Calculating B-meson decay constants using domain-wall light quarks and nonperturbatively tuned relativistic b-quarks

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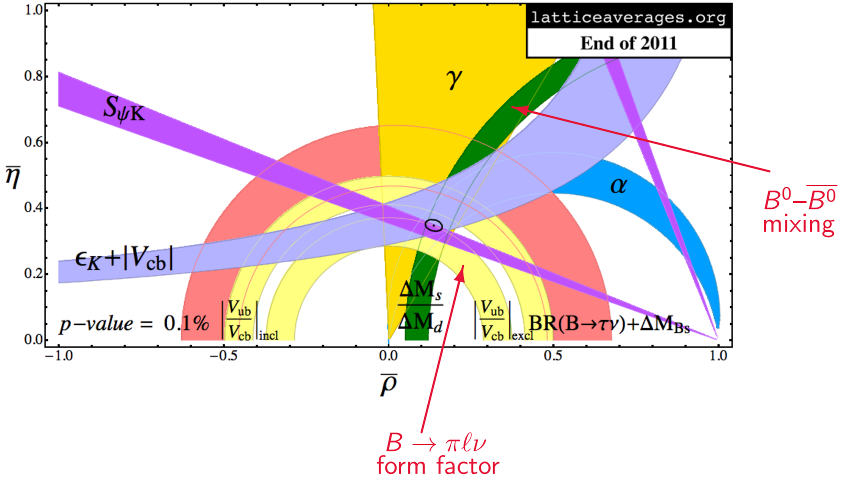
Phenomenological Importance

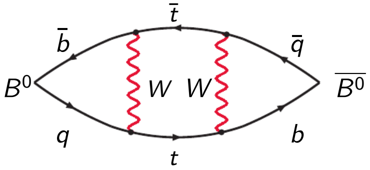


- Only two 2+1 flavor lattice determinations (HPQCD and Fermilab/MILC) both based on the asqtad-improved staggered ensembles generated by MILC

B^0–$\overline{B^0}$ Mixing

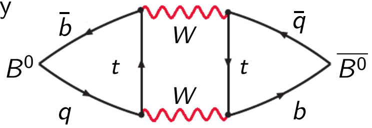
- Allows us to determine the CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

$$\left.\begin{array}{r}|V_{td}^* V_{tb}|\,\text{for}\,B_d-\text{mixing}\\ |V_{ts}^* V_{tb}|\,\text{for}\,B_s-\text{mixing}\end{array}\right\}\Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2}\eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$



- Nonperturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$
- CKM matrix elements are extracted by

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



- Experimental error of ΔM_q is better than a percent; lattice uncertainty for ξ is about 3%

$B \to \pi l \nu$ form factor

- Allows to determine the CKM matrix element V_{ub} from the experimental branching ratio

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} \left[(M_B^2 + M_\pi^2 - q^2)^2 - 4 M_B^2 M_\pi^2 \right]^{3/2} |f_+(q^2)|^2$$

- Tension between exclusive determination and inclusive determinations of V_{ub} is greater than 3σ

Possible Deviations from the Standard Model

[Lunghi and Soni 2010/11]

- Experimental value for $\sin(2\beta)$ is 3.3σ lower than SM expectation
- Measured value for $\mathrm{BR}(B \to \pi l \nu)$ is 2.8σ lower than predicted
- Most likely source of deviation in $B_{d(s)}$ mixing and $\sin(2\beta)$; less likely in $B \to \tau\nu$

[Laiho, Lunghi and Van de Water 2012, http://www.latticeaverages.org]

- Scenario in which new physics is in $B \to \tau\nu$ decay and/or in B_d-mixing preferred
- If tension is taken at face value, points to physics at a few-GeV mass scale

See also: http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it

Our Project

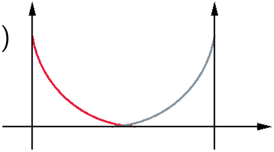
- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - B^0–$\overline{B^0}$ mixing
  - Decay constants f_B and f_{B_s} [this talk]
  - $B \to \pi \ell \nu$ form factor [Talk by T. Kawanai, today 5:40 PM]

