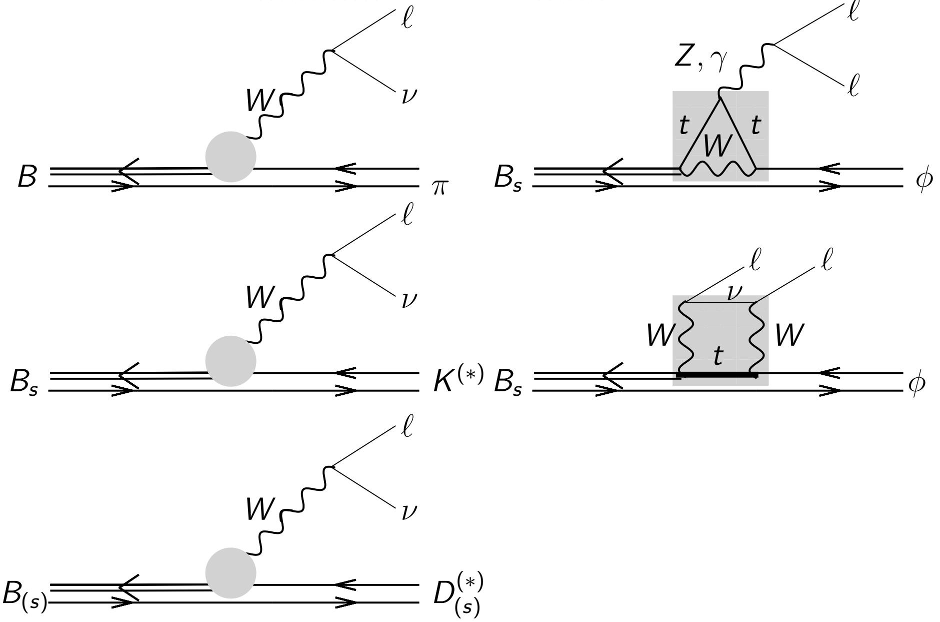
Renwick Hudspith

+ Ruth Van de Water (Fermilab)

# introduction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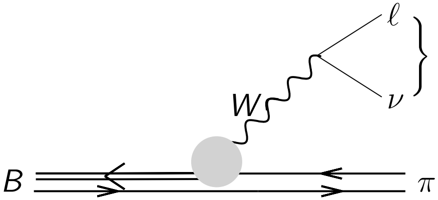
# flavor changing charged currents

(tree-level in the Standard Model)

# Example: $|V_{ub}|$ from exclusive semileptonic $B \to \pi \ell \nu$ decay




$$q^2 = M_B^2 + M_\pi^2 - 2M_B E_\pi$$

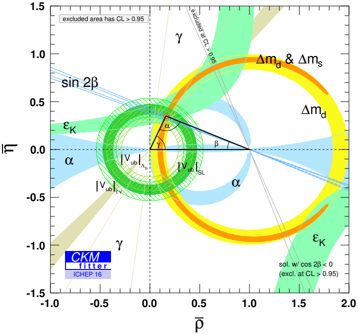
- Conventionally parametrized by

$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2 + M_\pi^2 - q^2\right)^2 - 4M_B^2 M_\pi^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment known nonperturbative input CKM

# Semi-leptonic decays: $|V_{ub}|$

- $|V_{ub}|$ is another constrain of the apex of the CKM unitarity triangle
- Longstanding $2-3\sigma$ discrepancy between exclusive $(B \to \pi\ell\nu)$ and inclusive $(B \to X_u\ell\nu)$ measurements
- Alternative, exclusive $(\Lambda_b \to p\ell\nu)$ determination
  [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- $B \to \tau\nu$ has larger error
- $B_s \to K\ell\nu$ not (yet) measured




[http://ckmfitter.in2p3.fr]

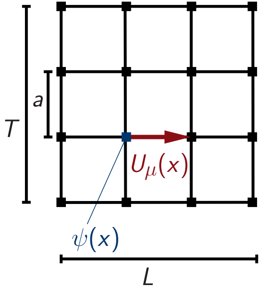
# Nonperturbative input: form factors

Form factors:

- Parametrize interactions due to the (nonperturbative) strong force
  $\rightsquigarrow$ Lattice QCD calculation

# Lattice QCD

- Wick rotation of Minkowski to Euclidean time
- Discretize space-time on a 4-d hypercube with extent $L^3 \times T$ and lattice spacing $a$ [fm] $\rightsquigarrow 1/a$ is the cutoff [GeV]
- Quark fields $\psi(x)$ live on the lattice sites, gauge fields $U_\mu(x)$ on the links




- Different discretizations for fermion (Wilson, Staggered, DWF, ...) and gauge actions (Wilson plaquette, Iwasaki, Symanzik, ...)
- Numerically solve path integral using Markov chain Monte Carlo simulations with importance sampling ($\rightsquigarrow$ supercomputers)

