B-meson decay constants with domain-wall light quarks and nonperturbatively tuned relativistic b-quarks

RBC and UKQCD collaborations

Oliver Witzel
Center for Computational Science

BOSTON UNIVERSITY

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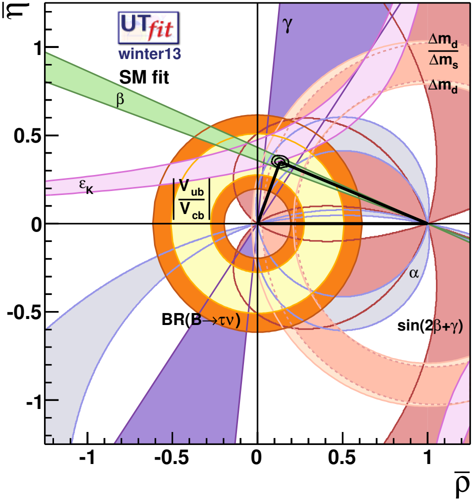
Motivation: CKM unitarity triangle fit



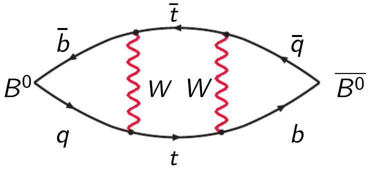
[http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it, http://www.latticeaverages.org]

Motivation: B^0–$\overline{B^0}$ Mixing

- Allows us to determine the CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

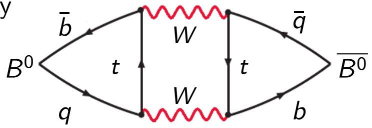
$$\left.\begin{array}{r}|V_{td}^* V_{tb}| \text{ for } B_d-\text{mixing} \\ |V_{ts}^* V_{tb}| \text{ for } B_s-\text{mixing}\end{array}\right\} \Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

- Nonperturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$



- CKM matrix elements are extracted by

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



- Experimental error of ΔM_q is better than a percent; lattice uncertainty for ξ is about 3%

Motivation: Rare B-decays

$B \to \tau\nu$ [UTfit Phys.Lett. B687 (2010) 61]

- f_B is needed for the Standard-Model prediction of $BR(B \to \tau\nu)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Helicity suppressed charged current decays: potential sensitivity to tree-level effects of new scalar particles (charged Higgs bosons in multi-Higgs extensions of the SM, e.g. type-II Two Higgs Doublet Model or MSSM)

$B_s \to \mu_+\mu_-$ [Buras et al. Eur.Phys.J. C72 (2012) 2172, Buras et al. arXiv:1303.3820 [hep-ph]]

- f_{B_s} is needed for Standard-Model prediction of $BR(B_s \to \mu_+\mu_-)$
- Measured by LHCb with 3.5σ significance [LHCb Phys.Rev.Lett. 110 (2013) 02180], at EPS2013: combination of LHCb and CMS results gives $> 5\sigma$ significance — in agreement with SM

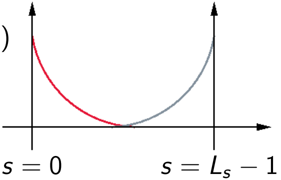
Both are sensitive to new physics!

Our Project

- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - B^0–$\overline{B^0}$ mixing
  - Decay constants f_B and f_{B_s}
  - $B \to \pi \ell \nu$ form factor [T. Kawanai, Tue 14:20 Room C]
  - $g_{B^* B\pi}$ coupling constant [B. Samways, Tue 16:40 Room C]
- Tuned RHQ parameters using bottom-strange states and high statistics
- Validated tuning procedure by computing $b\bar{b}$ masses and splittings
- Use mostly-nonperturbative renormalization scheme for f_B, f_{B_s} and $B \to \pi \ell \nu$
- Use one-loop mean-field improved lattice perturbation theory for small correction, and to renormalize B-mixing matrix elements [http://physyhcal.lhnr.de] [C. Lehner, Tue 14:40 Room C]

