B-physics with dynamical domain-wall light quarks and relativistic b-quarks

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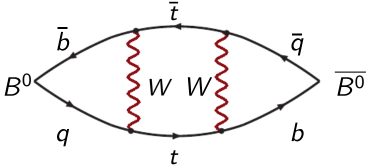
Determination of CKM Matrix Elements

- $B - \bar{B}$-mixing allows us to determine CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

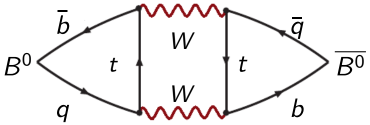
$$\left.\begin{array}{r} |V_{td}^* V_{tb}| \text{ for } B_d\text{−mixing} \\ |V_{ts}^* V_{tb}| \text{ for } B_s\text{−mixing} \end{array}\right\} \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

- Non-perturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$



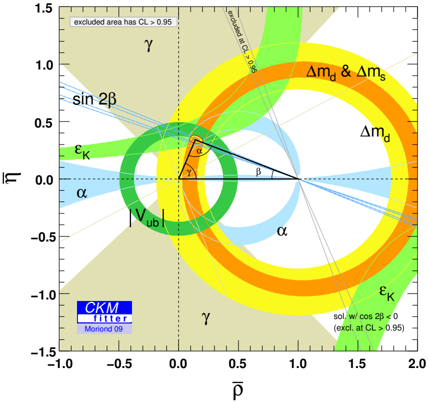
- CKM matrix elements are extracted by

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



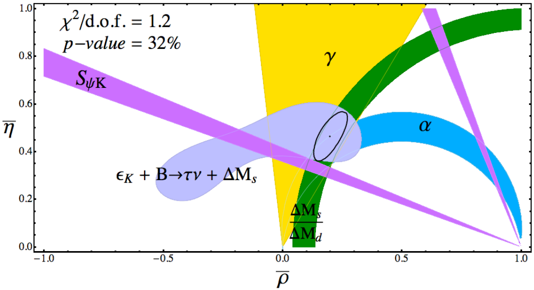
Constraining the CKM Unitarity Triangle

- The apex of the unitarity triangle is constrained by the ratio of B_s to B_d oscillation frequencies (Δm_q)
- Δm_q is experimentally measured to better than a percent [BABAR, Belle, CDF]
- Dominant error comes from the uncertainty on the lattice QCD calculation of the ratio ξ ($\sim 3\%$)
- A precise determination is needed to help constrain physics beyond the Standard Model

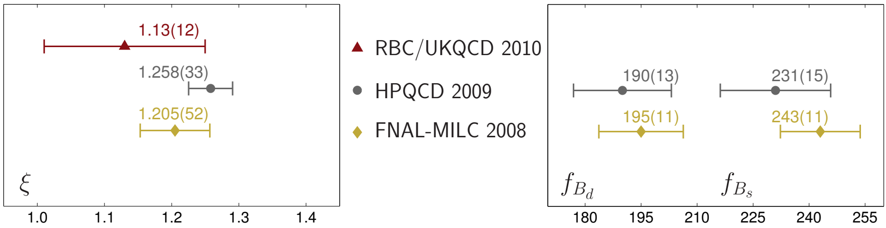


Unitarity Fit without Semileptonic Decays

- A unitarity fit without V_{ub} or V_{cb} is possible [Lunghi and Soni 2009]
- Avoids 1-2 σ tension between inclusive and exclusive determinations of both V_{ub} and V_{cb}
- Requires precise determination of f_B (and also of $B \to \tau\nu$ and ΔM_s)



Lattice Calculations of *B*-meson Parameters



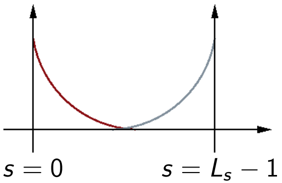
- HPQCD and FNAL-MILC result both based on the asqtad-improved staggered ensembles generated by MILC
- RBC/UKQCD result only exploratory study computed on 16^3 domain-wall fermion lattices and using static approximation for the *b*-quarks

Our Current B-Physics Projects

- Computation of $B-\bar{B}$-mixing and B-meson decay constants in the static limit [Talk by Y. Aoki, next]
- Tuning parameters for the relativistic heavy quark action (32^3) [Talk by H. Peng, Thu, 17:20]
- Determining the $B^*B\pi$ coupling using a relativistic heavy quark action [Talk by P. Fritzsch, Tue, 9:30]
- Computation of $B-\bar{B}$-mixing and B-meson decay constants using a relativistic heavy quark action

Light Quark and Gluon Action

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992 and Shamir 1993]
  - Five dimensional formulation with an approximate chiral symmetry
  - Left-handed modes are bound to 4-d brane at $s = 0$, right-handed modes to a 4-d brane at $s = L_s - 1$
  - Overlap exponentially suppressed
  - Renormalization simplified due to reduced operator mixing



