Lattice 2012
Cairns, Australia

Cairns Convention Centre, Cairns, Australia
Sunday, June 24 — Friday, June 29

The $B \longrightarrow \pi$ form factor from domain-wall light quarks and relativistic b-quarks

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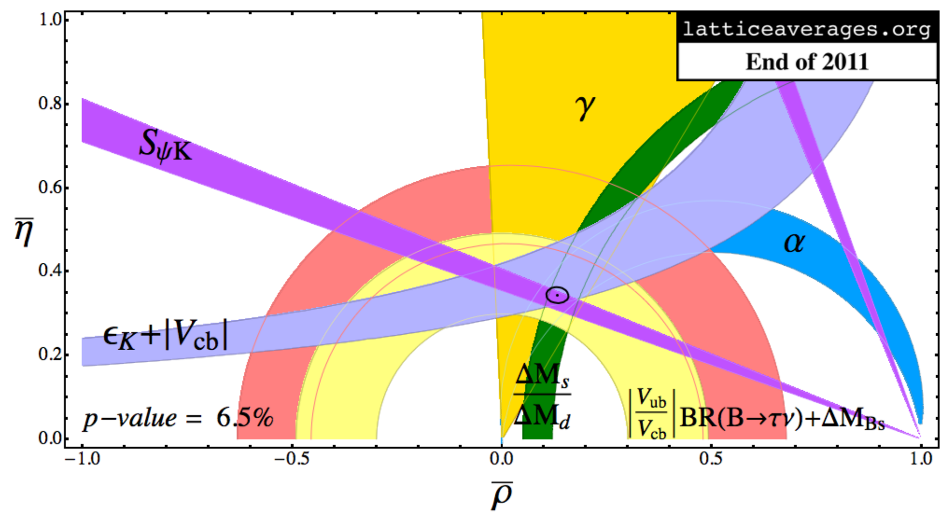
Why we compute the $B \to \pi$ form factor on the Lattice

A precise determination of V_{ub} allows a strong test of the standard model

The constraint on the apex $(\bar{\rho},\ \bar{\eta})$ of the CKM triangle from $|V_{ub}|$ will strengthen tests of the Standard-Model CKM framework.

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2+\bar{\eta}^2}$$

- $\lambda = |V_{ub}|$ known to $\sim 1\ \%$
- $|V_{cb}|$ known to $\sim 2\ \%$



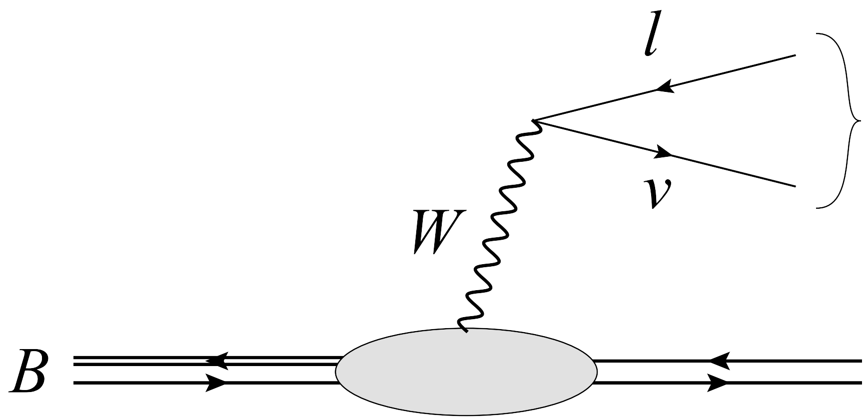
Dominant error (yellow ring) comes from the uncertainty of $|V_{ub}|$ ~7% inclusive / ~10% exclusive

There has been a long standing puzzle in the determination of $|V_{ub}|$

- $\sim 3\sigma$ discrepancy between exclusive ($B\to\pi l\nu$) and inclusive ($B\to X_u l\nu$) determination.
  J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. D81, 034503 (2010)
- BR($B\to\tau\nu$) leads to larger $|V_{ub}|$ which disagrees with an average of $|V_{ub}|_{\mathrm{excl}}$ and $|V_{ub}|_{\mathrm{incl}}$ by more than 2σ.
  E. Lunghi and A. Soni, Phys.Lett. B697, 323 (2011)

Why we compute the $B \to \pi$ form factor on the Lattice

$f_+(q^2)$ is crucial for the determination of the CKM matrix element $|V_{ub}|$



$q^2 = m_B{}^2 + m_\pi{}^2 - 2m_B E_\pi$

- The exclusive $B \to \pi l v$ semileptonic decay allows the determination of |Vub| via:

Goal

$$\boxed{\frac{d\Gamma}{dq^2}} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} \quad \times \quad |\mathbf{f_+(q^2)}|^2 \quad \times \quad |\mathbf{V_{ub}}|^2$$

Experiment Known factor Hadronic part CKM matrix

- Experiment can only measure the CKM matrix element times Hadronic form factor.
- The hadronic form factor must be computed nonperturbatively via lattice QCD.

How to calculate $f_+(q^2)$ from Lattice QCD

- Non-perturbative form factor $f_+(q^2)$ parametrizes the hadronic matrix element of the $b \to u$ quark flavor-changing vector current V_μ .

$$\langle \pi | V_\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0 \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