2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s)
  [Kaplan Phys.Lett. B288 (1992) 342]
  [Shamir Nucl.Phys. B406 (1993) 90]
- Iwasaki gauge action [Iwasaki UTHEP-118(1983)]
- Configurations generated by RBC and UKQCD collaborations [C. Allton et al. Phys.Rev. D78 (2008) 114509, Y. Aoki et al. Phys.Rev. D83 (2011) 074508]



L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.	# time sources
24	≈ 0.11	0.005	0.040	331	1636	1
24	≈ 0.11	0.010	0.040	419	1419	1
32	≈ 0.08	0.004	0.030	307	628	2
32	≈ 0.08	0.006	0.030	366	889	2
32	≈ 0.08	0.008	0.030	418	544	2

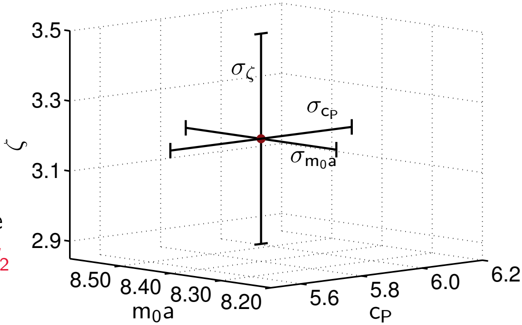
Relativistic Heavy Quark Action for the *b*-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin
  [Christ et al. Phys.Rev. D76 (2007) 074505; Lin and Christ Phys.Rev. D76 (2007) 074506]
- Builds upon Fermilab approach [El-Khadra et al. Phys.Rev. D55 (1997) 3933] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation
  [S. Aoki et al. Prog.Theor.Phys. 109 (2003) 383]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

Nonperturbative Tuning of the RHQ Action Parameters

[Phys.Rev. D86 (2012) 116003]

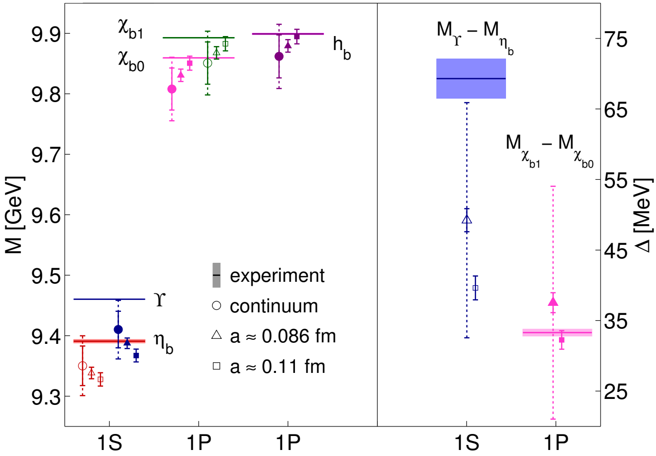
- Start from an educated guess for our three parameters m_0a, c_P, and ζ
- Probe parameter space at seven points by measuring
  spin-averaged mass: $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$
  hyperfine-splitting: $\Delta_M = M_{B_s^*} - M_{B_s}$
  ratio: $M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}}$
- Assume linearity to relate parameters and observables
- Obtain tuned parameters corresponding to physical b-quarks by requiring that $\overline{M}$ and Δ_M agree with experiment and that $M_1 = M_2$



Predictions for the Heavy-Heavy States

[Phys.Rev. D86 (2012) 116003]

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s-system we can predict bottomonium states and mass splittings and thereby test the method
- We find good agreement with experiment within errors



