Automated (L)PT and RHQ in the Columbia formulation

Christoph Lehner
RIKEN/BNL Research Center

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RBC/UKQCD B physics update

$B \to \tau\nu\ (f_{B_d})$

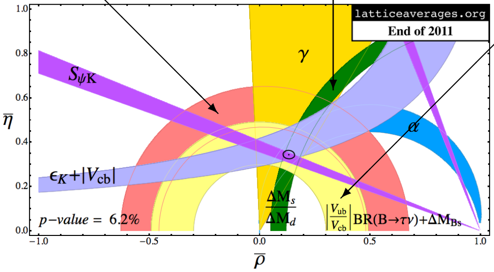
(Talk by O. Witzel)

$$\xi = \frac{f_{B_s}\sqrt{\hat{B}_s}}{f_{B_d}\sqrt{\hat{B}_d}}$$

(Talk by O. Witzel)

$V_{ub}\ (B \to \pi l \nu)$

(Talk by T. Kawanai)



DWF light quarks, RHQ heavy quarks

Tuning of the Columbia-RHQ action

Relativistic heavy quarks

(El-Khadra et al. 1997)
(S. Aoki et al. 2003) (Christ et al. 2006)

- Anisotropic (no $t \leftrightarrow x$ symmetry) clover-improved Wilson action

- Columbia formulation:

$$S = \sum_x \overline{Q}(x)\left((\gamma_0 D_0 - \frac{1}{2} D_0^2) + \zeta \sum_{i=1}^{3} (\gamma_i D_i - \frac{1}{2} D_i^2) + m_0 + c_P \sum_{\mu,\nu=0}^{3} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}(x) \right) Q(x)$$

- Tune coefficients of dimension 4 and 5 operators to remove $|a\vec{p}|$, $(am)^n$, $|a\vec{p}|(am)^n$ errors in on-shell quantities:

$$m_0,\ \zeta,\ c_P$$

Perturbative tuning

Match on-shell quantities after field rotation

$$Q'(x) = Q(x) + d_1 \sum_{i=1,2,3} \gamma_i D_i Q(x)$$

with parameter d_1.

Determine d_1, am_0, and ζ by matching of the bilinear

$$S(p) = \langle Q'(p)\overline{Q}'(-p)\rangle$$

to the continuum for on-shell momenta:

- am_0 from position of the pole
- ζ from dispersion relation
- d_1 from spinor structure

Perturbative tuning

Determine c_P by matching to the continuum the three-point function

$$\begin{aligned}\Lambda^a_\mu(p,q) &= \langle Q'(p) A^a_\mu(-p-q) \overline{Q}'(q)\rangle_{\mathrm{amp}} \\ &= \langle Q'(p)\overline{Q}'(-p)\rangle^{-1} \langle Q'(p) A^b_\nu(-p-q)\overline{Q}'(q)\rangle \\ &\quad \times \langle Q'(-q)\overline{Q}'(q)\rangle^{-1} [D(p+q)^{-1}]^{ba}_{\nu\mu}\,,\end{aligned}$$

where

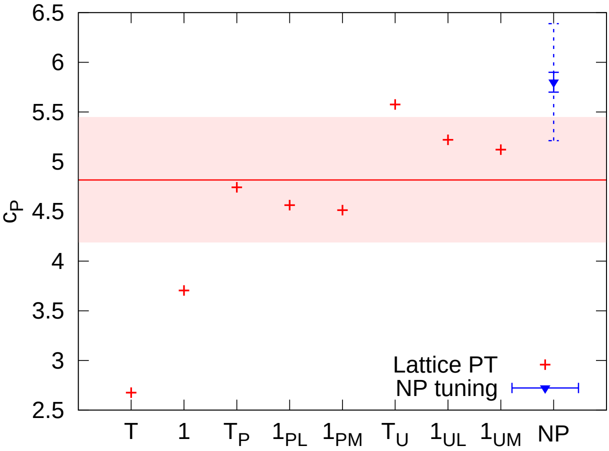
$$D(k)^{ab}_{\mu\nu} = \langle A^a_\mu(k) A^b_\nu(-k)\rangle^{-1}\,.$$

Non-perturbative tuning

- Since d_1 only enters through the field rotation, it has no effect on the hadron spectrum of the theory.

