23-28 June 2014 Columbia University, NY, USA
Friday (Parallel Session 6) 16:50-17:10

The $B \to \pi l\nu$ and $B_s \to K l\nu$ form factors from 2+1 flavors of domain-wall fermions and relativistic b-quarks

Taichi Kawanai (RBC-UKQCD collaboration)

Motivation

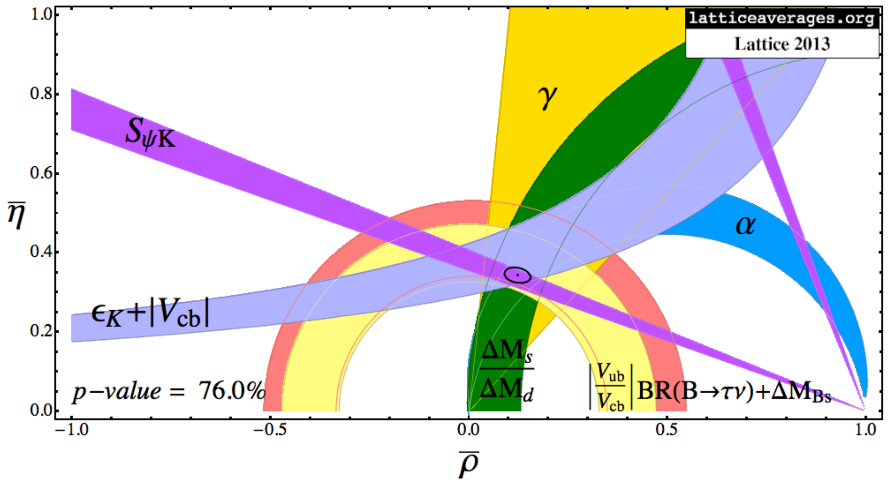
A precise determination of $|V_{ub}|$ allows a strong test of the standard model.

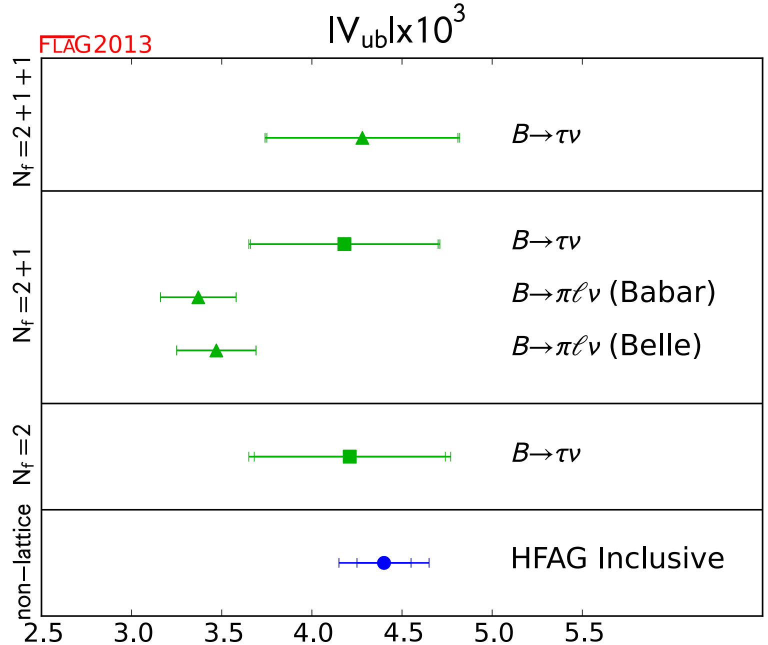
The constraint on the apex $(\bar{\rho},\ \bar{\eta})$ of the CKM triangle from $|V_{ub}|$ will strengthen tests of the Standard-Model CKM framework.

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2+\bar{\eta}^2}$$

- $\lambda = |V_{ud}|$ known to $\sim 1\ \%$
- $|V_{cb}|$ known to $\sim 2\ \%$

Dominant error (yellow ring) comes from the uncertainty of $|V_{ub}|$ (~6%).



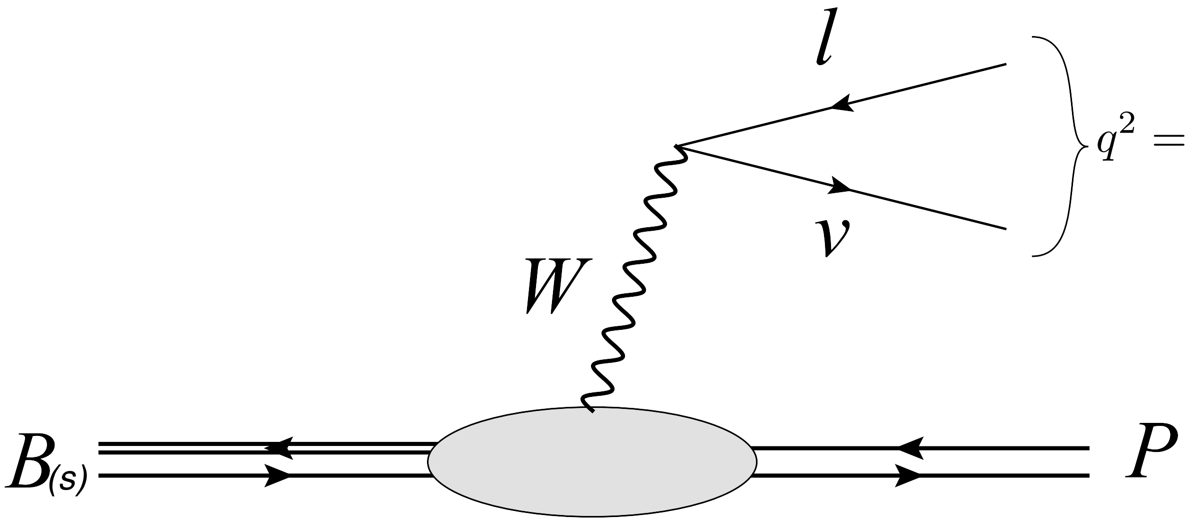


There has been a long standing puzzle in the determination of $|V_{ub}|$.

$\sim 3\sigma$ discrepancy between exclusive ($B\to\pi l\nu$) and inclusive ($B\to X_u l\nu$) determination.