$\Upsilon = 9410(30)(38)$ MeV

$\eta_b = 9350(33)(37)$ MeV

$\chi_{b1} = 9851(35)(39)$ MeV

$\chi_{b0} = 9808(35)(39)$ MeV

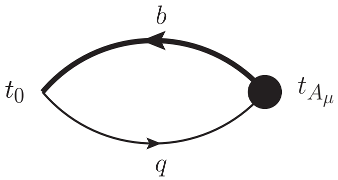
$h_b = 9862(36)(39)$ MeV

$M_\Upsilon - M_{\eta_b}$
$= 49(02)(17)$ MeV

$M_{\chi_{b1}} - M_{\chi_{b0}}$
$= 38(01)(16)$

B-meson Decay Constant Calculation

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- Computation performed with seven parameter box and interpolated to the tuned RHQ parameters
- Axial current is 1-loop $O(a)$ improved
- Use mostly nonperturbative renormalization
- Combined chiral and continuum extrapolation using heavy meson χPT



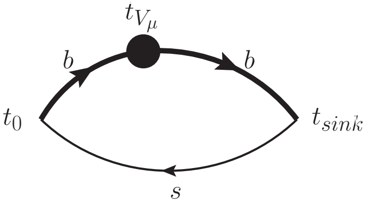
Mostly Nonperturbative Renormalization

For f_B, f_{B_s} and $B \to \pi$ we compute mostly non-perturbative renormalization factors á la [El-Khadra et al. Phys.Rev. D64 (2001) 014502]

$$Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}$$

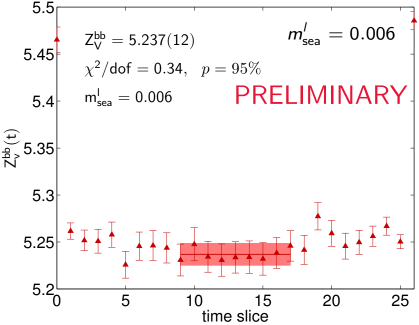
- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative series of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. Phys.Rev. D83 (2011) 074508] and compute Z_v^{bb} ourselves

Determination of Z_v^{bb}



$$Z_V^{bb} \times \langle B|V^{bb,0}|B\rangle = 2m_B$$

$$\frac{C_2^B(T)}{C_3^{B\to B}(T,t)} \lim_{T,t\to\infty} Z_V^{bb}$$

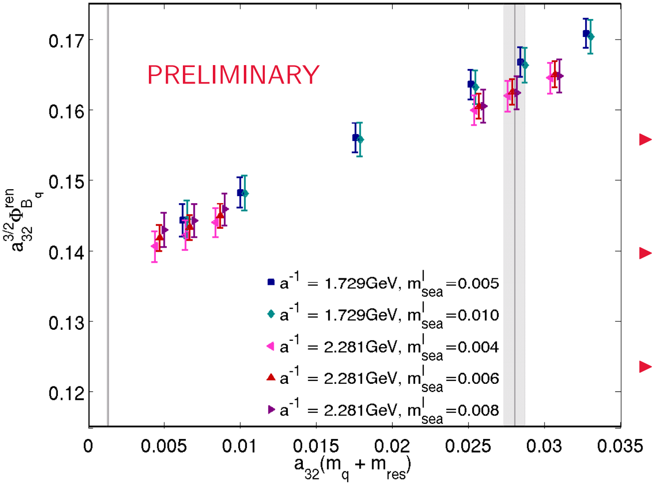


PRELIMINARY

$a_{24} m^l_{sea}$	Z_v^{bb}	$a_{32} m^l_{sea}$	Z_v^{bb}
0.005	10.037(34)	0.004	5.270(13)
0.010	10.042(37)	0.006	5.237(12)
		0.008	5.267(15)
Avg.$^{(24)}$	10.093(25)	Avg.$^{(32)}$	5.2560(76)
$\text{PT}^{(24)}_{\text{1-loop}}$	10.72(16)(0)	$\text{PT}^{(32)}_{\text{1-loop}}$	5.725(74)(1)