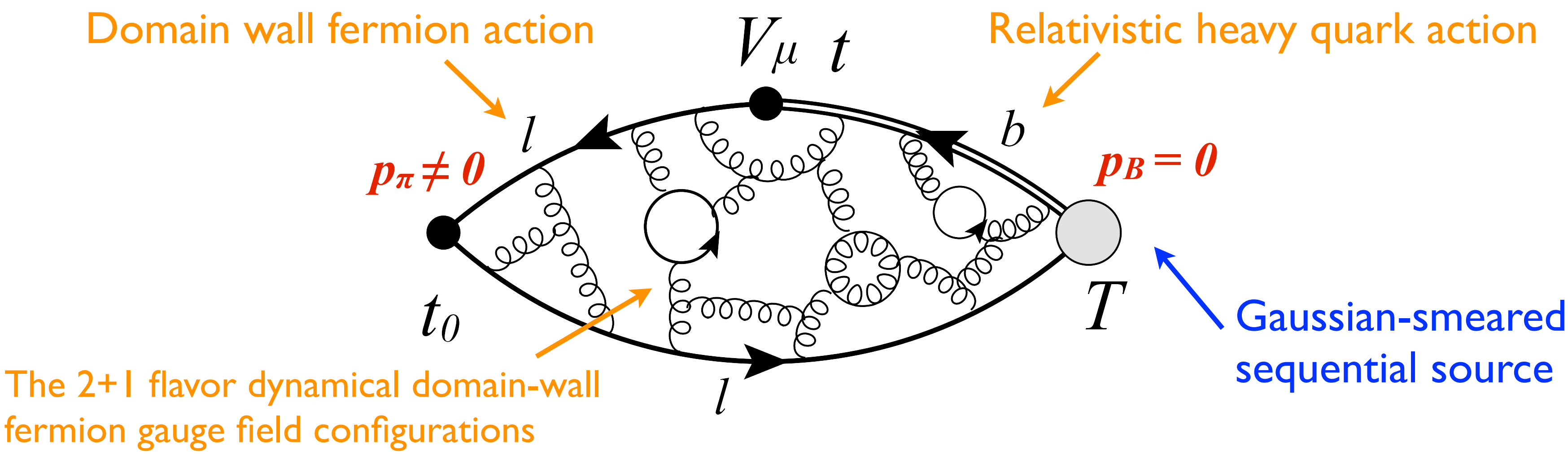
- On the lattice, we calculate the form factors $f_{||}$ and $f_\perp$.
  - ▶ Proportional to vector current matrix elements in the B-meson rest frame:

$$f_{||}(E_\pi) = \langle \pi | V_0 | B \rangle / \sqrt{2 m_B}$$
$$f_\perp(E_\pi) p_i = \langle \pi | V_i | B \rangle / \sqrt{2 m_B}$$

  - ▶ Easy to relate to the desired form factor $f_+(q^2)$.

$$f_+(q^2) = \frac{1}{\sqrt{2 m_B}} [f_{||}(E_\pi) + (m_B - E_\pi) f_\perp(E_\pi)]$$

How to calculate $f_+(q^2)$ from Lattice QCD



- Extract the lattice form factor from the ratio of the 3pt function to 2pt functions:

J. A. Bailey et al. (MILC Collaborations), Phys. Rev. D79, 054507 (2009).

$$R_{3,\mu}^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{\sqrt{C_2^\pi(t)C_2^B(T-t)}}\sqrt{\frac{2E_0^\pi}{e^{-E_0^\pi t}e^{-m_0^B t}}}$$

$$f_\parallel^{\rm lat} = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_\perp^{\rm lat} = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

Heavy-light current renormalization

The lattice amplitude must be multiplied by the appropriate renormalization factor.

$$\langle \pi | V_\mu | B \rangle = Z^{bl}_{V_\mu} \times \langle \pi | V^{\mathrm{lat}}_\mu | B \rangle$$

$Z_{V_\mu}{}^{bl}$ can be calculated via the mostly nonperturbative method.

A. X. El-Khadra et al. Phys.Rev. D64, 014502 (2001)

compute with 1-loop lattice perturbation theory

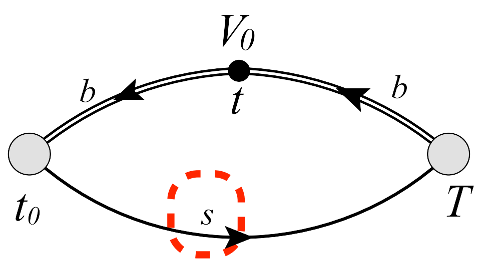
$$Z^{bl}_{V_\mu} = \underbrace{\boxed{\rho^{bl}_{V_\mu}}}_{\approx 1} \sqrt{\boxed{Z^{bb}_V Z^{ll}_V}}$$

compute nonperturbatively

[See talk by C.Lehner Thursday]

$$Z^{bb}_V \times \langle B | V^{bb,0} | B \rangle = 2 m_B$$

- Most of the heavy-light current renormalization factor comes from $Z_V{}^{bb}$ and $Z_V{}^{ll}$, such that ρ is expected to be close to unity
- $Z_V{}^{ll}$ has been obtained by the RBC/UKQCD Collaborations, where we use the fact Z_A=Z_V for domain-wall fermions. Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)
- We compute the matrix element of the $b \to b$ vector current between two B_s mesons.



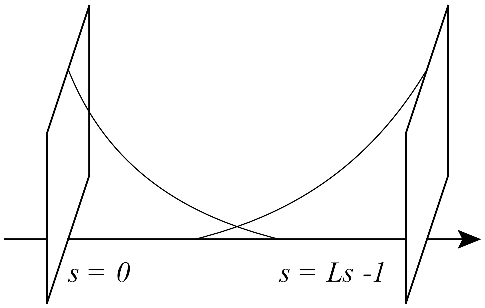
$Z_V{}^{bb}$ is independent of the light "spectator" quark mass

2+1 flavor domain wall gauge configurations

- We use the 2+1 flavor dynamical domain-wall fermion gauge field configurations generated by the RBC/UKQCD Collaborations.

C. Allton et al. (RBC-UKQCD), Phys. Rev. D78, 114509 (2008)
Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)

- domain-wall fermion for the light quarks
  - fermion fields have a 5th dimension of extent L_s
  - left and right handed fermions on slice 0 and L_s-1
  - chiral symmetry breaking under control



- Iwasaki gauge action

$L \times T$	a [fm]	m^{ud}_{sea}	m^{s}_{sea}	m^{π}_{sea} [MeV]	# of configs.	# of sources
32 × 64	≈ 0.08	0.004	0.030	289	628	2
32 × 64	≈ 0.08	0.006	0.030	345	445	2
32 × 64	≈ 0.08	0.008	0.030	394	544	2
24 × 64	≈ 0.11	0.005	0.040	329	1636	1
24 × 64	≈ 0.11	0.010	0.040	422	1419	1

This talk

Relativistic heavy quark action for b-quarks

Heavy quark mass introduces discretization errors of $O((ma)^n)$.

- At bottom quark mass, it becomes severe: $m_b \sim 4$ GeV and $1/a \sim 2$ GeV, then $m_b a > O(1)$.