- Tune RHQ parameters using bottom-strange states and high statistics
  - Improve upon exploratory studies and verify made assumptions
  - Validate tuning procedure by computing $b\bar{b}$ masses and splittings

- Derive lattice perturbation theory for matching lattice results to continuum 1-loop in tadpole-improved lattice perturbation [Talk by C. Lehner, Thursday, 3:50 PM]
  - Improve matching using a mostly-nonperturbative scheme for f_B, f_{B_s} and $B \to \pi \ell \nu$

2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992, Shamir 1993]
- Iwasaki gauge action [Iwasaki 1983]
- Configurations generated by RBC and UKQCD collaborations [C. Allton et al. 2008], [Y. Aoki et al. 2010]



L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.	# time sources
24	$\approx$ 0.11	0.005	0.040	331	1636	1
24	$\approx$ 0.11	0.010	0.040	419	1419	1
32	$\approx$ 0.08	0.004	0.030	307	628	2
32	$\approx$ 0.08	0.006	0.030	366	889	2
32	$\approx$ 0.08	0.008	0.030	418	544	2

Relativistic Heavy Quark Action for the *b*-Quarks

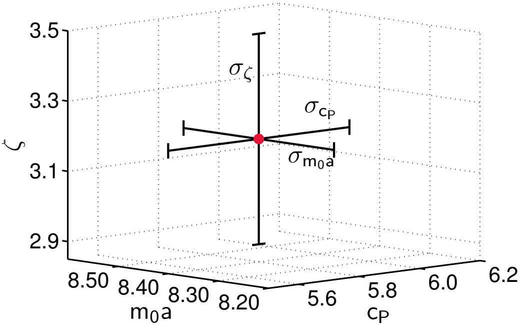
- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the *b*-quarks in 2-point and 3-point correlation functions [Christ, Li, Lin 2007; Lin and Christ 2007]
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie 1997] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation [Aoki, Kuramashi, Tominaga 2003]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

Tuning the Parameters of the RHQ Action

$$S = \sum_{n,n'} \bar{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a\left(\vec{D}\right)^2}{2} \right] - a \sum_{\mu\nu} \frac{i c_P}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \Psi_{n'}$$

- Start from an educated guess for $m_0 a$, c_P, and ζ

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix} \pm \begin{bmatrix} \sigma_{m_0 a} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sigma_{c_P} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \sigma_\zeta \end{bmatrix}$$



- Compute for all seven parameter sets

spin-averaged mass	$\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$	$\rightarrow$	5403.1(1.1) MeV
hyperfine-splitting	$\Delta_M = (M_{B_s^*} - M_{B_s})$	$\rightarrow$	49.0(1.5) MeV
ratio	$\frac{M_1}{M_2} = M_{\text{rest}}/M_{\text{kinetic}}$	$\rightarrow$	1

- Assuming linearity

$$Y_r = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_r = J^{(3\times 3)} \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_r + A^{(3\times 1)} \qquad (r = 1, \ldots, 7)$$

and defining

$$J = \left[\frac{Y_3 - Y_2}{2\sigma_{m_0 a}}, \frac{Y_5 - Y_4}{2\sigma_{c_P}}, \frac{Y_7 - Y_6}{2\sigma_\zeta} \right] \qquad A = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_1 - J \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_1$$

- We extract the RHQ parameters and iterate until result is inside uncertainties

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\text{RHQ}} = J^{-1} \times \left(\begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}^{\text{PDG}} - A \right)$$

Nonperturbatively Tuned Parameters of the RHQ Action

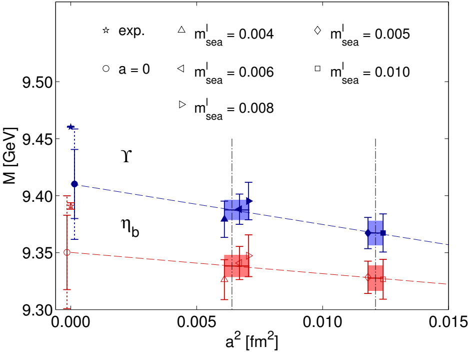
m^l_{sea}	$m_0 a$	c_P	ζ
0.005	8.43(7)	5.7(2)	3.11(9)
0.010	8.47(9)	5.8(2)	3.1(2)
average	8.45(6)	5.8(1)	3.10(7)