Exclusive determination of $|V_{ub}|$

$f_+(q^2)$ is crucial for the determination of the CKM matrix element $|V_{ub}|$.



$$q^2 = m_{B_{(s)}}^2 + m_P^2 - 2m_{B_{(s)}} E_P$$

- The exclusive $B \to \pi l \nu$ semileptonic decay allows the determination of $|V_{ub}|$ via:

Goal

$$\boxed{\frac{d\Gamma}{dq^2}} = \frac{G_F^2}{192\pi^3 m_{B_{(s)}}^3} \left[(m_{B_{(s)}}^2 + m_P^2 - q^2)^2 - 4m_{B_{(s)}}^2 m_P^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|$$

Experiment — Known factor — Hadronic part — CKM matrix

- Experiment can only measure the CKM matrix element times hadronic form factor.
- The hadronic form factor must be computed nonperturbatively via lattice QCD.

Form-factor definitions

- Non-perturbative form factors $f_+(q^2)$ and $f_0(q^2)$ parametrize the hadronic matrix element of the $b \to u$ quark flavor-changing vector current V_μ .

$$\langle P|V_\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu$$

- On the lattice, we calculate the form factors $f_{||}$ and $f_\perp$.
  - ▶ Proportional to vector current matrix elements in the $B_{(s)}$ meson rest frame:

$$\begin{aligned} f_{||}(E_P) &= \langle P|V_0|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \\ f_\perp(E_P)p_i &= \langle P|V_i|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \end{aligned}$$

  - ▶ Easy to relate to the desired form factor $f_+(q^2)$ and $f_0(q^2)$.

$$\begin{aligned} f_0(q^2) &= \frac{\sqrt{2m_{B_{(s)}}}}{m^2_{B_{(s)}} - m^2_P}\left[(m_{B_{(s)}} - E_P)f_{||}(E_P) + (E^2_P - m^2_P)f_\perp(E_P)\right] \\ f_+(q^2) &= \frac{1}{\sqrt{2m_{B_{(s)}}}}\left[f_{||}(E_P) + (m_{B_{(s)}} - E_P)f_\perp(E_P)\right] \end{aligned}$$

Lattice actions and parameters

- We use the 2+1 flavor dynamical **domain-wall** fermion gauge field configurations generated by the RBC/UKQCD Collaborations.

C. Allton et al. (RBC/UKQCD Collaboration), Phys. Rev. D78, 114509 (2008)
Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)

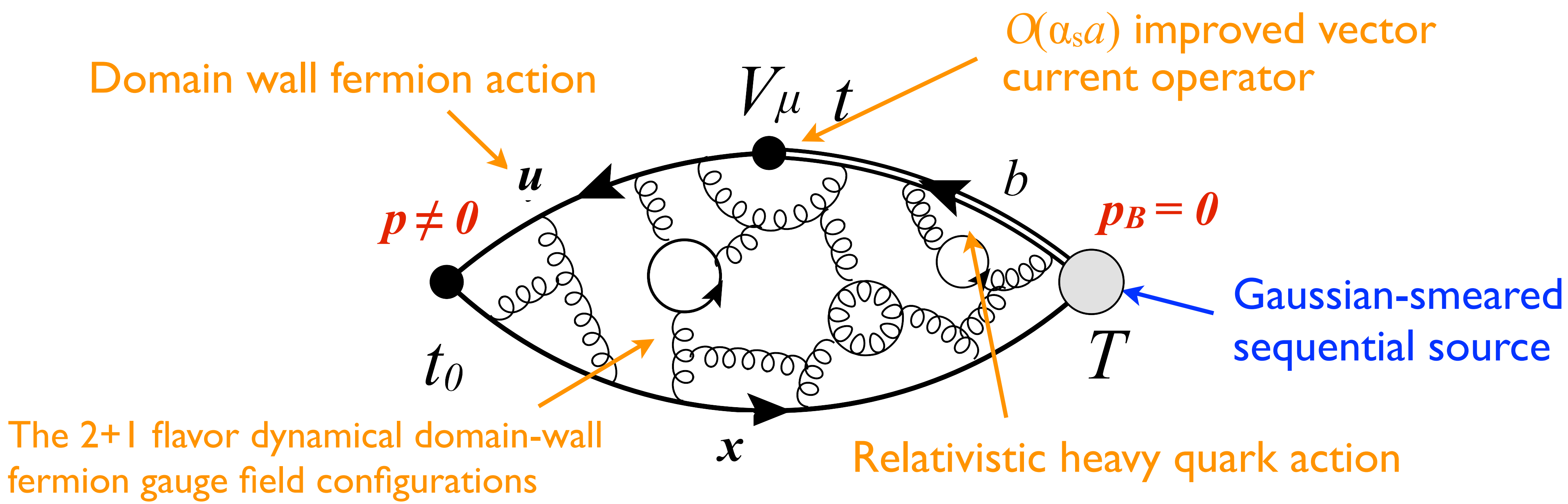
	$L{\times}T$	a [fm]	m_{ud}	m_s	m_π [MeV]	# of configs.	# of sources
Fine Lattice	32 × 64	≈ 0.08	0.004	0.03	289	628	2
	32 × 64	≈ 0.08	0.006	0.03	345	445	2
	32 × 64	≈ 0.08	0.008	0.03	394	544	2
Coarse Lattice	24 × 64	≈ 0.11	0.005	0.04	329	1636	1
	24 × 64	≈ 0.11	0.01	0.04	422	1419	1

- For the b-quark we use the relativistic heavy quark (RHQ) action developed by Li, Lin, and Christ in Refs. N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
  H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

- We use the nonperturbative determinations of the parameters of the RHQ action obtained in Y.Aoki et. al Phys. Rev. D 86, 116003 (2012).

- Provides important cross-check of existing $N_f = 2{+}1$ calculations using the MILC staggered ensembles.

Calculation of lattice form factors



- Extract the lattice form factor from the ratio of the 3pt function to 2pt functions:

J. A. Bailey et al. (Fermilab Lattice and MILC), Phys. Rev. D79, 054507 (2009).

$$R_{3,\mu}^{B_{(s)}\to P}(t,T) = \frac{C_{3,\mu}^{B_{(s)}\to P}(t,T)}{\sqrt{C_2^P(t)C_2^{B_{(s)}}(T-t)}}\sqrt{\frac{2E_P}{e^{-E_P t}e^{-m_{B_{(s)}}(T-t)}}}$$