# Lattice QCD

Typical steps of a calculation:

1) Generate gauge field configurations containing the QCD vacuum with "light" sea-quarks and gluons
   - $\rightsquigarrow$ Degenerate $u/d$ and $s$ quark: dynamical 2+1 flavor
   - $\rightsquigarrow$ $s$ quarks close to physical mass
   - $\rightsquigarrow$ $u/d$ quarks chirally extrapolated, now simulations at physical mass
   - $\rightsquigarrow$ Need experimental inputs to set quark masses, gauge coupling, $\theta$
2) Carry out valence quark measurements on gauge field configurations
3) Combine measurements on different ensembles, extrapolate to the continuum and physical quark masses
4) Match lattice calculation to $\overline{\text{MS}}$ scheme (renormalization)
5) Account for systematic effects

# Nonperturbative input: form factors

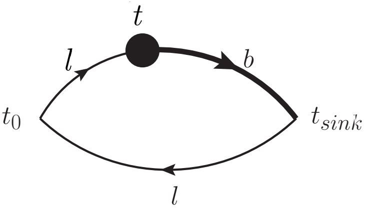
Form factors:

- Parametrize interactions due to the (nonperturbative) strong force
  ⇝ Lattice QCD calculation
- Predominantly short distance
  ⇝ Identify by operator product expansion (OPE)
  ⇝ Implement flavor changing currents as point-like operators
- Depend on the momentum transfer ($q^2$)
  ⇝ Lattice most precise at $q^2_{\mathrm{max}}$, experiment better at low $q^2$

# $B \to \pi \ell \nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current $V^\mu$ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left( p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$




- Calculate 3-point function by
  - → Inserting a quark source for a "light" propagator at $t_0$
  - → Allow it to propagate to $t_{sink}$, turn it into a sequential source for a $b$ quark
  - → Use another "light" quark propagating from $t_0$ and contract both at $t$

# Relating form factors $f_+$ and $f_0$ to $f_\parallel$ and $f_\perp$

- On the lattice we prefer using the $B$-meson rest frame and compute

$$f_\parallel(E_\pi) = \langle\pi|V^0|B\rangle/\sqrt{2M_B} \quad \text{and} \quad f_\perp(E_\pi)p_\pi^i = \langle\pi|V^i|B\rangle/\sqrt{2M_B}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2-M_\pi^2}\left[(M_B - E_\pi)f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2)f_\perp(E_\pi)\right]$$

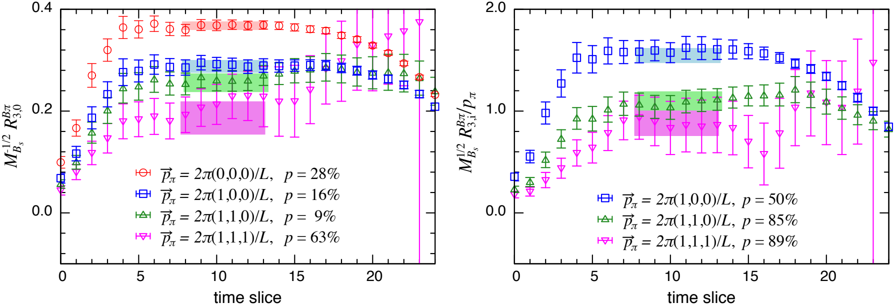
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(E_\pi) + (M_B - E_\pi)f_\perp(E_\pi)\right]$$

# Lattice results for form factors $f_\parallel$ and $f_\perp$ [PRD 91 (2015) 074510]

$$f_\parallel = \lim_{t, t_{\text{sink}} \to \infty} R_0^{B\to\pi}(t, t_{\text{sink}})$$

$$f_\perp = \lim_{t, t_{sink} \to \infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t, t_{\text{sink}})$$

$$R_\mu^{B\to\pi}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B\to\pi}(t, t_{\text{sink}})}{C_2^\pi(t) C_2^B(t_{\text{sink}} - t)} \sqrt{\frac{2E_\pi}{e^{-E_\pi t} e^{-M_B(t_{\text{sink}} - t)}}}$$