Relativistic heavy quark action (RHQ action)

$$S^{\mathrm{RHQ}} = \sum_{n,n'} \bar{\psi}_n \left\{ \mathbf{m_0} + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a\vec{D}^2}{2} \right] - a \sum_{\mu\nu} \frac{i\mathbf{c_P}}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \psi'_n$$

- The Fermilab group showed that you can remove all errors of $O((ma)^n)$ by appropriately tuning the parameters of the anisotropic clover action
  A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D55, 3933 (1997)

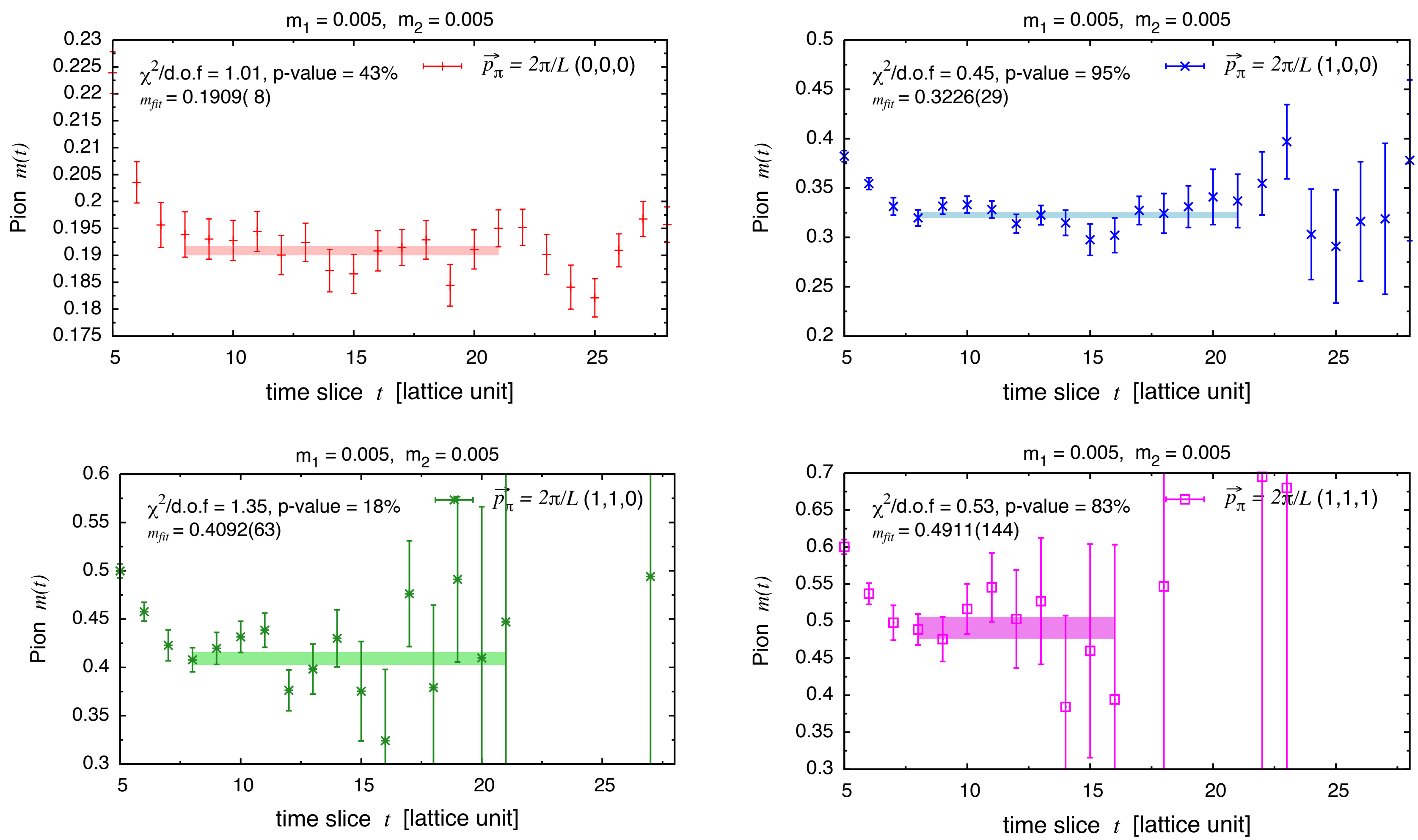
- Errors are of $O(a^2p^2)$.

- Li, Lin, and Christ showed that the parameters $\{m_0, \zeta, c_P\}$ can be tuned nonperturbatively.
  N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
  H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

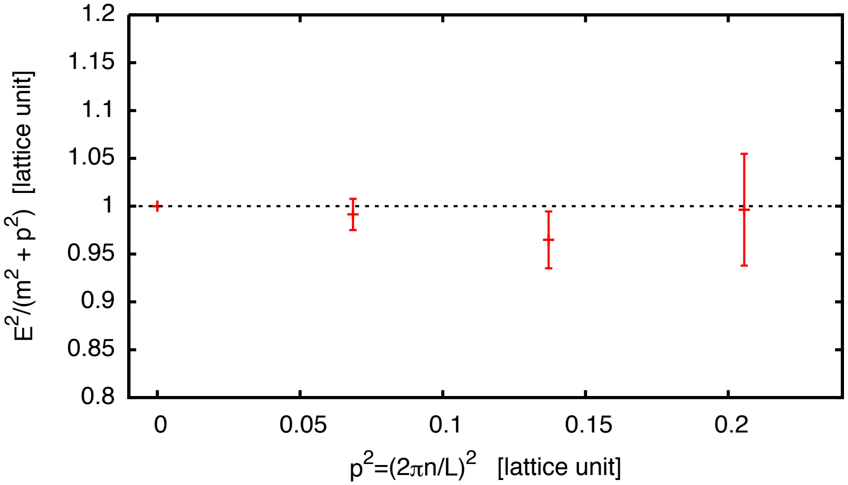
- We use the results for the parameters of the RHQ action obtained for b-quarks in Y. Aoki et. al arXiv:1206.2554. See talk by O.Witzel today.