- Iwasaki gauge action [Iwasaki 1983]
  - Improves chiral symmetry and reduces residual quark mass when combined with domain-wall sea quarks [Y. Aoki et al. 2004]

2+1 Flavor Domain-Wall Gauge Field Configurations

L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.
24	≈ 0.11	0.005	0.040	331	1640
24	≈ 0.11	0.010	0.040	419	1420
24	≈ 0.11	0.020	0.040	558	350
32	≈ 0.08	0.004	0.030	307	600
32	≈ 0.08	0.006	0.030	366	900
32	≈ 0.08	0.008	0.030	418	550

[C. Allton et al. 2008, RBC/UKQCD in preparation]

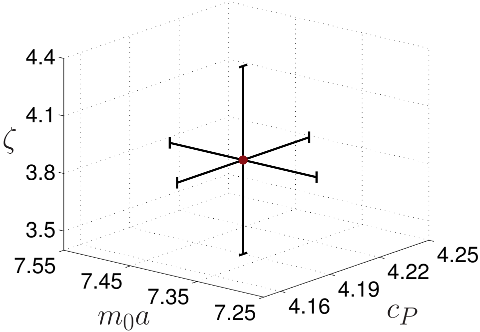
Relativistic Heavy Quark Action for the *b*-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the *b*-quarks in 2-point and 3-point correlation functions [Christ, Li, Lin 2007; Lin and Christ 2007]
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie 1997] (see also [Aoki, Kuramashi, Tominaga 2003])
- Parameters of the clover action are tuned non-perturbatively using the spin-averaged mass and the hyperfine-splitting for B_s mesons as well as the ratio $m_{\text{rest}}/m_{\text{kinetic}}$
- Once parameters are tuned for the heavy-light system, computations of the heavy-heavy system can be used to test the method
- RHQ action applicable for *c*-quarks, where calculations of leptonic decay constants f_D and f_{D_s} allow further checks of the method

Tuning the Parameters for the RHQ Action

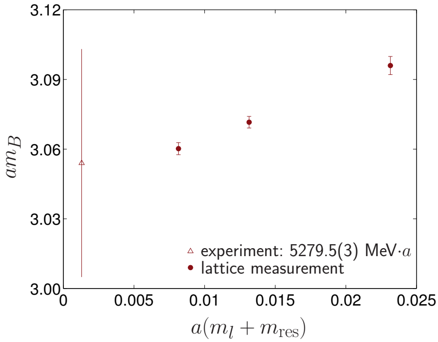
$$S = \sum_{n,n'} \bar{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a\left(\vec{D}\right)^2}{2} \right] - a \sum_{\mu\nu} \frac{i c_P}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \Psi_{n'}$$

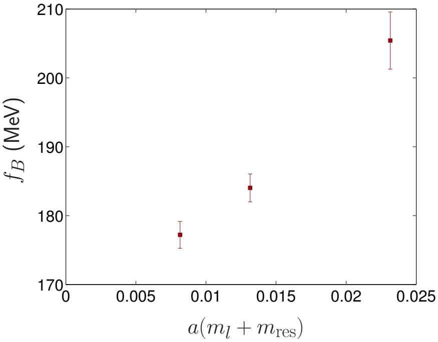
- Start from an educated guess for $(m_0 a,\ c_P,\ \zeta)$
- Compute
  spin-averaged mass $(m_{B_s} + 3m_{B_s^*})/4$
  hyperfine-splitting $(m_{B_s^*} - m_{B_s})$
  ratio $m_{B_s^{\text{rest}}}/m_{B_s^{\text{kinetic}}}$ or $m_{\Upsilon^{\text{rest}}}/m_{\Upsilon^{\text{kinetic}}}$
- Iterate until agreement with [PDG]
  spin-averaged mass 5403.1(1.1) MeV
  hyperfine-splitting 49.0(1.5) MeV
  ratio equals 1



- Chiral value on 24^3 (a = 0.11fm):
  $(m_0 a,\ c_P,\ \zeta) = (7.38(11),\ 3.89(49),\ 4.19(4))$ [M. Li 2009]

First Results for m_B and f_B on 24^3 ($a \approx 0.11$fm)