$$f_\parallel^{\rm lat} = \lim_{t,T\to\infty} R_0^{B_{(s)}\to P}(t,T)$$

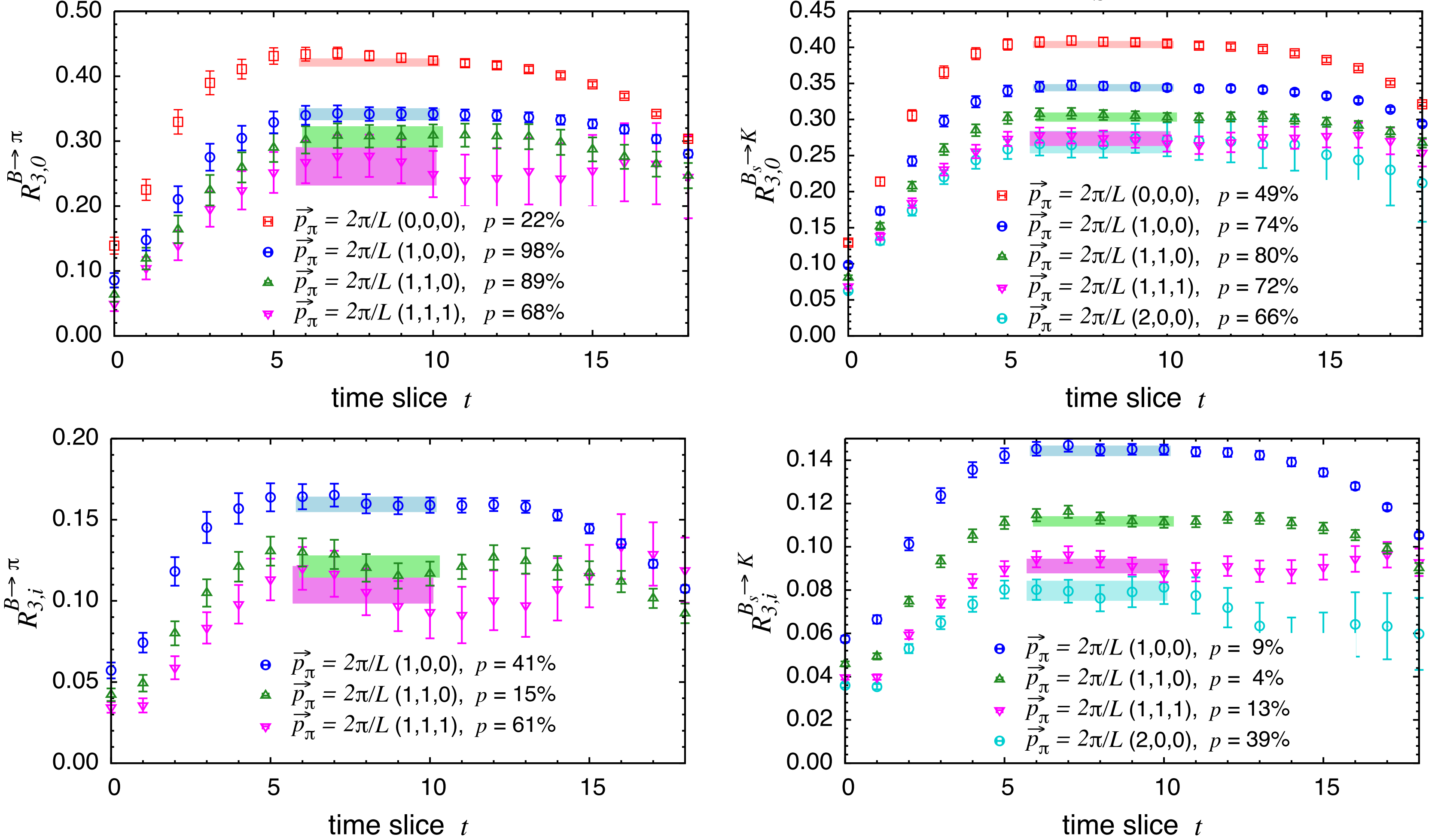
$$f_\perp^{\rm lat} = \lim_{t,T\to\infty} \frac{1}{p_P^i} R_i^{B_{(s)}\to P}(t,T)$$

Three-point correlator fits

$B \to \pi$ | $B_s \to K$

$f_\parallel$

$f_\perp$



- We use the lattice data up to $(1,1,1)$ for $B \to \pi$ and $(2,0,0)$ for $B_s \to K$.
- After a careful study, we fix source-sink separations $T = t_B - t_\pi$
- We fit the ratio to a plateau in the region $0 \ll t \ll T$.

Renormalization of lattice form factors

- The continuum form factors are given by

$$\boxed{\begin{aligned} f_{\|}(E_P) &= Z^{bl}_{V_0} \lim_{t,T\to\infty} R_0^{B_{(s)}\to P}(E_P,t,T) \\ f_{\perp}(E_P) &= Z^{bl}_{V_i} \lim_{t,T\to\infty} \frac{1}{p_P^i} R_i^{B_{(s)}\to P}(E_P,t,T) \end{aligned}}$$

- We calculate the heavy-light current renormalization factor Z_V^{bl} using the mostly nonperturbative method.

A. X. El-Khadra et al. Phys.Rev. D64, 014502 (2001)

$$Z^{bl}_{V_\mu} = \underbrace{\boxed{\rho^{bl}_{V_\mu}}}_{\approx 1} \sqrt{\boxed{Z_V^{bb} Z_V^{ll}}}$$

compute nonperturbatively

compute with 1-loop mean-field improved lattice perturbation theory

- ρ-factor calculated in PhySyHCAl (framework for automated lattice perturbation theory).

C. Lehner arXiv:1211.4013

- Z_V^{ll} obtained by the RBC/UKQCD collaborations by exploiting the fact $Z_A = Z_V$ for domain-wall fermions.

Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)

- Z_V^{bb} obtained from the matrix element of the $b \to b$ vector current between two B_s mesons.

N. H.Christ et al. (RBC/UKQCD Collaboration), arXiv:1404.4670

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

- Correlated simultaneous chiral-continuum fit ($m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$) to $f_\perp$ and $f_{||}$ data using Hard-pion NLO SU(2) χPT.
  - Strange quark integrated out
  - Applies to regime where $E_P >> m_\pi$