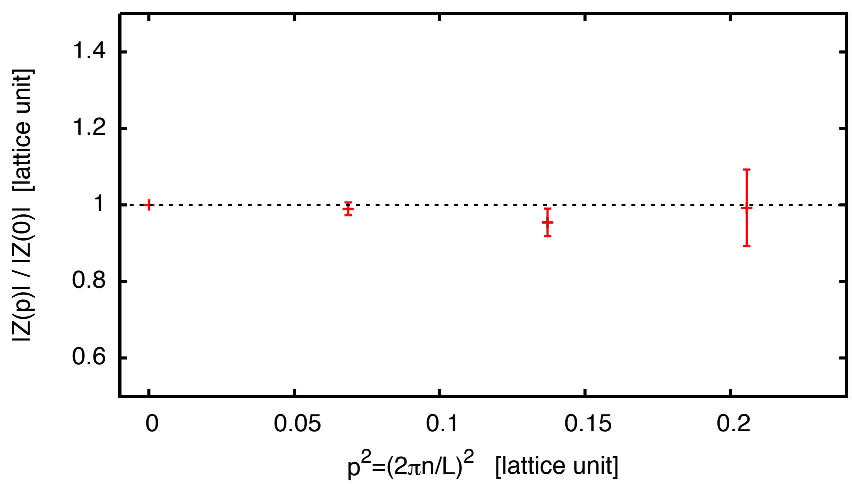
Effective mass plots of pion

$$E(t) = \cosh^{-1}\left\{\frac{C_2^\pi(t+2) + C_2^\pi(t)}{C_2^\pi(t+1)}\right\}$$



Dispersion relation and amplitude Z_π



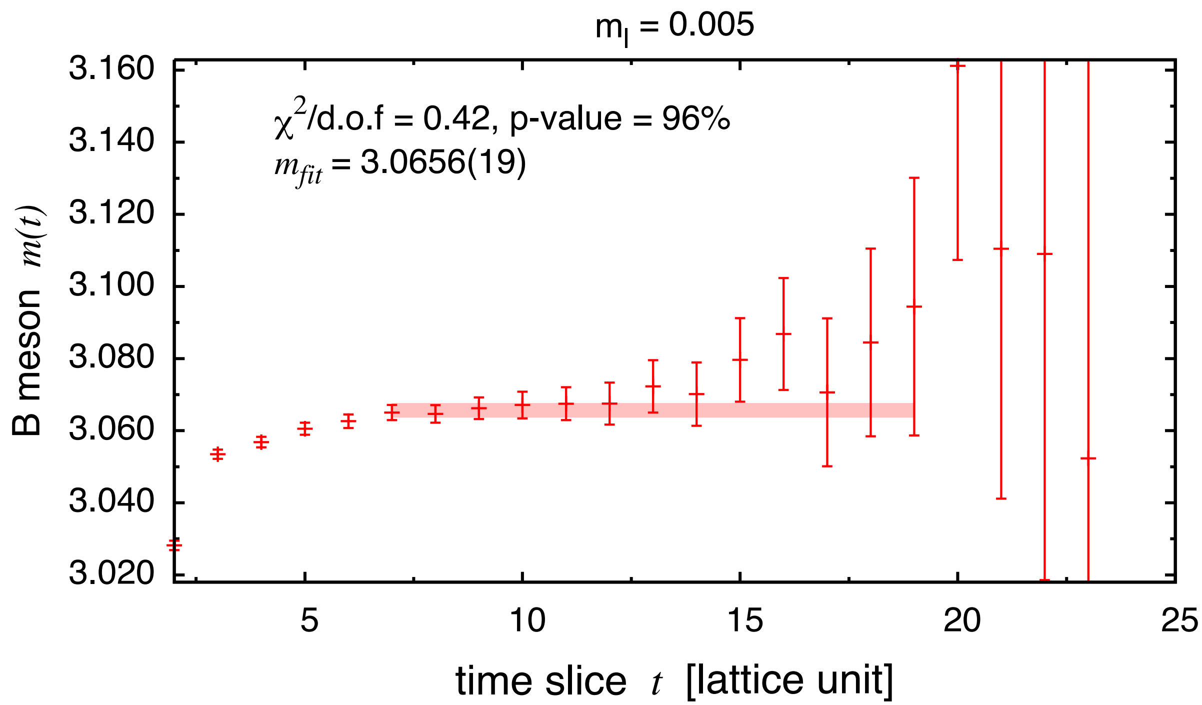


- The pion energy satisfy the continuum dispersion relation: $E_\pi^2 = |\vec{p}_\pi|^2 + m_\pi^2$

- The pion amplitude $Z_\pi = |\langle 0|\mathcal{O}_\pi|\pi\rangle|$ is independent of momentum

$$Z_\pi(E) = \lim_{t\to\infty} \left\{C_2^\pi(t) \times 2Ee^{Et}\right\}^{\frac{1}{2}}$$