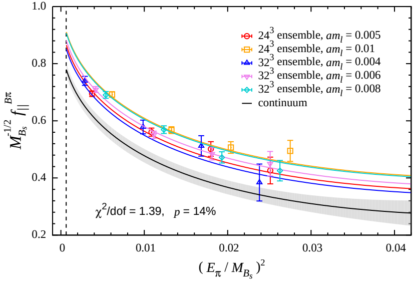


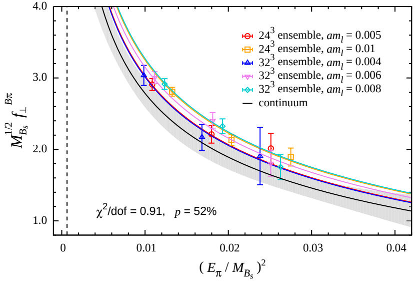
# Chiral-continuum extrapolation using SU(2) hard-pion $\chi$PT

$$f_{\parallel}(M_\pi, E_\pi, a^2) = c_{\parallel}^{(1)}\left[1 + \left(\frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)}\frac{M_\pi^2}{\Lambda^2} + c_{\parallel}^{(3)}\frac{E_\pi}{\Lambda} + c_{\parallel}^{(4)}\frac{E_\pi^2}{\Lambda^2} + c_{\parallel}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_{\perp}(M_\pi, E_\pi, a^2) = \frac{1}{E_\pi + \Delta} c_{\perp}^{(1)}\left[1 + \left(\frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)}\frac{M_\pi^2}{\Lambda^2} + c_{\perp}^{(3)}\frac{E_\pi}{\Lambda} + c_{\perp}^{(4)}\frac{E_\pi^2}{\Lambda^2} + c_{\perp}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

with $\delta f$ non-analytic logs of the pion mass and hard-pion limit is taken by $\frac{M_\pi}{E_\pi} \to 0$

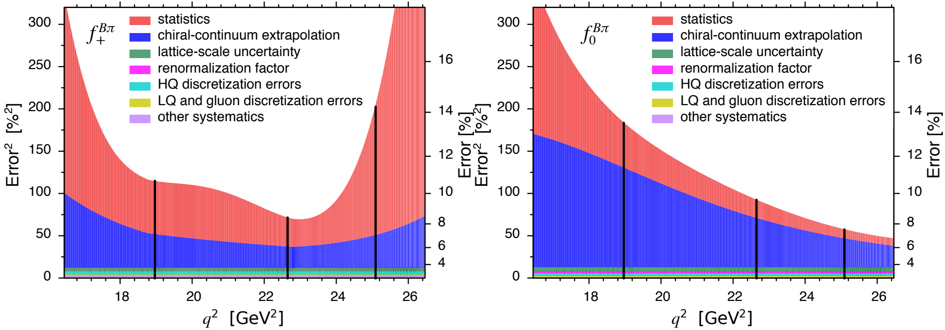



[PRD 91 (2015) 074510]

# Obtaining form factors $f_+$ and $f_0$ [PRD 91 (2015) 074510]

- Extract $f_\parallel$ and $f_\perp$ for three different $q^2$ values (synthetic data points)
- Estimate all systematic errors and them add in quadrature
- Convert results to $f_+$ and $f_0$



# $z$-expansion

- Use the model-independent $z$-expansion fit to extrapolate lattice results to the full kinematic range [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$$

with $t_\pm = (M_B \pm M_\pi)^2$ and $t_0 \equiv t_{\rm opt} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$

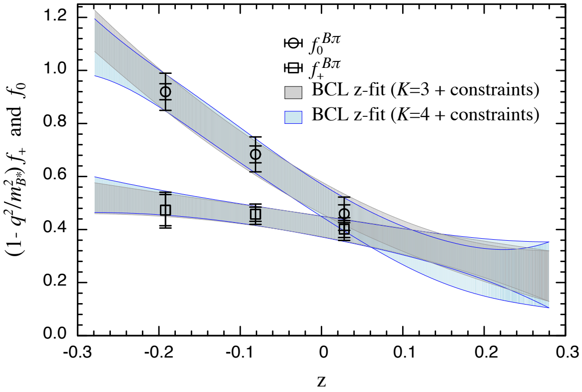
- Minimizes the magnitude of $z$ in the semi-leptonic region: $|z| \leq 0.279$
- $f_0(q^2)$ is analytic in the semi-leptonic region except at the $B^*$ pole
- $f_+(q^2)$ can be expressed as convergent power series

$$f_+(q^2) = \frac{1}{1-q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[ z^k - (-1)^{k-K} \frac{k}{K} z^k \right]$$

and use for $f_0(q^2)$ the functional form $f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$

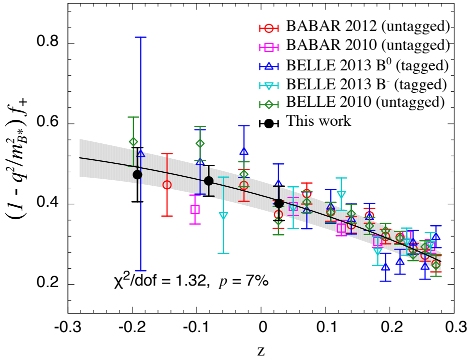
- Exploit the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$ and use HQ power counting to constrain the size of the $f_+$ coefficients