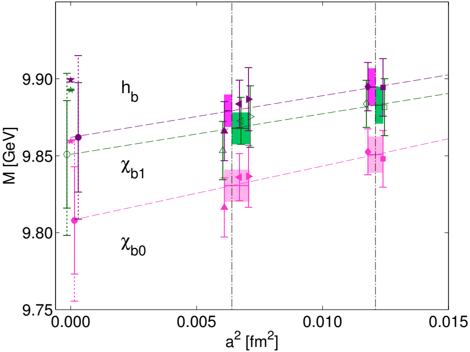
m^l_{sea}	$m_0 a$	c_P	ζ
0.004	4.07(6)	3.7(1)	1.86(8)
0.006	3.97(5)	3.5(1)	1.94(6)
0.008	3.95(6)	3.6(1)	1.99(8)
average	3.99(3)	3.57(7)	1.93(4)

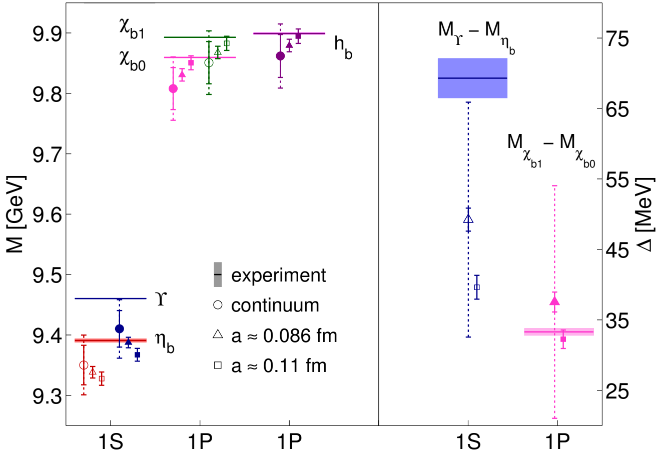
[Y. Aoki et al. 2012]

Predictions for the Heavy-Heavy States

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s-system we can predict bottomonium states and mass splittings







$\Upsilon = 9410(30)(38)$ MeV

$\eta_b = 9350(33)(37)$ MeV

$h_b = 9862(36)(39)$ MeV

$\chi_{b1} = 9851(35)(39)$ MeV

$\chi_{b0} = 9808(35)(39)$ MeV

$M_\Upsilon - M_{\eta_b} = 49(02)(17)$

$M_{\chi_{b1}} - M_{\chi_{b0}} = 38(01)(16)$

[Y. Aoki et al. 2012]

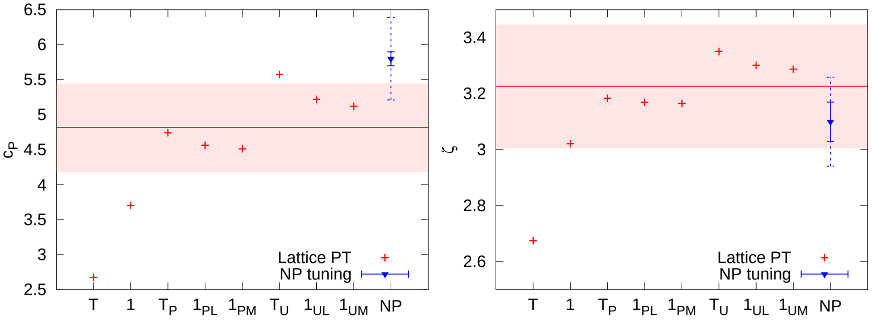
RHQ Lattice Perturbation Theory [C. Lehner]

Motivation

- Knowing the RHQ parameters nonperturbatively we can compare the outcome with lattice perturbation theory
- Helps to build confidence that lattice perturbation theory is working also in cases where we do not have fully non-perturbative matching (e.g. decay constants, form factors)