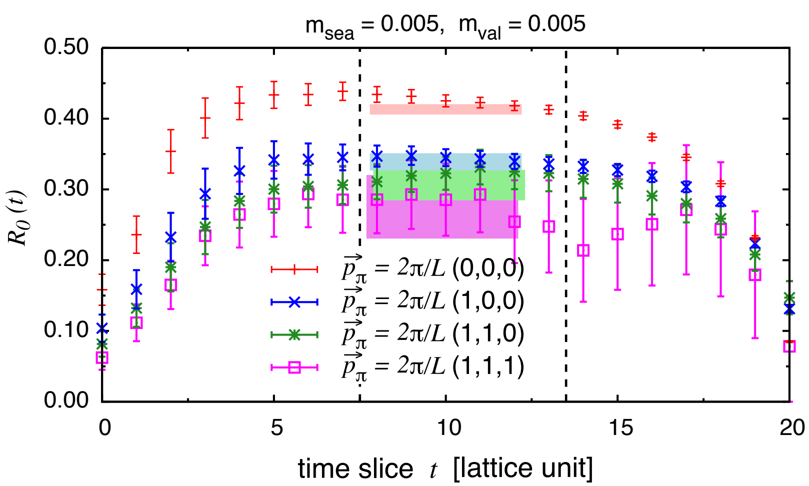
Effective mass plots of B meson

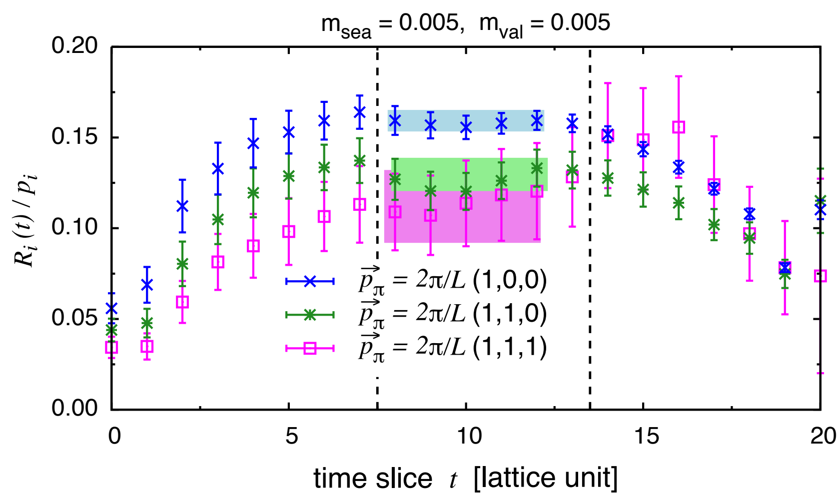


- Gauge-invariant Gaussian b-quark source and point sink
- Smearing the source succeeds in reducing excited state contamination.

The ratio of 3pt over 2pt functions

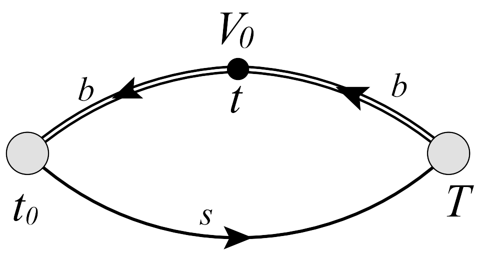
$$R_{3,\mu}^{B\to\pi}(t,T) \;=\; \frac{C_{3,\mu}^{B\to\pi}(t,T)}{\sqrt{C_2^{\pi}(t)C_2^{B}(T-t)}}\sqrt{\frac{2E_0^{\pi}}{e^{-E_0^{\pi}t}e^{-m_0^B t}}}, \qquad E_0^{\pi} = \sqrt{{m_0^{\pi}}^2 + \left(\frac{2\pi n}{L}\right)^2}$$



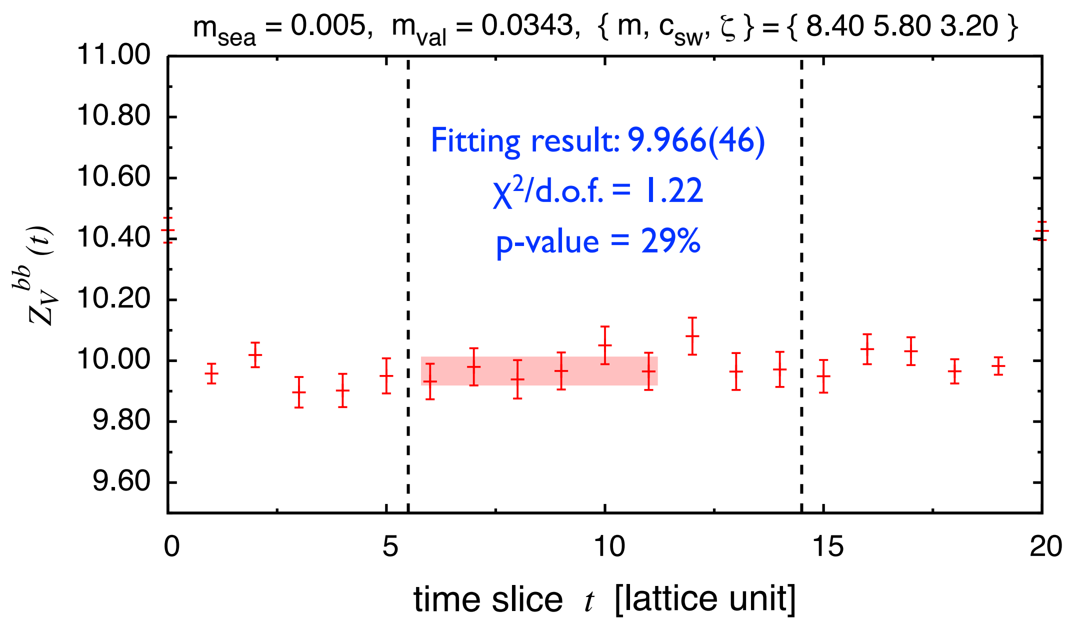


- source-sink separation $T = 20$
- We fit the ratio to a plateau in the region $0 \ll t \ll T$ where 2-point correlators indicate that excited state contributions can be neglected.

$Z_V{}^{bb}$



$$Z_V^{bb} \times \langle m_0^B | V_0^{bb} | m_0^B \rangle \quad = \quad 2m_B$$
$$\frac{C_2^B(T)}{C_{3,\mu}^{B\to B}(t,T)} \quad \xrightarrow{t,T\to\infty} \quad Z_V^{bb}$$



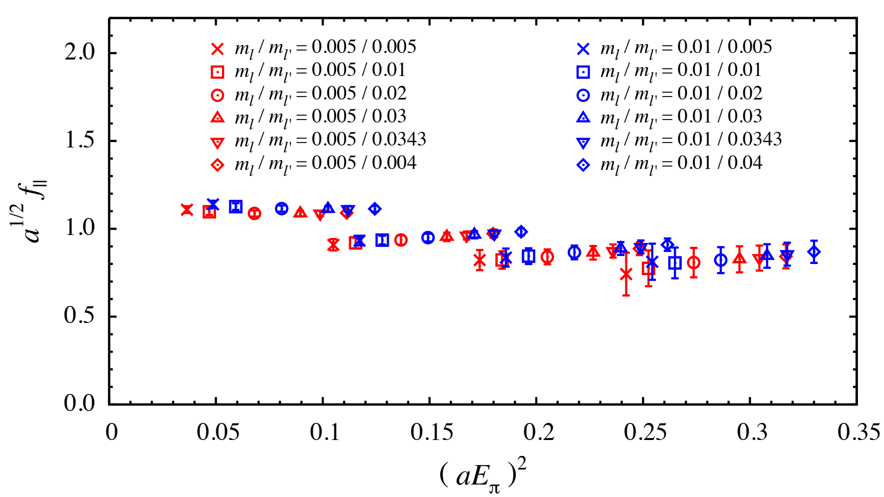
At tree level, the expression of $Z_V{}^{bb}$ is given by