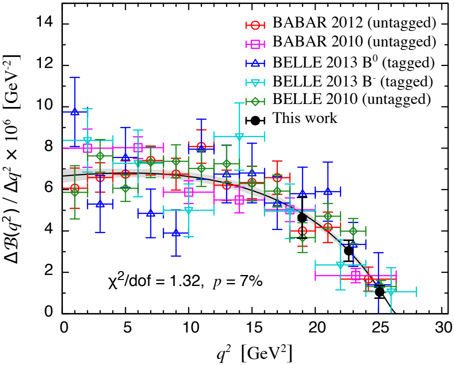
# $z$-expansion fit [PRD 91 (2015) 074510]



# Combine with experimental data to determine $|V_{ub}|$

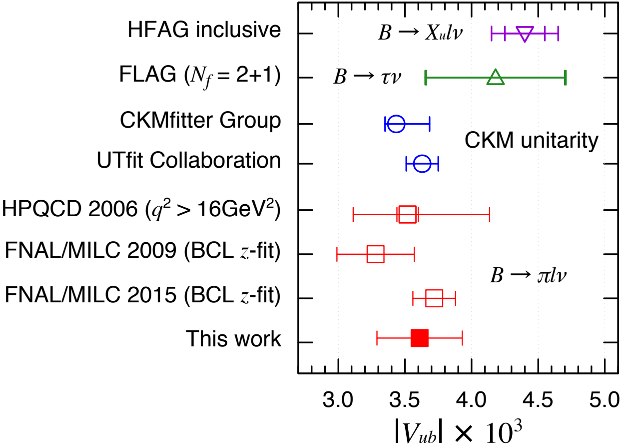
[PRD 91 (2015) 074510]




▶ Result: $|V_{ub}| = 3.61(32) \cdot 10^{-3}$

# Comparison with other determinations



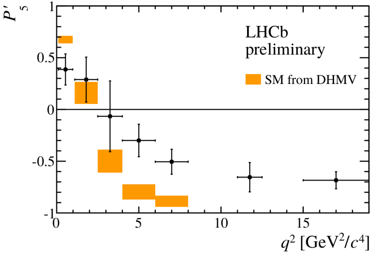

- In good agreement with existing and new FNAL/MILC result
- Result agrees with value obtained CKM unitarity
- Exhibits $2\sigma$ tension to inclusive results

# flavor changing neutral currents

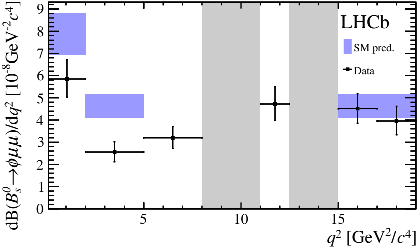
(loop-level in the Standard Model)

# Rare $B$ decays (FCNC)

- GIM suppressed in the Standard Model $\Rightarrow$ sensitive to new physics
- Angular observable $P_5'$ in $B \to K^* \mu^+ \mu^-$ received a lot of attention



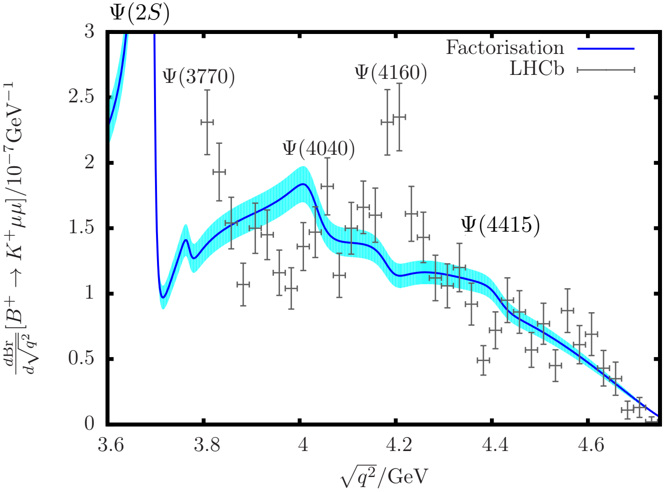

[LHCb-CONF-2015-002]



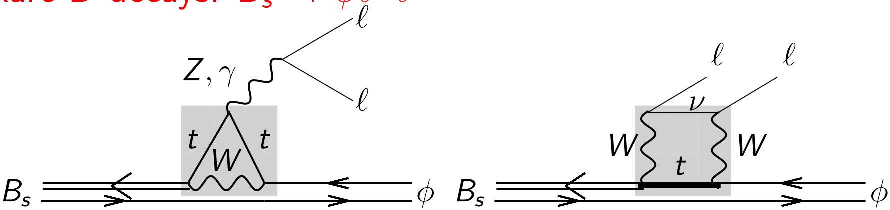

[LHCb JHEP 1509 (2015) 179]

- Lattice QCD: [Horgan et al. PRD 89 (2013) 094501]