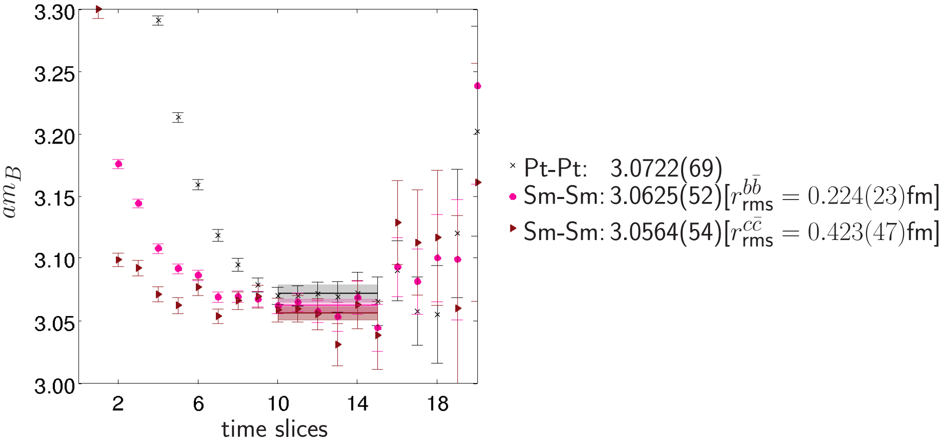




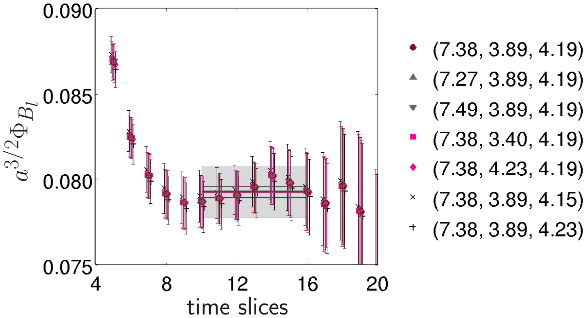
- Computation of m_B is a "prediction"
- Simplest test of the parameter tuning
- Statistical errors are small: m_B: 0.08% - 0.13% and Φ_B: 1.1% - 2.0%
- Result for f_B is multiplicatively renormalized (1-loop) [Yamada et al. 2005] but not $\mathcal{O}(a)$ improved

Improving the Signal by Smearing of Source and Sink

- Reduction of excited state contamination



Dependence on RHQ Parameters



- Decay amplitude computed on the $m_l = 0.005$ ensemble
- Varying each of the RHQ parameters by its statistical uncertainty
- No change within statistical uncertainties (of point-point data)
- Systematic uncertainty in RHQ parameters not yet estimated
- Probably a few percent uncertainty in f_B due to RHQ input parameters expected

Discretization Errors for Relativistic Heavy Quarks

- Matching of lattice action to continuum through $\mathcal{O}(pa)$
- Errors are of $\mathcal{O}(a^2 p^2)$
- Heavy quark mass is treated to all orders in $m_b a$
  $\Rightarrow$ coefficient of the $\mathcal{O}(a^2 p^2)$ error is a function of $m_b a$
- This function is bounded to be $\leq \mathcal{O}(1)$
  [El Khadra, Kronfeld, Mackenzie 1997]
- Improve heavy-light current by rotating of b-quark; rotation parameter d_1 is computed at tree-level in tadpole-improved lattice PT
- Heavy-light spectrum quantities can be computed with discretization errors of the same order as in light-light quantities

Further Uncertainties

Uncertainty in determination of s-quark mass

Controlled linear interpolation between two data points in the valence sector; sea-quark dependence expected to be small

Renormalization factors

Needed for matching lattice operator to continuum operator; computation will use 1-loop tadpole-improved lattice PT [Yamada et al. 2005]

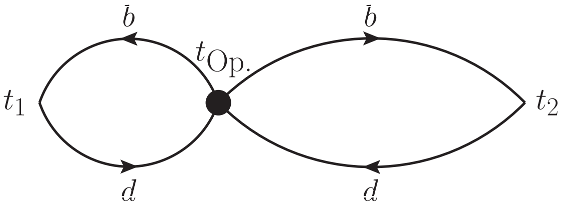
Chiral extrapolation

Performed using additional partially quenched data and heavy-light meson χPT

Continuum extrapolation

Use two different lattice spacings

$B^0 - \overline{B^0}$ mixing matrix element calculation



- Location of four-quark operator is fixed
- Location of B-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating form operator location
- Propagators can be used for B- and $\overline{B}$-meson
- Project out zero-momentum component using a Gaussian sink
- Generation of light quark propagators finished to more than 50%
- Computation of $\xi = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$ should be most reliable

Tentative Error Budget

	f_B	ξ
statistics	3%	3%
chiral extrapolation	3%	2%
uncertainty in $g_{B^*B\pi}$	1%	1%
renormalization factors	5%	2%
scale and quark mass uncertainties	2%	1%
finite volume error	1%	0.5%
(heavy-quark) discretization	2%	1%
total	7%	4%

- Conservative estimate based on comparison with static result and the work of other collaborations — hopefully we do even better
- Expect competitive results to [FNAL-MILC 2008] and [HPQCD 2009]

Conclusion

- This project aims for a precise determination of neutral B-meson mixing parameters and decay constants f_{B_d}, f_{B_s}
- Results will place an important constraint in the quark flavor sector when used in unitarity triangle analysis
- Work in progress and we expect to have preliminary results for f_{B_d}, f_{B_s} and ξ soon