J. Bijnens and I. Jemos, Nucl. Phys. B 840, 54 (2010)

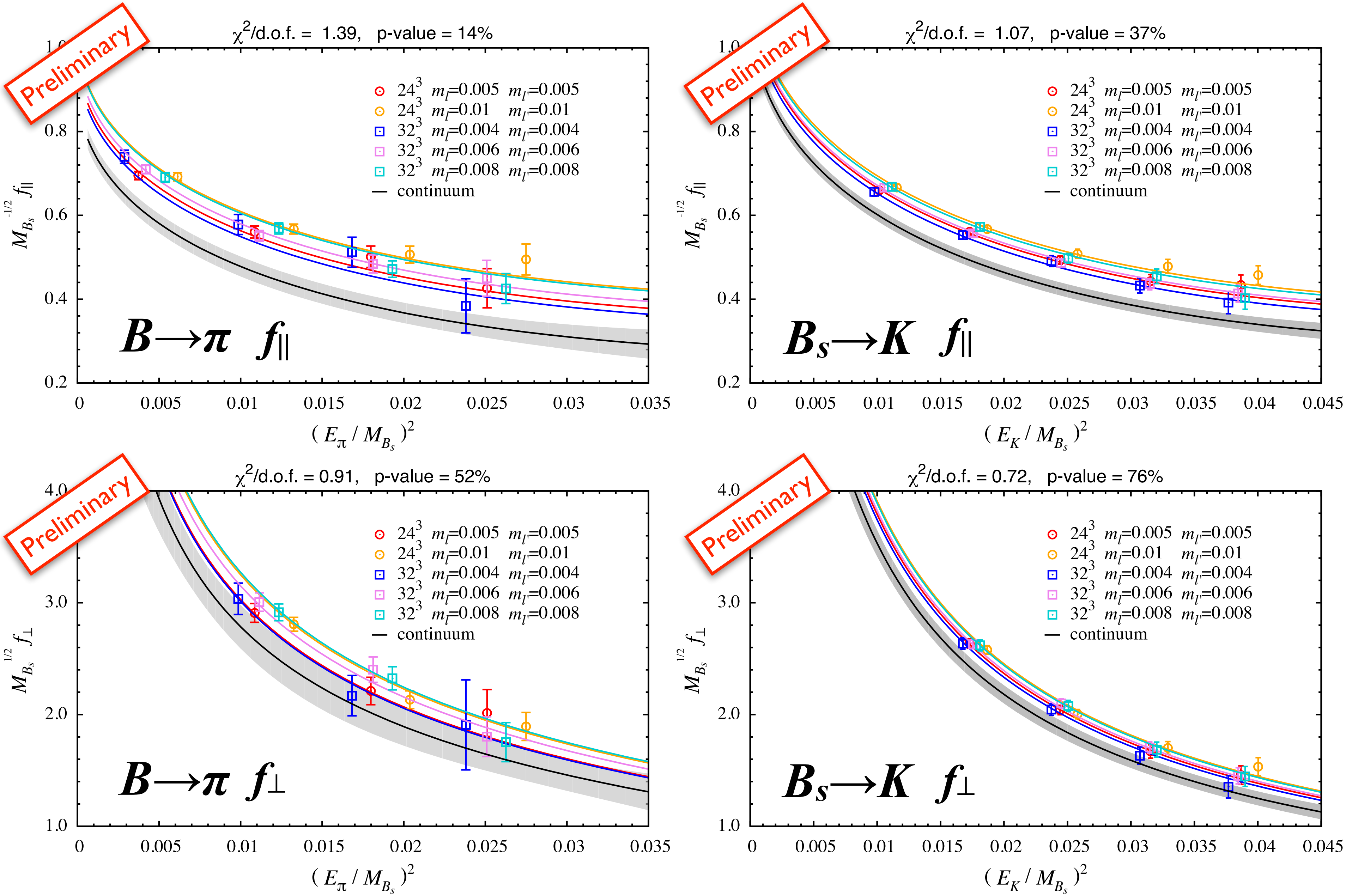
$$
\begin{aligned}
f_{\|}(m_\pi, E_P, a^2) &= c_{\|}^{(1)}\left(1 + (\delta f_{\|})^{\text{Hard-pion}} + c_{\|}^{(2)}\frac{m_\pi^2}{\Lambda^2} + c_{\|}^{(3)}\frac{E_P}{\Lambda} + c_{\|}^{(4)}\frac{E_P^2}{\Lambda^2} + c_{\|}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right) \\
f_{\perp}(m_\pi, E_P, a^2) &= \frac{1}{E + m_B^* - m_B} c_{\perp}^{(1)}\left(1 + (\delta f_{\perp})^{\text{Hard-pion}} + c_{\perp}^{(2)}\frac{m_\pi^2}{\Lambda^2} + c_{\perp}^{(3)}\frac{E_P}{\Lambda} + c_{\perp}^{(4)}\frac{E_P^2}{\Lambda^2} + c_{\perp}^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right).
\end{aligned}
$$

The function δf indicate non-analytic "log" functions of the pion mass.

- The hard-pion SU(2) logarithms are given by simply taking the limit $m_\pi / E_P \to 0$.

$$
\begin{aligned}
(4\pi f_\pi)^2 (\delta f_{\|,\perp}^{B\to\pi})^{\text{Hard-pion}} &= -\frac{3}{4}\left(3g^2 + 1\right) m_\pi^2 \log\left(\frac{m_\pi^2}{\Lambda^2}\right) \\
(4\pi f_\pi)^2 (\delta f_{\perp,\|}^{B_s\to K})^{\text{Hard-pion}} &= -\frac{3}{4} m_\pi^2 \log\left(\frac{m_\pi^2}{\Lambda^2}\right)
\end{aligned}
$$