- Charm resonances under control? [Lyon and Zwicky, arXiv:1406.0566]



# Rare $B$ decays: $B_s \to \phi\ell^+\ell^-$




- Pseudoscalar or vector final state (narrow width approximation)
- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(\prime)}$$

- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$
$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \ell$$
$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \bar{\ell}\gamma_\mu \gamma^5 \ell$$

# Seven form factors

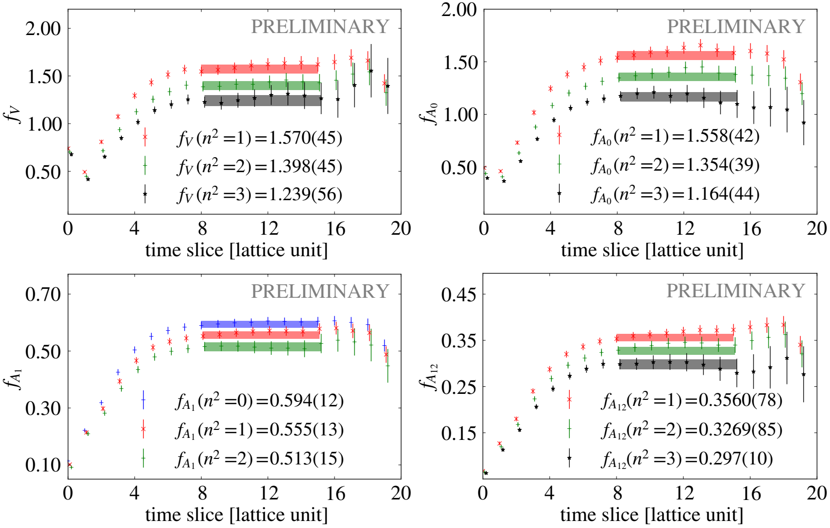
$$\langle \phi(k,\lambda)|\bar{s}\gamma^\mu b|B_s(p)\rangle = f_\phi(q^2)\frac{2i\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_{B_s}+M_\phi}$$

$$\begin{aligned}\langle \phi(k,\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B_s(p)\rangle &= f_{A_0}(q^2)\frac{2M_\phi\varepsilon^*\cdot q}{q^2}q^\mu \\ &+ f_{A_1}(q^2)(M_{B_s}+M_\phi)\left[\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ &- f_{A_2}(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_\phi}\left[k^\mu + p^\mu - \frac{M_{B_s}^2-M_\phi^2}{q^2}q^\mu\right]\end{aligned}$$

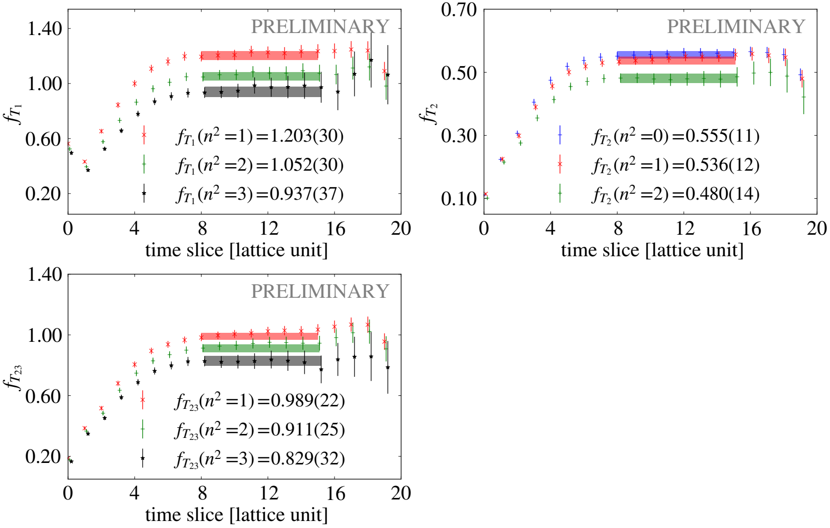
$$q_\nu\langle \phi(k,\lambda)|\bar{s}\sigma^{\nu\mu} b|B_s(p)\rangle = 2f_{T_1}(q^2)\epsilon^{\mu\rho\tau\sigma}\varepsilon^*_\rho k_\tau p_\sigma\ ,$$

$$\begin{aligned}q_\nu\langle \phi(k,\lambda)|\bar{s}\sigma^{\nu\mu}\gamma^5 b|B_s(p)\rangle &= if_{T_2}(q^2)\left[\varepsilon^{*\mu}(M_{B_s}^2-M_\phi^2) - (\varepsilon^*\cdot q)(p+k)^\mu\right] \\ &+ if_{T_3}(q^2)(\varepsilon^*\cdot q)\left[q^\mu - \frac{q^2}{M_{B_s}^2-M_\phi^2}(p+k)^\mu\right]\end{aligned}$$

