B-physics with dynamical domain-wall light quarks and relativistic b-quarks

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Determination of CKM Matrix Elements

- \( B - \bar{B} \)-mixing allows us to determine CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

\[
\begin{align*}
\left| V_{td}^* V_{tb} \right| & \text{ for } B_d - \text{mixing} \\
\left| V_{ts}^* V_{tb} \right| & \text{ for } B_s - \text{mixing}
\end{align*}
\]

\[
\Delta m_q = \frac{G_F m_W^2}{6\pi^2} \eta_B S_0 m_B q f_{B_q}^2 B_{B_q} \left| V_{ts}^* V_{tb} \right|^2
\]

- Non-perturbative contribution: \( f_q^2 B_{B_q} \)
- Define the \( SU(3) \) breaking ratio \( \xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d} \)

- CKM matrix elements are extracted by

\[
\begin{align*}
\frac{\Delta m_s}{\Delta m_d} &= \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| V_{ts} \right|^2 \\
\frac{\Delta m_s}{\Delta m_d} &= \frac{m_{B_d}}{m_{B_s}} \xi^2 \left| V_{td} \right|^2
\end{align*}
\]
Constraining the CKM Unitarity Triangle

- The apex of the unitarity triangle is constrained by the ratio of $B_s$ to $B_d$ oscillation frequencies ($\Delta m_q$)
- $\Delta m_q$ is experimentally measured to better than a percent [BABAR, Belle, CDF]
- Dominant error comes from the uncertainty on the lattice QCD calculation of the ratio $\xi$ ($\sim 3\%$)
- A precise determination is needed to help constrain physics beyond the Standard Model
A unitarity fit without $V_{ub}$ or $V_{cb}$ is possible [Lunghi and Soni 2009]
Avoids 1-2 $\sigma$ tension between inclusive and exclusive determinations of both $V_{ub}$ and $V_{cb}$
Requires precise determination of $f_B$ (and also of $B \rightarrow \tau \nu$ and $\Delta M_s$)
Lattice Calculations of $B$-meson Parameters

- HPQCD and FNAL-MILC result both based on the asqtad-improved staggered ensembles generated by MILC

- RBC/UKQCD result only exploratory study computed on $16^3$ domain-wall fermion lattices and using static approximation for the $b$-quarks
Our Current $B$-Physics Projects

- Computation of $B - \bar{B}$-mixing and $B$-meson decay constants in the static limit [Talk by Y. Aoki, next]

- Tuning parameters for the relativistic heavy quark action ($32^3$) [Talk by H. Peng, Thu, 17:20]

- Determining the $B^* B \pi$ coupling using a relativistic heavy quark action [Talk by P. Fritzsch, Tue, 9:30]

- Computation of $B - \bar{B}$-mixing and $B$-meson decay constants using a relativistic heavy quark action
Light Quark and Gluon Action

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992 and Shamir 1993]
  - Five dimensional formulation with an approximate chiral symmetry
  - Left-handed modes are bound to 4-d brane at $s = 0$,
    right-handed modes to a 4-d brane at $s = L_s - 1$
  - Overlap exponentially suppressed
  - Renormalization simplified due to reduced operator mixing

\[ s = 0 \quad \text{to} \quad s = L_s - 1 \]

- Iwasaki gauge action [Iwasaki 1983]
  - Improves chiral symmetry and reduces residual quark mass when combined with domain-wall sea quarks [Y. Aoki et al. 2004]
2+1 Flavor Domain-Wall Gauge Field Configurations

<table>
<thead>
<tr>
<th>( L )</th>
<th>( a(\text{fm}) )</th>
<th>( m_l )</th>
<th>( m_s )</th>
<th>( m_\pi(\text{MeV}) )</th>
<th># configs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>( \approx 0.11 )</td>
<td>0.005</td>
<td>0.040</td>
<td>331</td>
<td>1640</td>
</tr>
<tr>
<td>24</td>
<td>( \approx 0.11 )</td>
<td>0.010</td>
<td>0.040</td>
<td>419</td>
<td>1420</td>
</tr>
<tr>
<td>24</td>
<td>( \approx 0.11 )</td>
<td>0.020</td>
<td>0.040</td>
<td>558</td>
<td>350</td>
</tr>
<tr>
<td>32</td>
<td>( \approx 0.08 )</td>
<td>0.004</td>
<td>0.030</td>
<td>307</td>
<td>600</td>
</tr>
<tr>
<td>32</td>
<td>( \approx 0.08 )</td>
<td>0.006</td>
<td>0.030</td>
<td>366</td>
<td>900</td>
</tr>
<tr>
<td>32</td>
<td>( \approx 0.08 )</td>
<td>0.008</td>
<td>0.030</td>
<td>418</td>
<td>550</td>
</tr>
</tbody>
</table>

[C. Allton et al. 2008, RBC/UKQCD in preparation]
Relativistic Heavy Quark Action for the $b$-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the $b$-quarks in 2-point and 3-point correlation functions [Christ, Li, Lin 2007; Lin and Christ 2007]

- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie 1997] (see also [Aoki, Kuramashi, Tominaga 2003])

- Parameters of the clover action are tuned non-perturbatively using the spin-averaged mass and the hyperfine-splitting for $B_s$ mesons as well as the ratio $m_{\text{rest}}/m_{\text{kinetic}}$