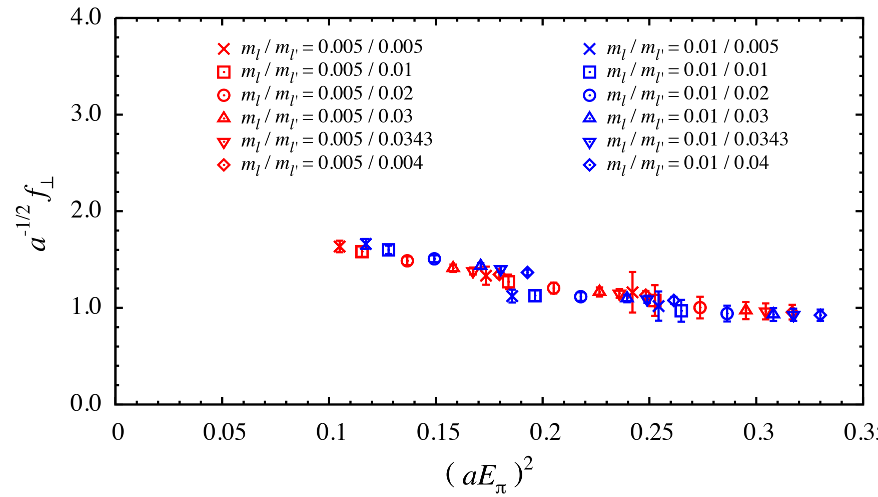
$$Z_V^{bb} \quad = \quad u_0 \exp(M_1), \quad M_1 = \log[1+\tilde{m}_0], \quad \tilde{m}_0 = \frac{m_0}{u_0} - (1+3\zeta)(1-\frac{1}{u_0})$$

Here $m_0 = 7.80$, $\zeta = 3.20$, $u_0 = 0.8757$.

$$\boxed{\begin{array}{rcl} \textbf{tree level} & : & Z_V^{bb} = 9.993 \\ \textbf{NP} & : & Z_V^{bb} = 9.966(46) \end{array}}$$

$f_{||}$ and $f_{\perp}$





- Will extrapolate to physical quark masses and continuum and interpolate in E_π^2 using chiral perturbation theory.

Conclusions and future prospects

- We are calculating the $B \to \pi$ form factors $f_{||}$ and $f_{\perp}$ using 2+1 flavor dynamical domain-wall fermion gauge field configurations with relativistic heavy quark action on $24^3 \times 64$ ($a \sim 0.11$fm) lattice.
- Implementing mostly nonperturbative renormalization.
- Will provide important independent check on existing calculations using staggered light quarks.

Work still in progress:

- underway on $32^3 \times 64$ (a~0.08fm) lattice
- Calculation of ρ-factors in lattice perturbation theory

[see talk by C. Lehner on Thursday]

- chiral extrapolation
- continuum limit
- Extrapolate to low momentum transfer (high E_π^2) using z-expansion and compare with experiment to obtain $|V_{ub}|$.

Back slides

Tuning RHQ parameters

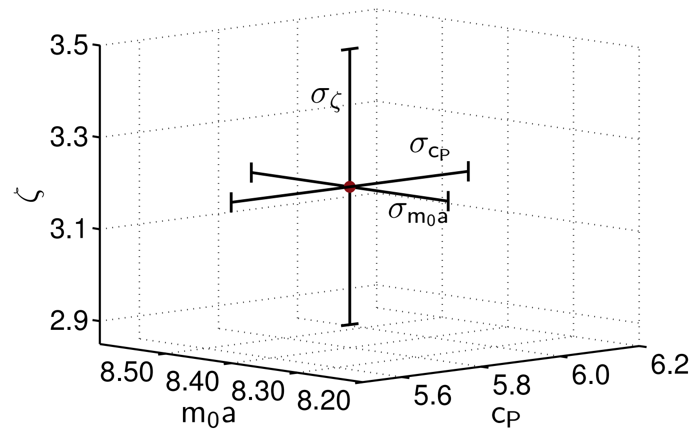
Oliver Witzel gives a talk on monday morning. Y. Aoki et al. arxiv:1206.2554

- Start from calculation of

$$Y_r = \left\{ \bar{M}_{B_s},\ \Delta M_{B_s},\ M_1^{B_s}/M_2^{B_s} \right\}$$

at an educated guess point ($r=1$) and at around 6 points ($r = 2,\ldots,7$):

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_1,\ \begin{bmatrix} m_0 a \pm \sigma_{m_0 a} \\ c_P \\ \zeta \end{bmatrix}_{2,3},\ \begin{bmatrix} m_0 a \\ c_P \pm \sigma_{c_P} \\ \zeta \end{bmatrix}_{4,5},\ \begin{bmatrix} m_0 a \\ c_P \\ \zeta \pm \sigma_\zeta \end{bmatrix}_{6,7}$$



- We extract the RHQ parameters and iterate until tuned values settle inside box.