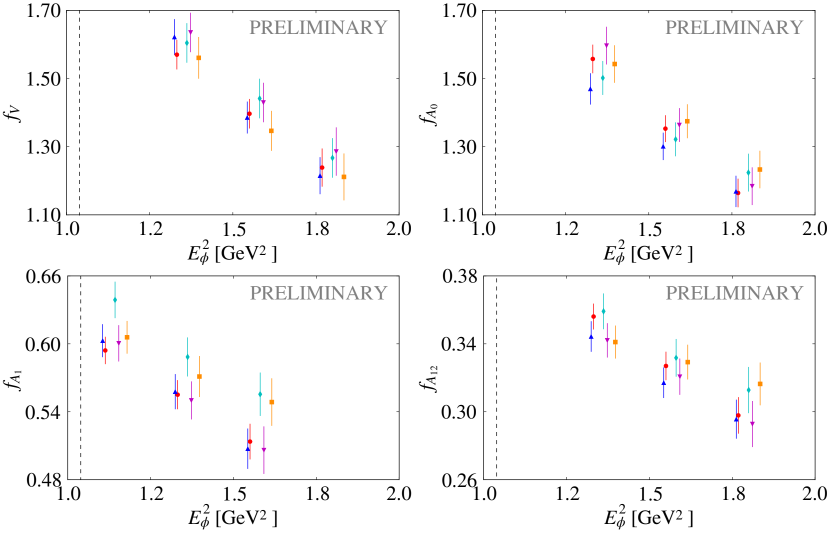
# Seven form factors [PoS(LATTICE2016)296]



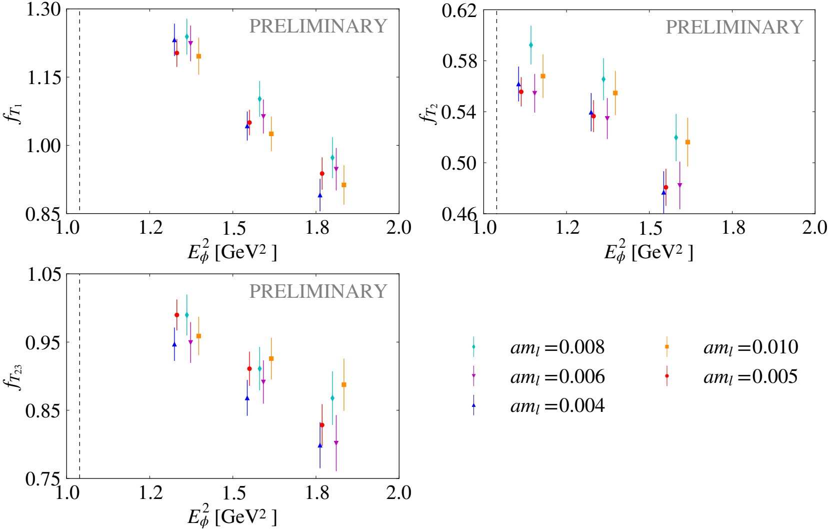
# Seven form factors [PoS(LATTICE2016)296]



# Seven form factors vs. $q^2$ [PoS(LATTICE2016)296]



# Seven form factors vs. $q^2$ [PoS(LATTICE2016)296]



# Conclusion

- Computational setup tested and verified
- Complete calculations for $B \to \pi\ell\nu$ and $B_s \to K\ell\nu$
  - $\to$ Dominant uncertainty: chiral- and continuum extrapolation
  - $\to$ Improvement in progress: 1) new simulation with physical light quarks
    2) new simulation at finer lattice spacing

- Additional operators for FCNC and (stable) vector final state implemented
- Numerical results for $B_s \to \phi\ell^+\ell^-$, extrapolation in progress
- We have more data: $B \to K^*\ell^+\ell^-$, ...

- Also determination of $B \to D\ell\nu$ and $B_s \to D_s\ell\nu$ in progress

# Appendix

# 2+1 Flavor Domain-Wall Iwasaki ensembles

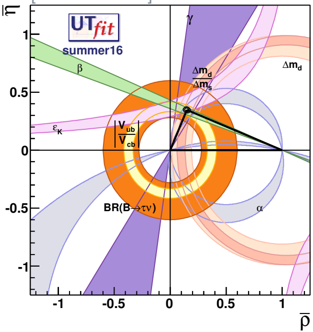
| L | $a^{-1}$(GeV) | $am_l$ | $am_s$ | $M_\pi$(MeV) | # configs. | #sources | |
|---|---|---|---|---|---|---|---|
| 24 | 1.784 | 0.005 | 0.040 | 338 | 1636 | 1 | [PRD 78 (2008) 114509] |
| 24 | 1.784 | 0.010 | 0.040 | 434 | 1419 | 1 | [PRD 78 (2008) 114509] |
| 32 | 2.383 | 0.004 | 0.030 | 301 | 628 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.006 | 0.030 | 362 | 889 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.008 | 0.030 | 411 | 544 | 2 | [PRD 83 (2011) 074508] |
| 48 | 1.730 | 0.00078 | 0.0362 | 139 | 40 | 81/1* | [PRD 93 (2016) 074505] |
| 64 | 2.359 | 0.000678 | 0.02661 | 139 | — | — | [PRD 93 (2016) 074505] |
| 48 | ∼2.7 | 0.002144 | 0.02144 | ∼250 | > 50 | 24 | [in progress] |