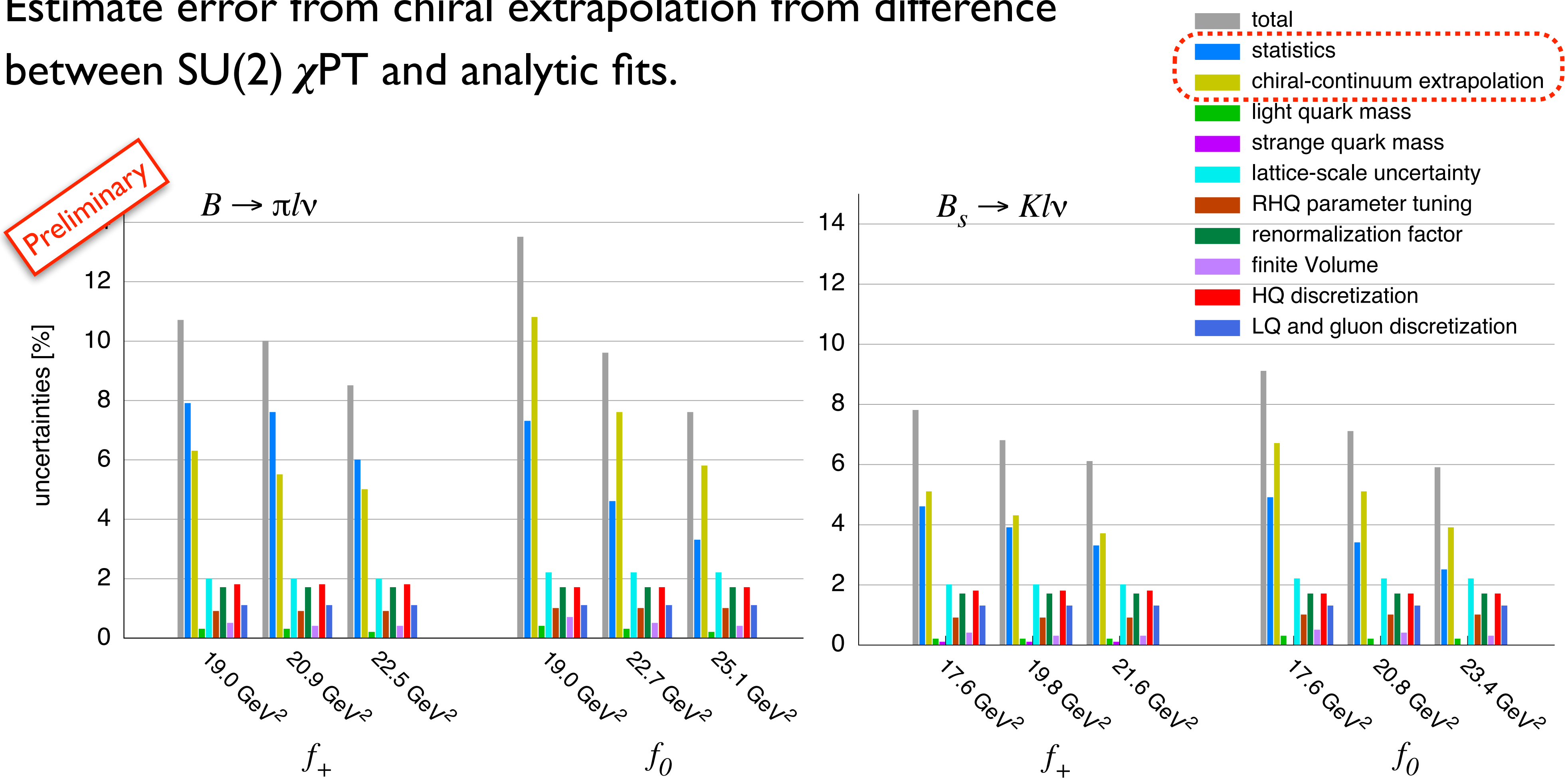
Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$



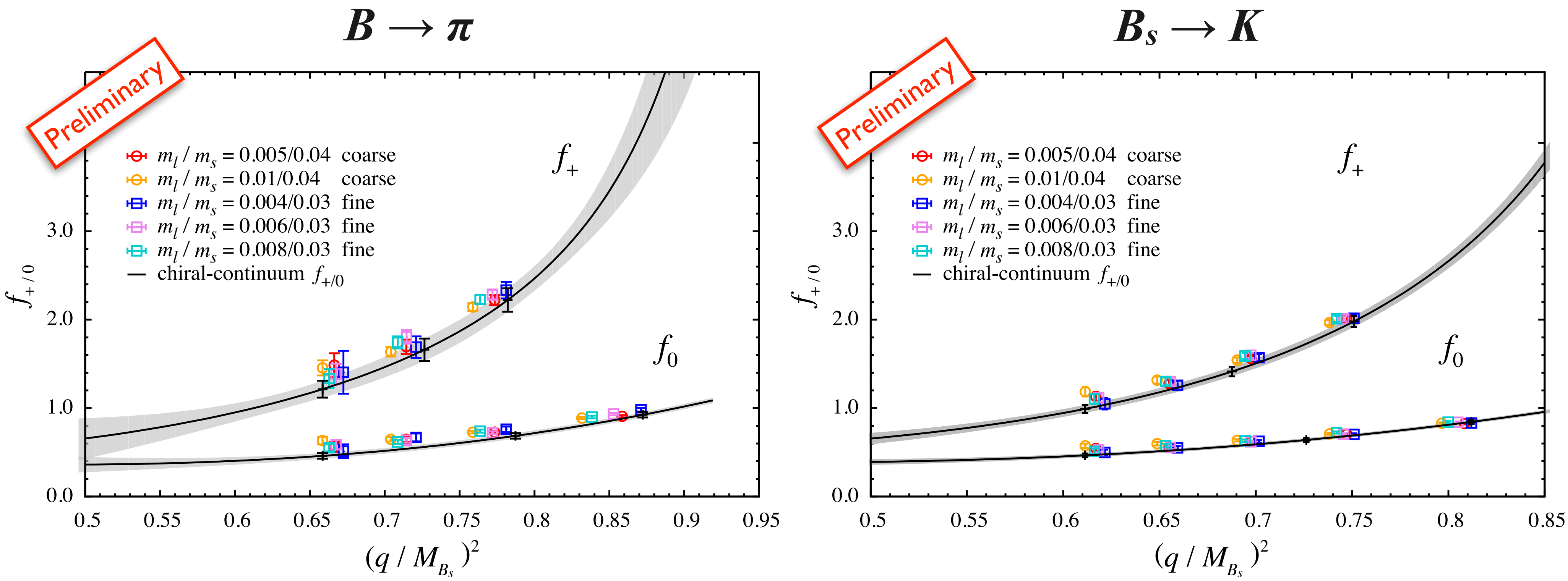
Black curves show chiral-continuum extrapolated $f_{||}$ and $f_{\perp}$ with statistical errors.

Preliminary error budgets

- Show error budgets for three q^2 points within the range of simulated lattice momenta.
- Dominant uncertainties from **statistics** and **chiral extrapolation.**
- Estimate error from chiral extrapolation from difference between SU(2) χPT and analytic fits.



Synthetic data for f_+ and f_0



Using the output of the chiral-continuum fit, we generate 3 synthetic data points for f_+ and f_0 (black) evenly spaced in the range of simulated z values to use in the extrapolation to q^2=0

z-expansion of f_+ and f_0

Boyd, Grinstein, Lebed, Phys.Rev.Lett. 74 (1995) 4603

We employ the model-independent z-expansion fit to extrapolate lattice results to full kinematic range.