- Once parameters are tuned for the heavy-light system, computations of the heavy-heavy system can be used to test the method

- RHQ action applicable for $c$-quarks, where calculations of leptonic decay constants $f_D$ and $f_{D_s}$ allow further checks of the method
Tuning the Parameters for the RHQ Action

\[ S = \sum_{n,n'} \tilde{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{a D_0^2}{2} + \zeta \left[ \vec{\gamma} \cdot D - \frac{a (\vec{D})^2}{2} \right] - a \sum_{\mu\nu} \frac{i c_P}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\} \psi_{n'} \]

- Start from an educated guess for \((m_0 a, c_P, \zeta)\)
- Compute
  - spin-averaged mass \((m_{B_s} + 3m_{B_s^*})/4\)
  - hyperfine-splitting \((m_{B_s^*} - m_{B_s})\)
  - ratio \(m_{B_s^{\text{rest}}}/m_{B_s^{\text{kinetic}}}\) or \(m_{\gamma^{\text{rest}}}/m_{\gamma^{\text{kinetic}}}\)
- Iterate until agreement with [PDG]
  - spin-averaged mass 5403.1(1.1) MeV
  - hyperfine-splitting 49.0(1.5) MeV
  - ratio equals 1
- Chiral value on \(24^3\) \((a = 0.11\text{fm})\):
  \((m_0 a, c_P, \zeta) = (7.38(11), 3.89(49), 4.19(4))\) [M. Li 2009]
First Results for $m_B$ and $f_B$ on $24^3$ ($a \approx 0.11\text{fm}$)

- Computation of $m_B$ is a “prediction”
- Simplest test of the parameter tuning
- Statistical errors are small: $m_B$: 0.08% - 0.13% and $\Phi_B$: 1.1% - 2.0%
- Result for $f_B$ is multiplicatively renormalized (1-loop)

[Yamada et al. 2005] but not $\mathcal{O}(a)$ improved
Improving the Signal by Smearing of Source and Sink

▶ Reduction of excited state contamination

- Pt-Pt: $3.0722(69)$
- Sm-Sm: $3.0625(52)$ [$r_{bb}^{\text{rms}} = 0.224(23)\text{fm}$]
- Sm-Sm: $3.0564(54)$ [$r_{cc}^{\text{rms}} = 0.423(47)\text{fm}$]
Dependence on RHQ Parameters

- Decay amplitude computed on the $m_l = 0.005$ ensemble
- Varying each of the RHQ parameters by its statistical uncertainty
- No change within statistical uncertainties (of point-point data)
- Systematic uncertainty in RHQ parameters not yet estimated
- Probably a few percent uncertainty in $f_B$ due to RHQ input parameters expected
Discretization Errors for Relativistic Heavy Quarks

- Matching of lattice action to continuum through $O(pa)$
- Errors are of $O(a^2 p^2)$
- Heavy quark mass is treated to all orders in $m_b a$
  $\Rightarrow$ coefficient of the $O(a^2 p^2)$ error is a function of $m_b a$
- This function is bounded to be $\leq O(1)$
  [El Khadra, Kronfeld, Mackenzie 1997]
- Improve heavy-light current by rotating of $b$-quark; rotation parameter $d_1$ is computed at tree-level in tadpole-improved lattice PT
- Heavy-light spectrum quantities can be computed with discretization errors of the same order as in light-light quantities
Further Uncertainties

Uncertainty in determination of $s$-quark mass
Controlled linear interpolation between two data points in the valence sector; sea-quark dependence expected to be small

Renormalization factors
Needed for matching lattice operator to continuum operator; computation will use 1-loop tadpole-improved lattice PT [Yamada et al. 2005]

Chiral extrapolation
Performed using additional partially quenched data and heavy-light meson $\chi$PT

Continuum extrapolation
Use two different lattice spacings
$B^0 - \bar{B}^0$ mixing matrix element calculation

- Location of four-quark operator is fixed
- Location of $B$-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating from operator location
- Propagators can be used for $B$- and $\bar{B}$-meson
- Project out zero-momentum component using a Gaussian sink
- Generation of light quark propagators finished to more than 50%
- Computation of $\xi = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$ should be most reliable
### Tentative Error Budget

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>$f_B$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>statistics</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>chiral extrapolation</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>uncertainty in $g_{B^* B \pi}$</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>renormalization factors</td>
<td>5%</td>
<td>2%</td>
</tr>
<tr>
<td>scale and quark mass uncertainties</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>finite volume error</td>
<td>1%</td>
<td>0.5%</td>
</tr>
<tr>
<td>(heavy-quark) discretization</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7%</td>
<td>4%</td>
</tr>
</tbody>
</table>

- Conservative estimate based on comparison with static result and the work of other collaborations — hopefully we do even better
- Expect competitive results to [FNAL-MILC 2008] and [HPQCD 2009]
Conclusion

- This project aims for a precise determination of neutral $B$-meson mixing parameters and decay constants $f_{B_d}$, $f_{B_s}$

- Results will place an important constraint in the quark flavor sector when used in unitarity triangle analysis

- Work in progress and we expect to have preliminary results for $f_{B_d}$, $f_{B_s}$ and $\xi$ soon