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\mathrm{RHQ}} = J^{-1} \times \left(\begin{bmatrix} \bar{M}_{B_s} \\ \Delta M_{B_s} \\ \frac{M_1^{B_s}}{M_2^{B_s}} \end{bmatrix}^{\mathrm{PDG}} - A \right) \quad \text{where} \quad J = \left[\frac{Y_3 - Y_2}{2\sigma_{m_0 a}},\ \frac{Y_5 - Y_4}{2\sigma_{c_P}}.\ \frac{Y_7 - Y_6}{2\sigma_\zeta} \right]$$

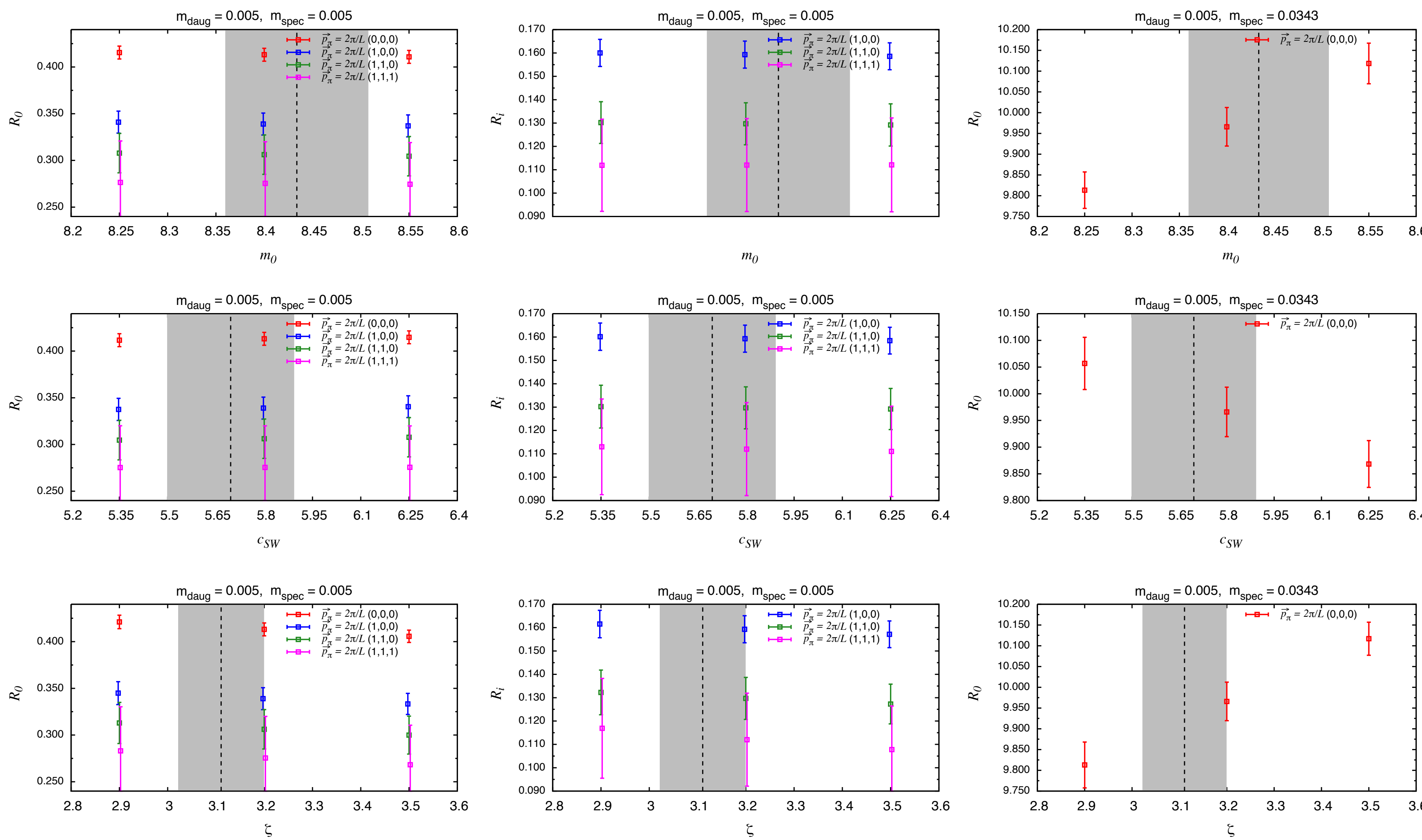
$$A = Y_1 - J \times [m_0 a,\ c_P,\ \zeta]^t$$

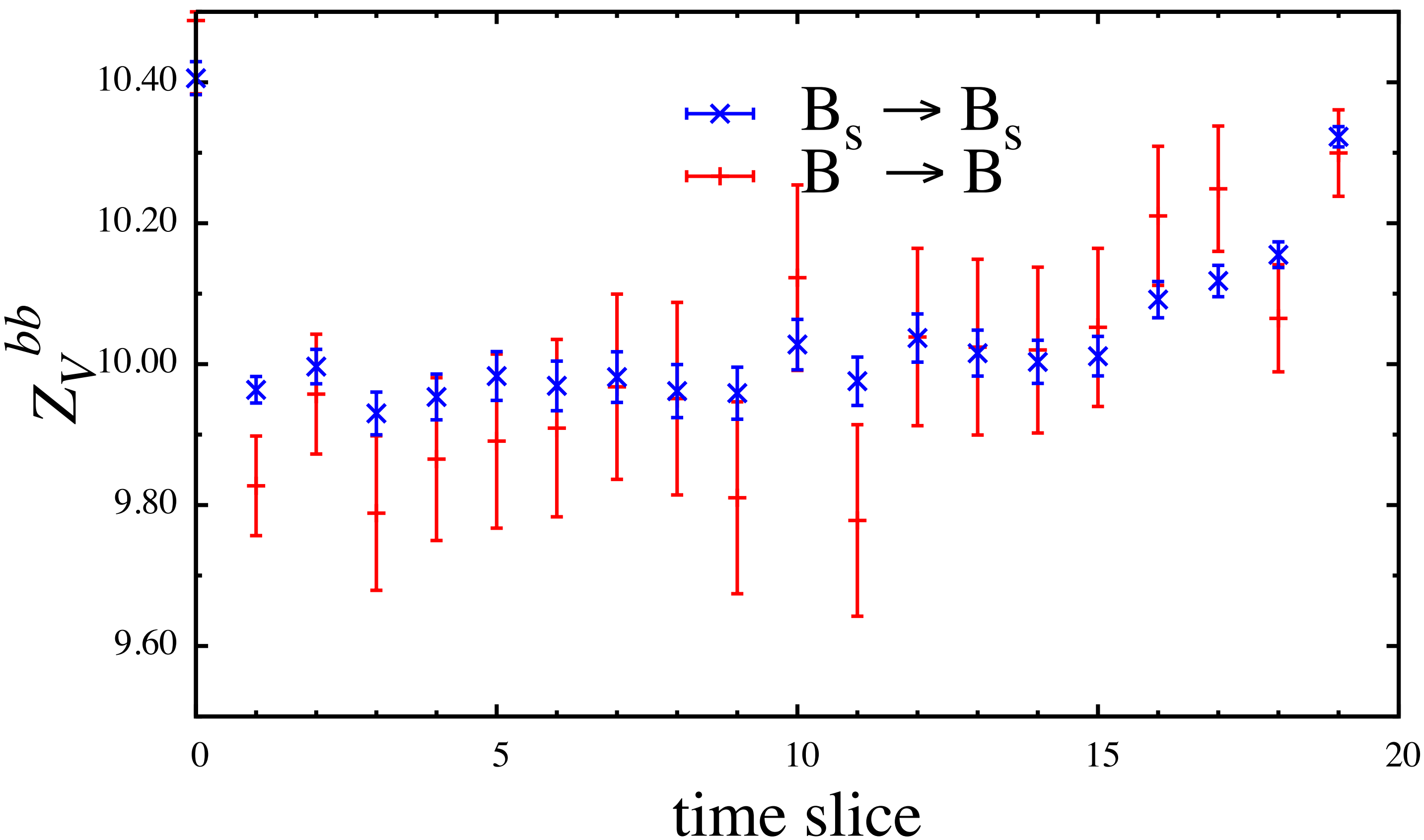
- The other quantity $\mathcal{O}$ at the tuned RHQ parameters can be obtained as