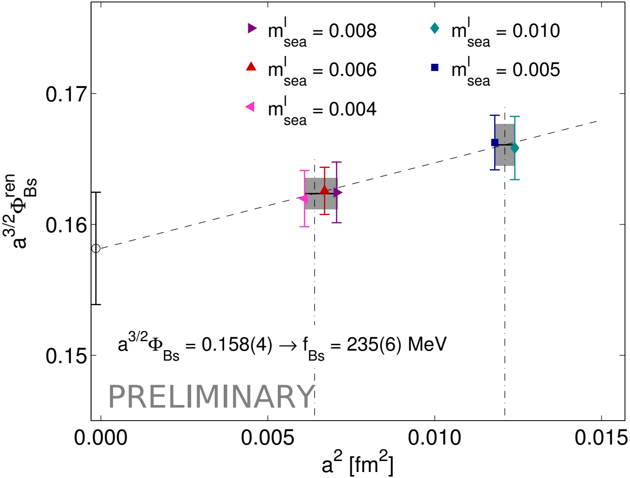
PT values: http://physyhcal.lhnr.de

Preliminary Results for f_B and f_{B_s}



- On the lattice we compute Φ_{B_q}
  $$f_B = \Phi^{\text{ren}}_{B_q} \cdot a_{32}^{-3/2} / \sqrt{M_{B_q}}$$
- Partially quenched data are highly correlated
- Variance-covariance matrix is statistically well resolved
- Linearly interpolate to get f_{B_s} and fit to extrapolate to f_B

Preliminary Results Φ_{B_s}

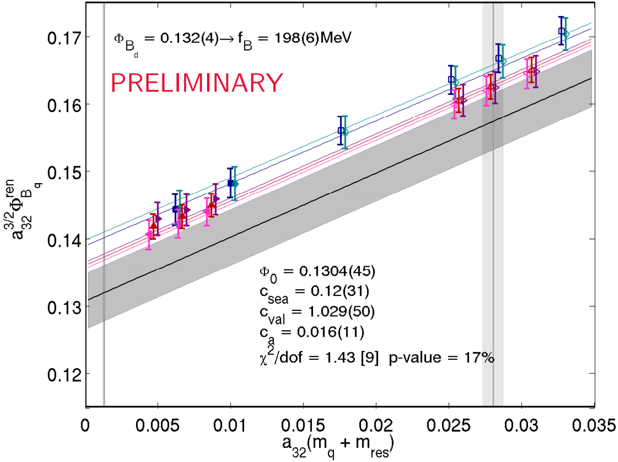


- Data for Φ_{B_s} show no sea-quark mass dependence
- Average data at same lattice spacing and assume a^2 scaling to remove light-quark and gluon discretization errors
- Remaining heavy-quark discretization errors will be estimated with heavy-quark power counting and included in the systematic error budget

Preliminary Results Φ_{B_d}

- Fit only "chiral" data i.e. $a_{24} m_q < 0.01$ ($m_\pi < 420$ MeV) using an analytic function in the quark masses and lattice spacing

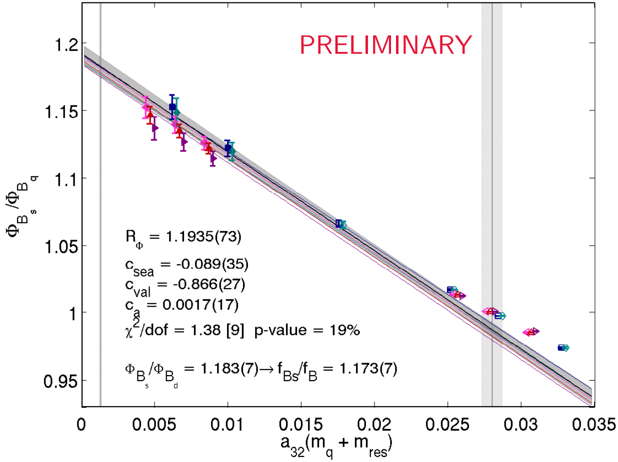
$$\Phi_B = \Phi_0 \left[1 + c_{sea} m^l_{sea} 2B/(4\pi f)^2 + c_{val} m_{val} 2B/(4\pi f)^2 + c_a a^2/(a_{32}^2 4\pi f)^2\right]$$