- We can tune m_0, ζ, and c_P non-perturbatively using meson masses and make predictions about the mass spectrum without knowledge of d_1 (talk by O. Witzel on Monday).

For (axial-)vector operators or four-quark operators we need LPT to determine higher-dimensional correction terms.



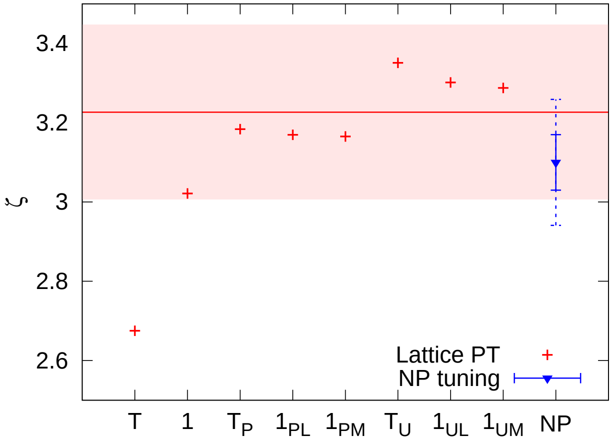
24^3 ensembles

- (T)ree level
- (1) loop
- (P)laquette MF improvement
- Landau gauge (U) link MF improvement
- Expansion in (L)attice coupling
- Expansion in (M)Sbar coupling, $\mu = 1/a$
- NP: non-perturbative tuning result

PT result $= (1_{PM} + 1_{UM})/2$

- PT error is maximum of naive $\alpha_s^2 \sim 5\%$ error and $(1_{UM} - 1_{PM})$
- NP error is stat. (inner) and stat. $+$ syst. in quadrature (outer)

24^3 ensembles



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A general framework for automated (L)PT

Framework for (L)PT

(CL 2012)

- New computer algebra system (CAS) as a C++ library
- Direct access to parsed expression tree in C++
- Speed comparable to FORM, for some applications faster
- Some special features: function map, optimized series expansion, hooks

On top of new CAS: unified LPT, continuum PT framework (RI $\to \overline{\text{MS}}$)

Excerpt of RHQ tuning code

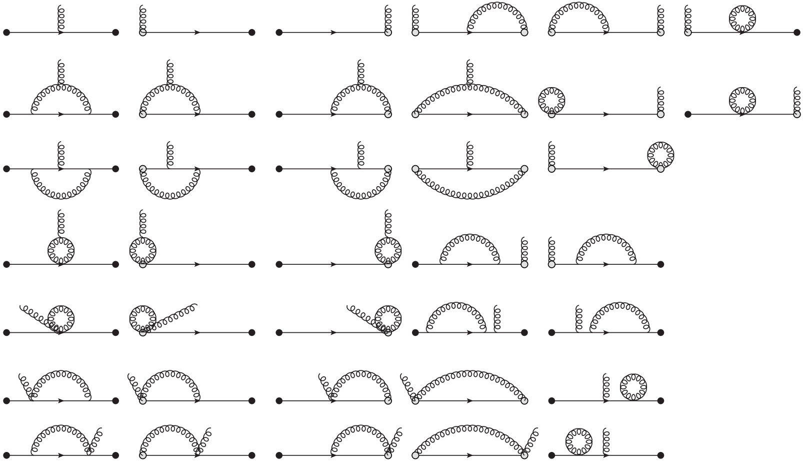
```
Context c;

// use rhq + gauge action
ActionRHQ rhq(&c,"Q");
ActionGAUGE gauge(&c);

// define field rotations
c.coefficients << "d1FT";
const char* QimpD =
 "(1 + sum(i,4)*d1FT(i)*Ngamma(i)*aD(i,x))*Q(x)";
FieldRotationRHQ Qimp(&c,"Q","QimpmomT",QimpD);
const char* QbimpD =
 "Qb(x)*(1 - sum(j,4)*d1FT(j)*Ngamma(j)*aDl(j,x))";
FieldRotationRHQ Qbimp(&c,"Qb","QbimpmomT",QbimpD);

// perform wick contractions
Wick w(&c);
w << rhq << gauge << Qimp << Qbimp;
Expression* vertex = w.contract(
 "sum(k,mom)*QimpmomT(q)*aACmom(mu1,a1,k)*QbimpmomT(-p)",3);
Expression* prop = w.contract(
 "sum(q,mom)*QimpmomT(p)*QbimpmomT(q)",2);
```