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

- Consider mapping the variable q^2 onto a new variable z.

  semileptonic region

  $0 < q^2 < t_-$ $\rightarrow$ $-0.34 < z < 0.22$ (when we choose $t_0 = 0.65 t_+$)

- The form factor $f(q^2)$ is analytic in the semileptonic region except at B^* pole.
  → $f(q^2)$ can be expressed as convergent power series.

$$f(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a^{(k)}(t_0) z(q^2, t_0)^k$$

contains subthreshold poles

Arbitrary analytic function which affects the numerical values of the series coefficients

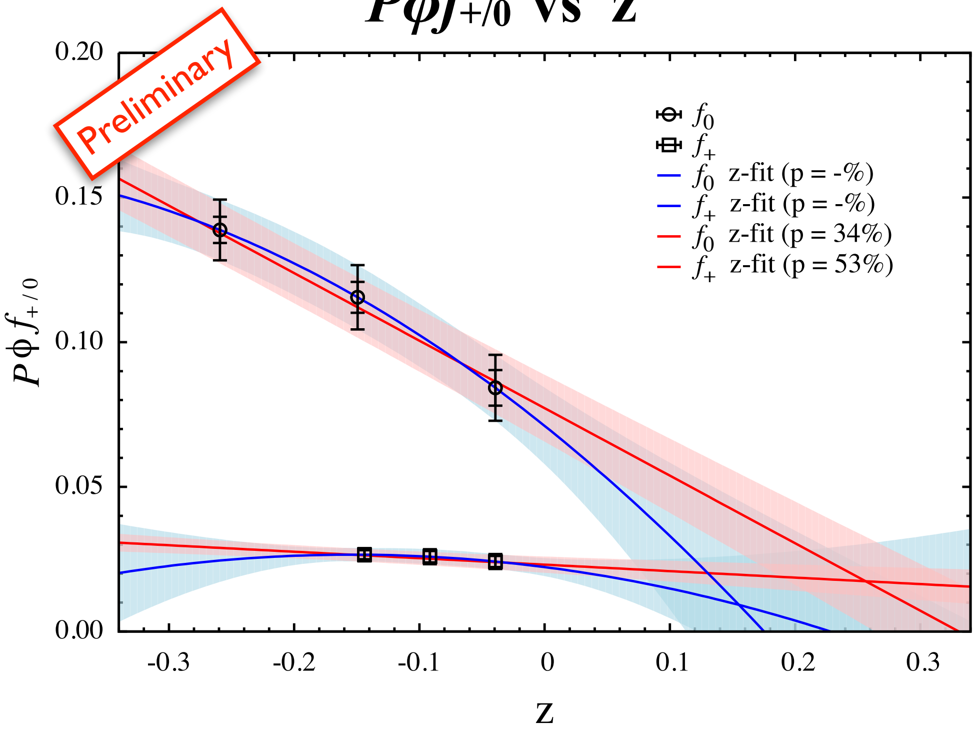
- The sum of the series coefficients is bounded by unitarity.

$$\sum_{k=0}^{N} {a^{(k)}}^2 \leq 1$$

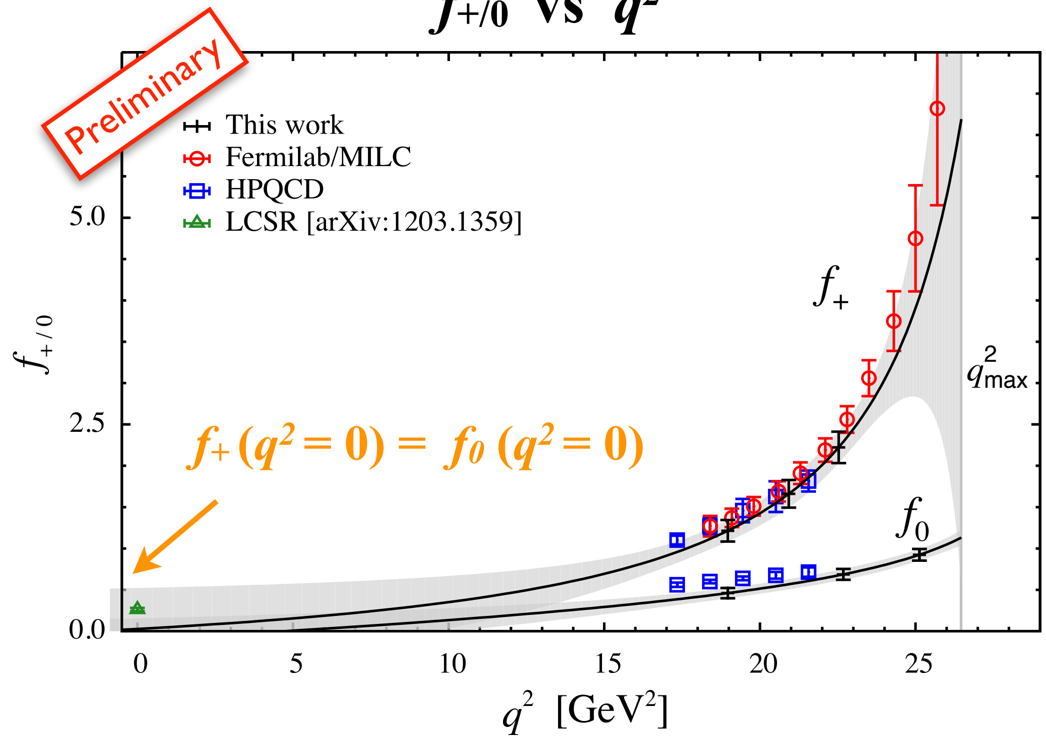
- Therefore this bound combined with the small $|z|$ ensures that only a small number of terms is needed to accurately describe the shape of the form factor.

z-expansion of f_+ and f_0

$P\phi f_{+/0}$ vs z



$f_{+/0}$ vs q^2



Our data determines normalization and slope, but only loosely constrains curvature.