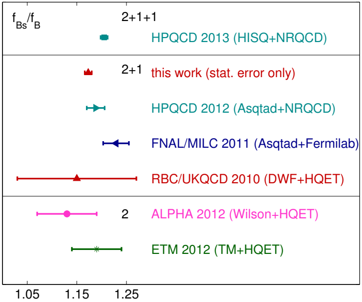
Preliminary Results Φ_{B_s}/Φ_{B_d}

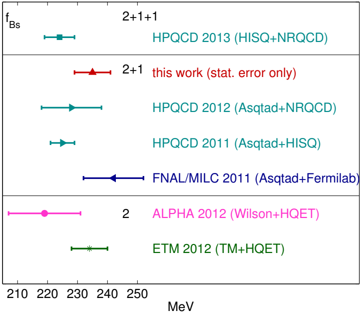
- Fit only "chiral" data i.e. $a_{24} m_q < 0.01$ ($m_\pi < 420$ MeV) using an analytic function in the quark masses and lattice spacing

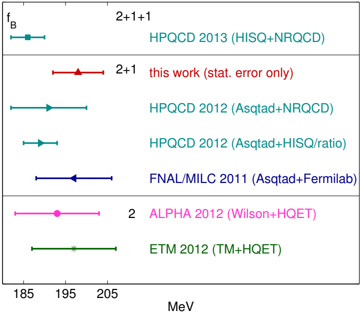
$$\Phi_{B_s}/\Phi_B = R_\Phi \left[1 + c_{\text{sea}} m^l_{\text{sea}} 2B/(4\pi f)^2 + c_{\text{val}} m_{\text{val}} 2B/(4\pi f)^2 + c_a a^2/(a_{32}^2 4\pi f)^2\right]$$



Comparison


f_{Bs}/f_B
2+1+1
HPQCD 2013 (HISQ+NRQCD)
2+1
this work (stat. error only)
HPQCD 2012 (Asqtad+NRQCD)
FNAL/MILC 2011 (Asqtad+Fermilab)
RBC/UKQCD 2010 (DWF+HQET)
2
ALPHA 2012 (Wilson+HQET)
ETM 2012 (TM+HQET)
1.05
1.15
1.25


f_{Bs}
2+1+1
HPQCD 2013 (HISQ+NRQCD)
2+1
this work (stat. error only)
HPQCD 2012 (Asqtad+NRQCD)
HPQCD 2011 (Asqtad+HISQ)
FNAL/MILC 2011 (Asqtad+Fermilab)
2
ALPHA 2012 (Wilson+HQET)
ETM 2012 (TM+HQET)
210 220 230 240 250
MeV


f_B
2+1+1
HPQCD 2013 (HISQ+NRQCD)
2+1
this work (stat. error only)
HPQCD 2012 (Asqtad+NRQCD)
HPQCD 2012 (Asqtad+HISQ/ratio)
FNAL/MILC 2011 (Asqtad+Fermilab)
2
ALPHA 2012 (Wilson+HQET)
ETM 2012 (TM+HQET)
185 195 205
MeV

Observations

- SU(2) HMχPT is valid for $m_{u,d} \ll m_s$. Are our data "chiral" enough?
- Our data do not show visible signs of SU(2) chiral logarithms.
- Strong correlations among partially quenched data are troublesome. Are light valence-quark masses too close to each other?

Preliminary Results

- $f_{B_s} = 235(6)$ MeV
- $f_B = 198(6)$ MeV $\Rightarrow f_{B_s}/f_B = 1.19(5)$
- $f_{B_s}/f_B = 1.173(7) \Rightarrow f_B = 200(5)$ MeV
- Overall consistent results

Statistical errors only!

Derived (gray) results neglect correlations!

Outlook

- We are finalizing the analysis of f_B, f_{B_s} and f_{B_s}/f_B
- Next we start the computation of $B^0 - \overline{B^0}$ mixing
- Future data will be obtained at physical pions on the $48^3 \times 96$ and $64^3 \times 128$ Möbius domain-wall ensembles