$B$-meson decay constants with domain-wall light quarks and nonperturbatively tuned relativistic $b$-quarks

RBC and UKQCD collaborations

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Center for Computational Science

Lattice 2013, Mainz, Germany
Motivation: CKM unitarity triangle fit
**Motivation: $B^0 - \overline{B^0}$ Mixing**

- Allows us to determine the CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

$$\Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{Bq} f_{Bq}^2 B_{Bq} |V_{tq}^* V_{tb}|^2$$

- Nonperturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio
  $$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$$

- CKM matrix elements are extracted by
  $$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

- Experimental error of $\Delta M_q$ is better than a percent; lattice uncertainty for $\xi$ is about 3%
Motivation: Rare $B$-decays

$B \rightarrow \tau \nu$ [UTfit Phys.Lett. B687 (2010) 61]

- $f_B$ is needed for the Standard-Model prediction of $BR(B \rightarrow \tau \nu)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Helicity suppressed charged current decays: potential sensitivity to tree-level effects of new scalar particles (charged Higgs bosons in multi-Higgs extensions of the SM, e.g. type-II Two Higgs Doublet Model or MSSM)


- $f_{B_s}$ is needed for Standard-Model prediction of $BR(B_s \rightarrow \mu^+ \mu^-)$
- Measured by LHCb with $3.5\sigma$ significance [LHCb Phys.Rev.Lett. 110 (2013) 02180], at EPS2013: combination of LHCb and CMS results gives $> 5\sigma$ significance — in agreement with SM

Both are sensitive to new physics!
Our Project

- Use domain-wall light quarks and nonperturbatively tuned relativistic $b$-quarks to compute at few-percent precision
  - $B^0$–$\overline{B}^0$ mixing
  - Decay constants $f_B$ and $f_{B_s}$
  - $B \rightarrow \pi \ell \nu$ form factor [T. Kawanai, Tue 14:20 Room C]
  - $g_{B^*B\pi}$ coupling constant [B. Samways, Tue 16:40 Room C]
- Tuned RHQ parameters using bottom-strange states and high statistics
- Validated tuning procedure by computing $b\overline{b}$ masses and splittings
- Use mostly-nonperturbative renormalization scheme for $f_B$, $f_{B_s}$ and $B \rightarrow \pi \ell \nu$
- Use one-loop mean-field improved lattice perturbation theory for small correction, and to renormalize B-mixing matrix elements
[http://physyhcal.lhnrd.de] [C. Lehner, Tue 14:40 Room C]
**2+1 Flavor Domain-Wall Gauge Field Configurations**

- **Domain-wall fermions for the light quarks (u, d, s)**
  
  

- **Iwasaki gauge action**
  
  [Iwasaki UTlEP-118(1983)]

- **Configurations generated by RBC and UKQCD collaborations**
  

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<thead>
<tr>
<th>L</th>
<th>a(fm)</th>
<th>m_l</th>
<th>m_s</th>
<th>m_π (MeV)</th>
<th>approx. # configs.</th>
<th># time sources</th>
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Relativistic Heavy Quark Action for the $b$-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin


- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\bar{p} a)$
  - Resulting errors will be of $O(\bar{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities

- Applies for all values of the quark mass
- Has a smooth continuum limit
Nonperturbative Tuning of the RHQ Action Parameters

> Start from an educated guess for our three parameters $m_0 a$, $c_P$, and $\zeta$

> Probe parameter space at seven points by measuring
  - spin-averaged mass: $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$
  - hyperfine-splitting: $\Delta_M = M_{B_s^*} - M_{B_s}$
  - ratio: $M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}}$

> Assume linearity to relate parameters and observables

> Obtain tuned parameters corresponding to physical $b$-quarks by requiring that $\overline{M}$ and $\Delta_M$ agree with experiment and that $M_1 = M_2$
Predictions for the Heavy-Heavy States

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the $B_s$-system we can predict bottomonium states and mass splittings and thereby test the method
- We find good agreement with experiment within errors

\[ \Upsilon = 9410(30)(38) \text{ MeV} \]
\[ \eta_b = 9350(33)(37) \text{ MeV} \]
\[ \chi_{b1} = 9851(35)(39) \text{ MeV} \]
\[ \chi_{b0} = 9808(35)(39) \text{ MeV} \]
\[ h_b = 9862(36)(39) \text{ MeV} \]

\[ M_{\Upsilon} - M_{\eta_b} = 49(02)(17) \text{ MeV} \]
\[ M_{\chi_{b1}} - M_{\chi_{b0}} = 38(01)(16) \]

\[ M [\text{GeV}] \quad \Delta [\text{MeV}] \]

- $\Upsilon$
- $\eta_b$
- $\chi_{b1}$
- $\chi_{b0}$
- $h_b$
**B-meson Decay Constant Calculation**

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- Computation performed with seven parameter box and interpolated to the tuned RHQ parameters
- Axial current is 1-loop $O(a)$ improved
- Use mostly nonperturbative renormalization
- Combined chiral and continuum extrapolation using heavy meson $\chi$PT
Mostly Nonperturbative Renormalization