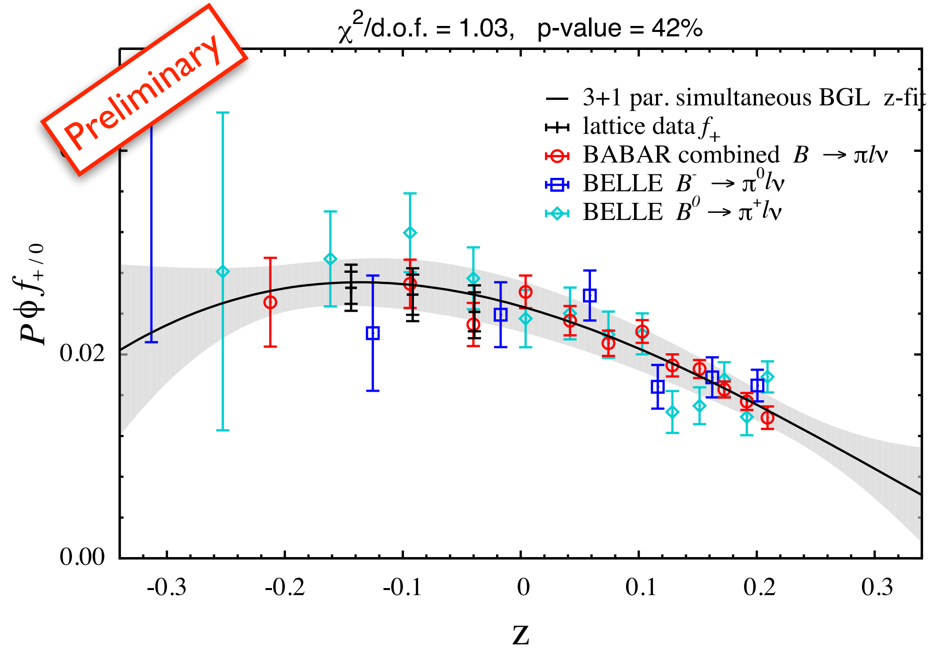
Preliminary

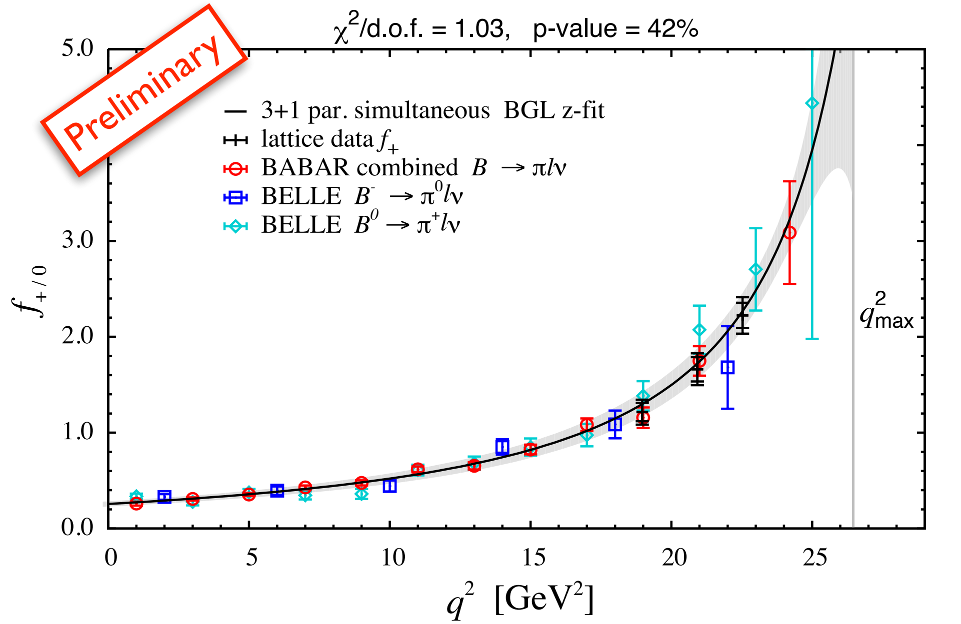
# of par.	$a_+^{(0)}$	$a_+^{(1)}/a_+^{(0)}$	$a_+^{(2)}/a_+^{(0)}$	χ^2/d.o.f	p-value	$a_0^{(0)}$	$a_0^{(1)}/a_0^{(0)}$	$a_0^{(2)}/a_0^{(0)}$	χ^2/d.o.f	p-value
2	0.0231(28)	-0.97(50)		0.40	53%	0.077(12)	-3.02(55)		0.91	34%
3	0.0223(31)	-2.5(2.5)	-8(13)	—	–%	0.071(13)	-4.9(1.9)	-4.7(5.0)	—	–%

Determination of $|V_{ub}|$

Now add experimental data to z-fit to obtain $|V_{ub}|$.

BaBar Collaboration, Phys. Rev. D 86, 092004 (2012) [arXiv:1208.1253 [hep-ex]]
Belle Collaboration, Phys. Rev. D 88, no. 3, 032005 (2013) [arXiv:1306.2781 [hep-ex]]





- q^2 dependence of lattice form factor agrees well with experiment.
- Experimental measurements determine both slope and curvature well.
- Error on normalization (and hence $|V_{ub}|$) saturates with 3-parameter z-fit.

Preliminary

# of par.	$a_+^{(1)}/a_+^{(0)}$	$a_+^{(2)}/a_+^{(0)}$	$a_+^{(3)}/a_+^{(0)}$	$\lvert V_{ub}\rvert \times 10^3$	χ^2/d.o.f	p-value
2+1	-1.76(22)			4.20(37)	1.42	6%
3+1	-1.22(19)	-3.6(1.2)		3.54(36)	1.03	42%
4+1	-1.32(30)	-4.0(1.5)	4(8)	3.53(36)	1.06	38%

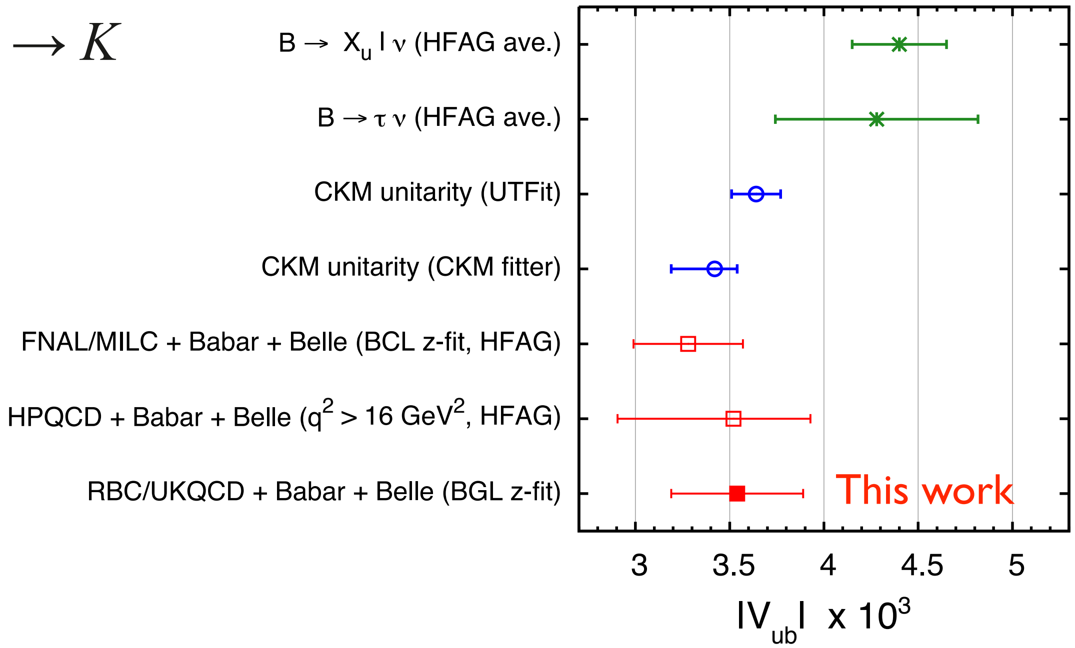
Conclusions and future prospects

- We have calculated the $B \to \pi$ and $B_s \to K$ form factors using 2+1 flavor dynamical domain-wall fermion gauge field configurations with relativistic heavy quark action.