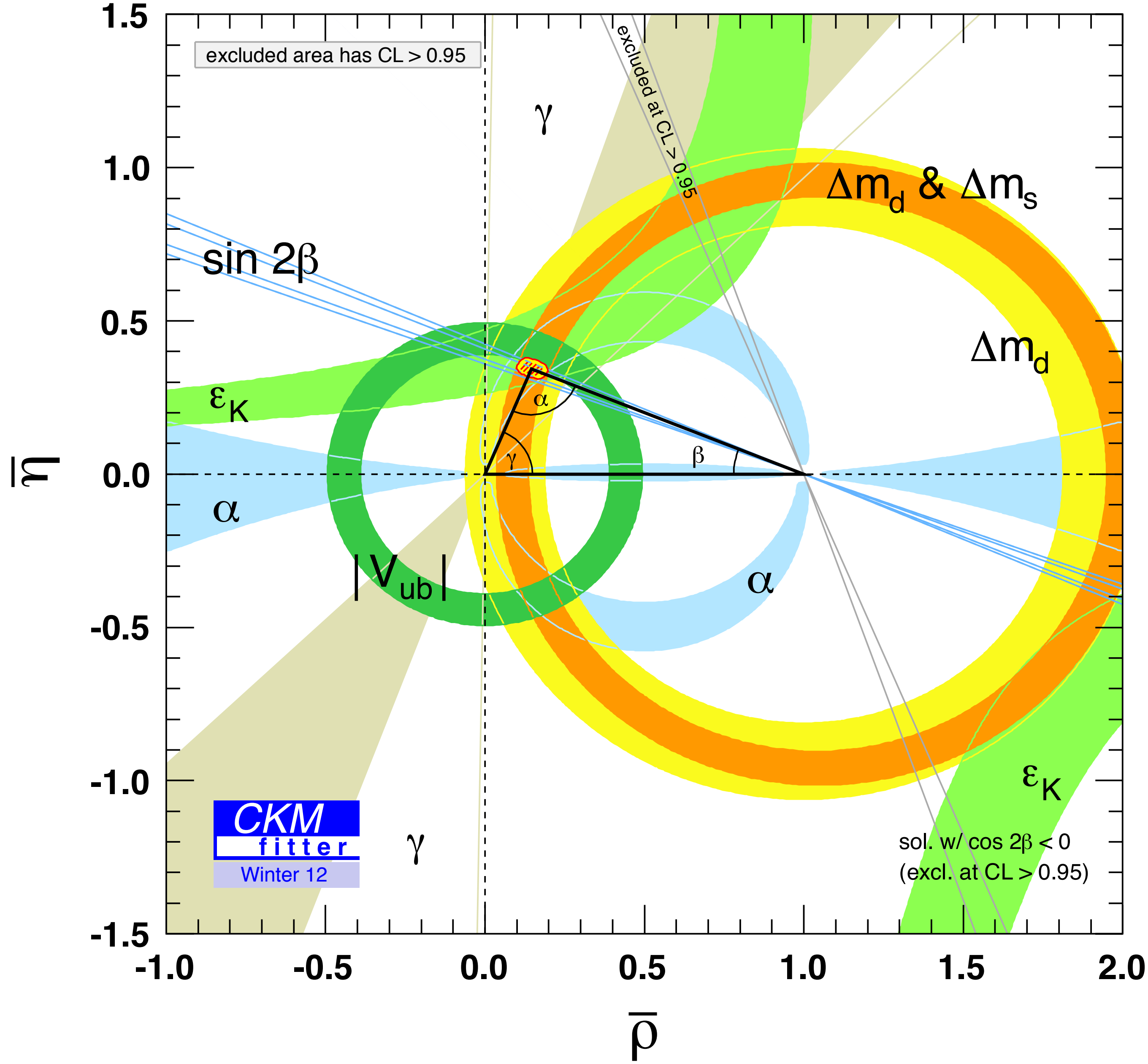
$$\mathcal{O}^{\mathrm{RHQ}} = J \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\mathrm{RHQ}} + A_{\mathcal{O}} \quad \text{where} \quad J_{\mathcal{O}} = \left[\frac{\mathcal{O}_3 - \mathcal{O}_2}{2\sigma_{m_0 a}},\ \frac{\mathcal{O}_5 - \mathcal{O}_4}{2\sigma_{c_P}}.\ \frac{\mathcal{O}_7 - \mathcal{O}_6}{2\sigma_\zeta} \right]$$

$$A_{\mathcal{O}} = \mathcal{O}_1 - J_{\mathcal{O}} \times [m_0 a,\ c_P,\ \zeta]^t$$

Interpolation to tuned RHQ parameter




10.40
10.20
10.00
9.80
9.60
0
5
10
15
20
time slice
Z_V^{bb}
$B_s \to B_s$
$B \to B$



excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
α
$|V_{ub}|$
α
ε_K
CKM fitter
Winter 12
γ
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
$\bar{\eta}$
$\bar{\rho}$