For $f_B$, $f_{B_S}$ and $B \to \pi$ we compute mostly non-perturbative renormalization factors \textit{à la} [El-Khadra et al. Phys.Rev. D64 (2001) 014502]

$$Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}$$

- Compute $Z_V^{ll}$ and $Z_V^{bb}$ non-perturbatively and only $\varrho^{bl}$ perturbatively
- Enhanced convergence of perturbative series of $\varrho^{bl}$ w.r.t. $Z_V^{bl}$ because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- $\varrho^{bl}$ is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know $Z_V^{ll}$ [Y. Aoki et al. Phys.Rev. D83 (2011) 074508] and compute $Z_V^{bb}$ ourselves
Motivation

Decay Constant

Results

Conclusion

Determination of $Z_{V}^{bb}$

$Z_{V}^{bb} \times \langle B \mid V^{bb,0} \mid B \rangle = 2m_{B}$

$\frac{C_{2}^{B}(T)}{C_{3}^{B \rightarrow B}(T,t)} \lim_{T,t \to \infty} Z_{V}^{bb}$

$Z_{V}^{bb} = 5.237(12)$

$m_{\text{sea}} = 0.006$

$\chi^{2}/\text{dof} = 0.34, \quad p = 95\%$

$m_{\text{sea}}^{l} = 0.006$

PRELIMINARY

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<tr>
<th>$a_{24} m_{\text{sea}}^{l}$</th>
<th>$Z_{V}^{bb}$</th>
<th>$a_{32} m_{\text{sea}}^{l}$</th>
<th>$Z_{V}^{bb}$</th>
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<tr>
<td>0.010</td>
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<td>0.008</td>
<td>5.267(15)</td>
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Avg. (24) 10.093(25) Avg. (32) 5.2560(76)

PT (24) 1-loop 10.72(16)(0) PT (32) 1-loop 5.725(74)(1)

PT values: [http://physyhcal.lhnr.de](http://physyhcal.lhnr.de)
On the lattice we compute $\Phi_{Bq}$

$$f_B = \Phi_{Bq}^{\text{ren}} \cdot a_3^{3/2} \sqrt{M_{Bq}}$$

- Partially quenched data are highly correlated
- Variance-covariance matrix is statistically well resolved
- Linearly interpolate to get $f_{Bs}$ and fit to extrapolate to $f_B$
Preliminary Results $\Phi_{B_s}$

Data for $\Phi_{B_s}$ show no sea-quark mass dependence.

Average data at same lattice spacing and assume $a^2$ scaling to remove light-quark and gluon discretization errors.

Remaining heavy-quark discretization errors will be estimated with heavy-quark power counting and included in the systematic error budget.
Preliminary Results $\Phi_{B_d}$

- Fit only “chiral” data i.e. $a_{24} m_q < 0.01$ ($m_\pi < 420$ MeV)
  using an analytic function in the quark masses and lattice spacing
  $\Phi_B = \Phi_0 \left[ 1 + c_{sea} m_{sea}^l 2B/(4\pi f)^2 + c_{val} m_{val} 2B/(4\pi f)^2 + c_a a^2/(a_{32}^2 4\pi f)^2 \right]$

\[ \phi_{B_d} = 0.132(4) \rightarrow f_B = 198(8)\text{MeV} \]
Preliminary Results $\Phi_{B_s}/\Phi_{B_d}$

- Fit only “chiral” data i.e. $a_24m_q < 0.01$ ($m_\pi < 420$ MeV)
  using an analytic function in the quark masses and lattice spacing

$$\Phi_{B_s}/\Phi_B = R_\Phi \left[ 1 + c_{\text{sea}} m_{\text{sea}} l^2 B/(4\pi f)^2 + c_{\text{val}} m_{\text{val}} l^2 B/(4\pi f)^2 + c_a l^2/(a_{32}^2 4\pi f)^2 \right]$$

PRELIMINARY
Comparison

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<th>$f_{B_s}/f_B$</th>
<th>$2+1+1$</th>
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Observations

- SU(2) HMχPT is valid for $m_{u,d} \ll m_s$. Are our data “chiral” enough?
- Our data do not show visible signs of SU(2) chiral logarithms.
- Strong correlations among partially quenched data are troublesome.
  Are light valence-quark masses too close to each other?

Preliminary Results

- $f_{B_s} = 235(6)$ MeV
- $f_B = 198(6)$ MeV $\Rightarrow f_{B_s}/f_B = 1.19(5)$
- $f_{B_s}/f_B = 1.173(7) \Rightarrow f_B = 200(5)$ MeV
- Overall consistent results

Outlook

- We are finalizing the analysis of $f_B$, $f_{B_s}$ and $f_{B_s}/f_B$
- Next we start the computation of $B^0 - \overline{B^0}$ mixing
- Future data will be obtained at physical pions on the $48^3 \times 96$ and $64^3 \times 128$ Möbius domain-wall ensembles