* All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. PRD 88 (2012) 094503]

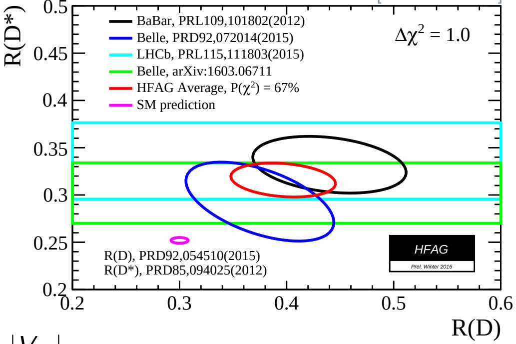
- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- $a$: $\sim 0.11$ fm, $\sim 0.08$ fm, $\sim 0.07$ fm

# $B \to D\ell\nu$ and $B \to D^*\ell\nu$

[UTfit website]




[HFAG website]




- Determine CKM matrix element $|V_{cb}|$
- Ratio of branching fractions

$$\mathcal{R}^{\tau/\mu}_{D^{(*)}} \equiv \frac{d\Gamma(B \to D^{(*)}\tau\nu_\tau)/d_q^2}{d\Gamma(B \to D^{(*)}\mu\nu_\mu)/d_q^2}$$

# Update: $B \to D\ell\nu$

- Two lattice form factor calculations with $q^2 \leq q^2_{\text{max}}$
  [Fermilab/MILC PRD92 (2015) 035606] [HPQCD PRD92 (2015) 054510]
- New more precise Belle measurement [Belle PRD93 (2016) 032006]

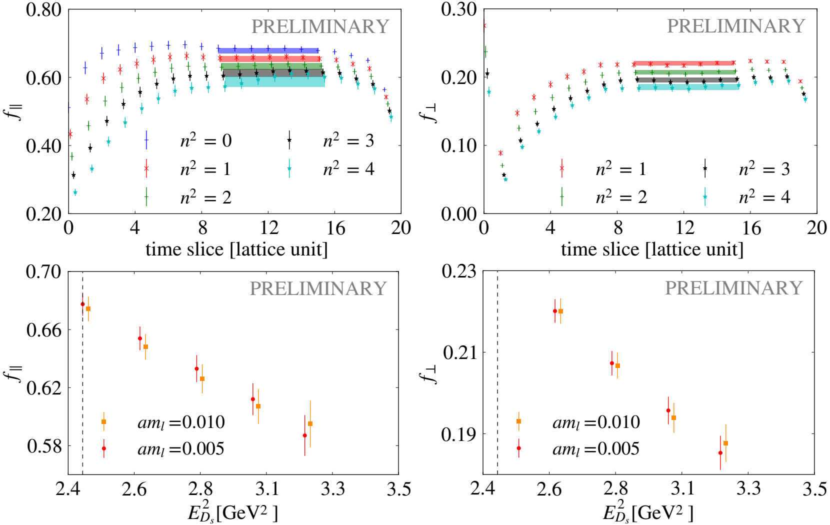
- Result of new analysis [Bigi and Gambino, arXiv:1606.08030]

$$|V_{cb}|^{\text{incl}} = 42.00(65) \cdot 10^{-3} \text{ vs. } |V_{cb}|^{\text{excl}} = 40.49(97) \cdot 10^{-3} \sim 1.5\sigma$$

$$R_D^{\text{exp}} = 0.397(49) \text{ vs. } R_D^{\text{SM}} = 0.299(3) \sim 2.0\sigma$$

# Our first results for $B_s \to D_s \ell \nu$

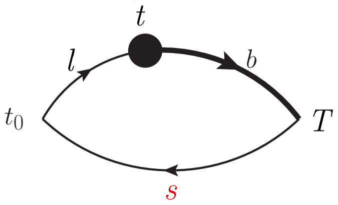
[PoS(LATTICE2016)296]



# $B_s \to K\ell\nu$

[PRD 91 (2015) 074510]