Method

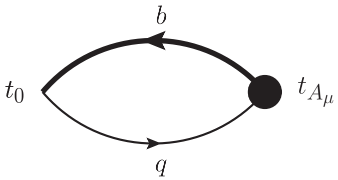
- Computation at 1-loop order
- Mean field improved
- Use nonperturbative inputs for $\langle P \rangle$, $\langle R \rangle$, $\langle L \rangle$ and $m_0 a$
- Predict: c_P and ζ
- Naive $\alpha_S^2 \sim 5\%$ power-counting estimate



- Central values: average of one-loop mean-field improved values computed with u_0 obtained from the plaquette and from the spatial Landau link
- Error on perturbative c_P: difference between mean field methods dominates
- Error on perturbative ζ: naive power-counting dominates
- Nonperturbative values include systematic errors from discretization errors in quantities used for tuning
- Agreement within errors $\Rightarrow$ MF-improved LPT can be trusted in situations for which NP matching factors are not available

B-meson Decay Constant Calculation

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- Computation performed with seven paramter box and interpolated to the tuned RHQ parameters
- Axial current will be 1-loop $O(a)$ improved
- Result will use mostly nonperturbative renormalization
- Combined chiral and continuum extrapolation using heavy meson χPT



Mostly Nonperturbative Renormalization

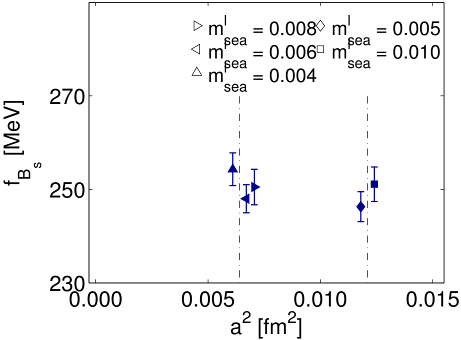
For f_B, f_{B_s} and $B \to \pi$ we plan to compute mostly non-perturbative renormalization factors á la [El Khadra et al. 2001]

$$\varrho^{bl} = \frac{Z_V^{bl}}{\sqrt{Z_V^{bb} Z_V^{ll}}}$$

- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative serious of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. 2011], Z_V^{bb} computed on 24^3 [Talk by T. Kawanai]

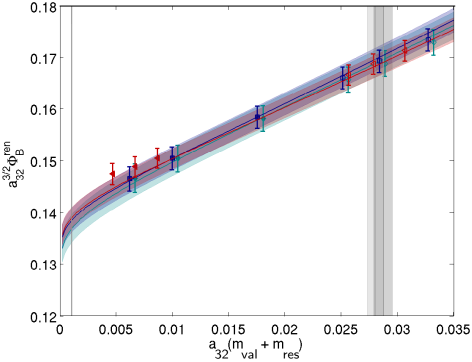
Very Preliminary Results — small scaling violations

- Results for f_{B_s} obtained by simulating close to the physical strange quark mass
- Renormalization and matching to be improved:
  nonperturbative Z_V^{ll}; perturbative Z_V^{bb}; (tree level, 20% error); $\varrho_{bl} = 1$
- Axial current only tree-level $O(a)$ improved



Preview on f_B

- Data obtained on three ensembles: red $m^l_{sea} = 0.006$ (32^3, $a \approx 0.08$fm); blue $m^l_{sea} = 0.005$, cyan $m^l_{sea} = 0.010$ (both 24^3, $a \approx 0.11$fm)
- Independent, correlated 2-parameter fit to the three lowest valence quark masses of each ensemble



- Awaiting proper mostly NPR
- Combined fit in progress
- More data underway

Conclusion

- We have completed tuning the parameters of the RHQ action for b-quarks, and find good agreement between our predictions for bottomonium masses and fine splittings with experiment.
- Given this success, we are now using this method for B-meson quantities such as decay constants, neutral B-meson mixing parameters and form factors, and expect to obtain errors competitive with other groups.
- The RHQ action can also be used for charm quarks, and Hao Peng is currently performing the necessary parameter tuning.
- We should have first results within the next year, and maybe sooner!