One loop vertex diagrams



- How to define a lattice action:

```
// create action from formula string
string sAction = " sum(x)*" + flavor + "b(x)*( "
      " + sum(i1,4)*zeta(i1)*Ngamma(i1)*aD(i1,x) + am0 "
      " - sum(i1,4)*r(i1)*aD(i1,i1,x)/2 "
      " + sum(i1,4)*sum(i2,4)*i_*cp(i1,i2)/4"
          "*Nsigma(i1,i2)*aF(i1,i2,x) "
  " )*" + flavor + "(x) ";
from(sAction.c_str());
```

- Simple example for CAS base layer:

```
Context c;
Expression* e = _("D*Sin(x + y)*Cos(x)*STOP");
_repeat();
_id(c,e,"D*Sin(x?)","Cos(x) + Sin(x)*D");
_id(c,e,"D*Cos(x?)","-Sin(x) + Cos(x)*D");
_id(c,e,"D*STOP","0");
_endrepeat();
_id(c,e,"STOP","1");
```

LPT at the precision frontier

- Speed: $\sim$ 10s for calculation of 1-loop corrections to propagator to 10^{-3} accuracy

- Two loop calculations feasible $\Rightarrow$ significantly reduce systematic error

- 1-loop PT tuning of all coefficients in an $O(ap)^2$ improved action (Oktay and Kronfeld '08 at tree level)

Concluding remarks

- Targets: f_B, ξ (talk by O. Witzel), $B \to \pi$ form factor (talk by T. Kawanai)
- RHQ action for bottom quarks tuned
- Automated (L)PT framework is in place
  - Very general setup (arbitrary action, field rotations; also continuum regulator)
  - RHQ, DWF, improved gauge, and continuum actions implemented
  - Should allow for rapid progress on similar calculations in the future
  - New CAS as a side result
- Operator matching in progress

Backup slides

Non-perturbative tuning - Methodology

(Y. Aoki, . . . , Izubuchi, CL, Soni, Van de Water, Witzel 2012)

- Tuning of RHQ bottom quark in the B_s (PS), B_s^* (V) system
- Match the following spectral quantities to their experimental values:
  - Spin-averaged mass: $\overline{m} = (m_{B_s} + 3m_{B_s^*})/4 \stackrel{!}{=} 5403.1(1.1)$ MeV
  - Hyperfine-splitting: $\Delta_m = m_{B_s^*} - m_{B_s} \stackrel{!}{=} 49.0(1.5)$ MeV
  - Speed of light (in B_s): $m_1/m_2 \stackrel{!}{=} 1$ with

$$E = m_1 + \frac{p^2}{2m_2} + \mathcal{O}(p^4)\,.$$

Only HL quantities

Lattice ensembles

(Allton et al. 2008, Y. Aoki et al. 2010)

$(L/a)^3 \times (T/a)$	$\approx a(\mathrm{fm})$	am_l	am_h	$M_\pi(\mathrm{MeV})$	# configs.	# time sources
$24^3 \times 64$	0.11	0.005	0.040	329	1636	1
$24^3 \times 64$	0.11	0.010	0.040	422	1419	1
$32^3 \times 64$	0.086	0.004	0.030	289	628	2
$32^3 \times 64$	0.086	0.006	0.030	345	889	2
$32^3 \times 64$	0.086	0.008	0.030	394	544	2

2+1 light DWF & Iwasaki gauge action & $L_t = 64$, $L_s = 16$