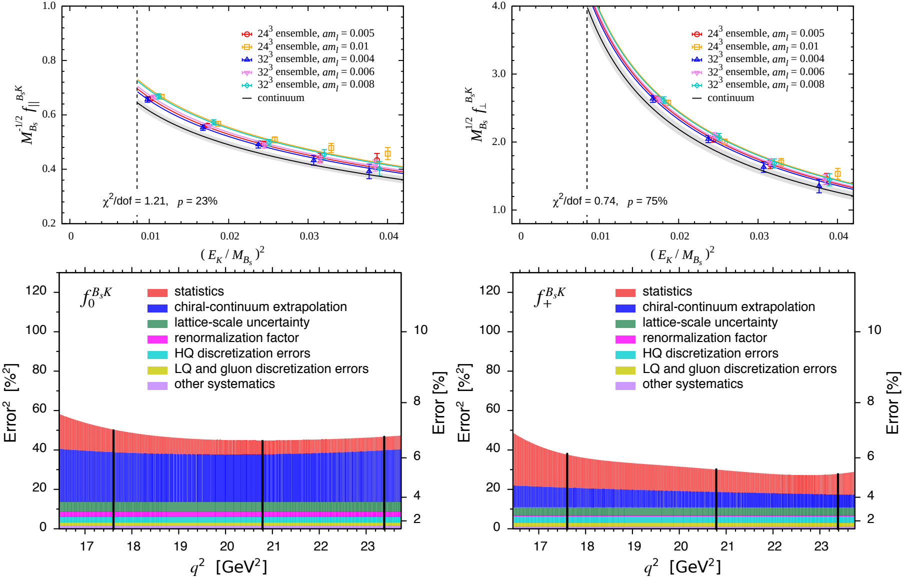
- Lattice calculation: replace light spectator quark with $s$-quark




- Chiral-continuum extrapolation is similar but pole masses change
- Smaller statistical and extrapolation errors
- Perform $z$-expansion
- Experimental results for $B_s \to K\ell\nu$ not (yet) available
- Can make phenomenological predictions to be compared with future measurements

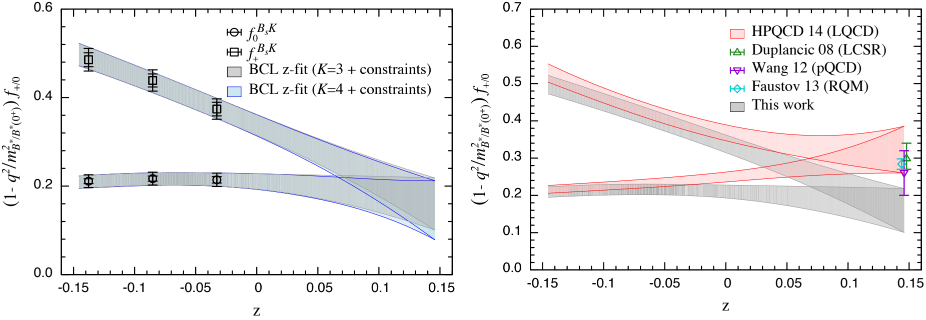
# Chiral-continuum extrapolation for $B_s \to K\ell\nu$

[PRD 91 (2015) 074510]



# $z$-expansion fit for $B_s \to K\ell\nu$ and comparisons

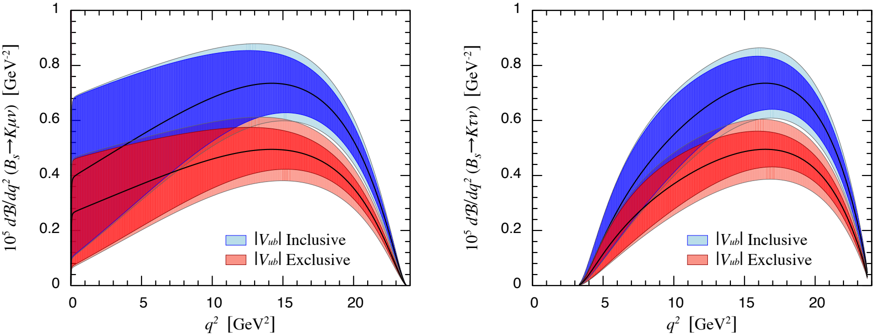
[PRD 91 (2015) 074510]




- [Bouchard et al., PRD 90 (2014) 054506]
- [Duplancic and Melic, PRD 78 (2008) 054015]
- [Faustov and Galkin, PRD 87 (2013) 094028]
- [Wang and Xiao, PRD 86 (2012) 114025]

# Phenomenological prediction

[PRD 91 (2015) 074510]




- Using our value for $|V_{ub}|$ we can make predictions for the $B_s \to K\ell\nu$ differential branching fraction for $\ell = \mu, \tau$
- Given an experimental measurement of branching fractions at $q^2 \gtrsim 13$ GeV one may distinguish between curves corresponding to $|V_{ub}|_{\text{excl}}$ and $|V_{ub}|_{\text{incl}}$