- Provide important independent check on existing calculations using staggered light quarks.

- Will present final results for $B \to \pi$ and $B_s \to K$ lattice form factors as coefficients of the z-expansion and their correlations.

- $|V_{ub}|$ is determined by combined z-fit with experimental data from Babar and Belle to about 10% precision.

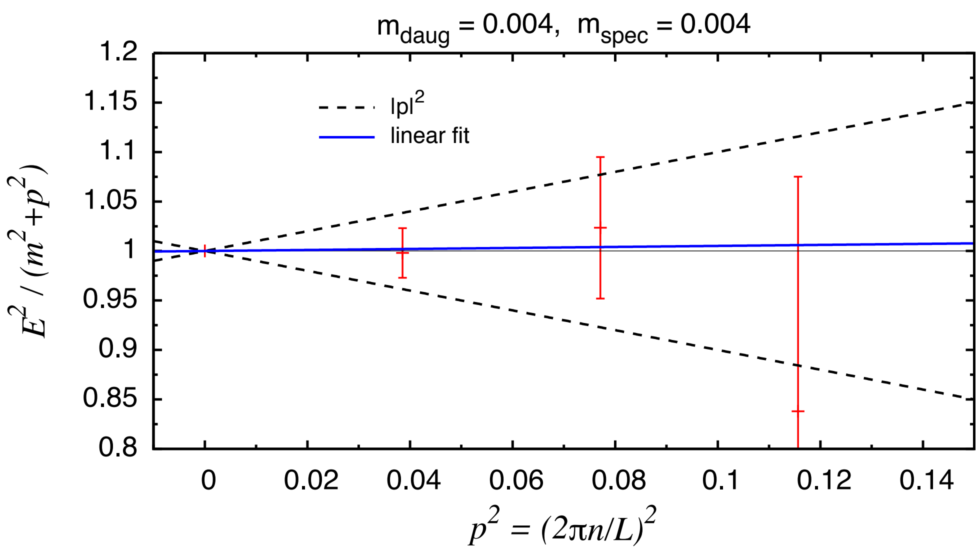


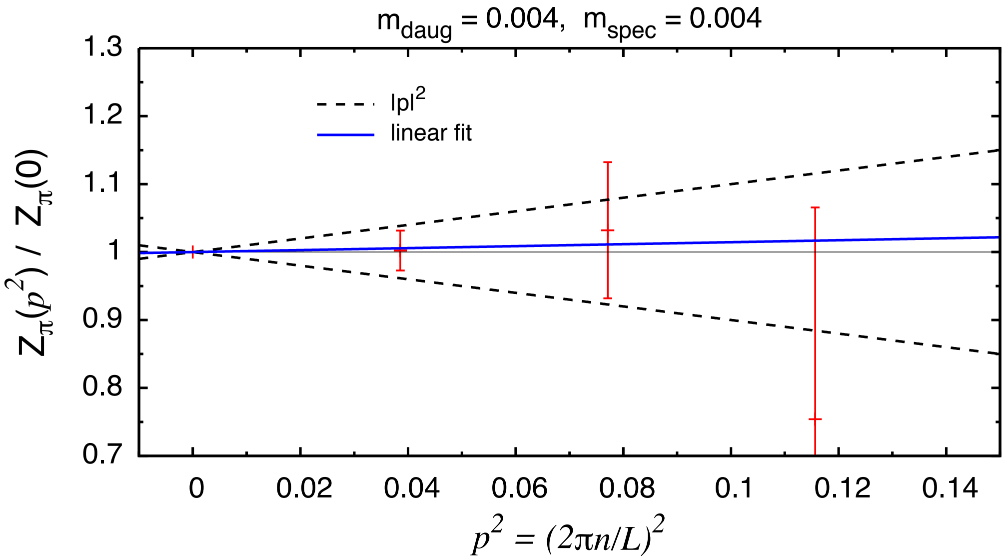
Still to do:

- Implement unitarity and heavy-quark constraints on sum of coefficients.
- Compare with result using BCL parameterization.

Backup slides

Dispersion relation and amplitude Z_π





- The pion energies satisfy the continuum dispersion relation: $E_\pi^2 = |\vec{p}_\pi|^2 + m_\pi^2$
- The pion amplitude $Z_\pi = |\langle 0|\mathcal{O}_\pi|\pi\rangle|$ is independent of momentum

$O(a)$ improved vector current operator

The heavy-light current operator at tree level is

$$V_{\mu,0}(x) = \bar{q}(x)\mathcal{O}_{\mu,0}Q(x), \quad \mathcal{O}_{\mu,0} = \gamma_\mu$$

Four single derivative operators are needed for O(a) improvement.

$$\boxed{\begin{array}{rcl} \mathcal{O}_{1,\mu} & = & 2\overrightarrow{D}_\mu \\ \mathcal{O}_{2,\mu} & = & 2\overleftarrow{D}_\mu \\ \mathcal{O}_{3,\mu} & = & 2\gamma_\mu\gamma_i\overrightarrow{D}_i \\ \mathcal{O}_{4,\mu} & = & 2\gamma_\mu\gamma_i\overleftarrow{D}_i \end{array}}$$

The O(a) improved vector current operator is given by

$$\begin{array}{rrcl} \text{temporal } (\mu = 0): & \mathcal{O}_0^{\mathrm{imp}} & = & \mathcal{O}_{0,0} + c_3^{V_0}\mathcal{O}_{0,3} + c_4^{V_0}\mathcal{O}_{0,4} \\ \text{spatial } (\mu = i): & \mathcal{O}_i^{\mathrm{imp}} & = & \mathcal{O}_{i,0} + c_1^{V_i}\mathcal{O}_{i,1} + c_2^{V_i}\mathcal{O}_{i,2} + c_3^{V_i}\mathcal{O}_{i,3} + c_4^{V_i}\mathcal{O}_{i,4} \end{array}$$

Coefficients are determined by 1-loop lattice perturbation theory.

Relativistic heavy quark action for b-quarks

Heavy quark mass introduces discretization errors of $O((ma)^n)$.

- At bottom quark mass, it becomes severe: $m_b \sim 4$ GeV and $1/a \sim 2$ GeV, then $m_b a > O(1)$.

Relativistic heavy quark action (RHQ action)

$$S^{\mathrm{RHQ}} = \sum_{n,n'} \bar{\psi}_n \left\{ \mathbf{m_0} + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a\vec{D}^2}{2} \right] - a \sum_{\mu\nu} \frac{i\mathbf{c_P}}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \psi'_n$$

- The Fermilab group showed that you can remove all errors of $O((ma)^n)$ by appropriately tuning the parameters of the anisotropic clover action

A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D55, 3933 (1997)

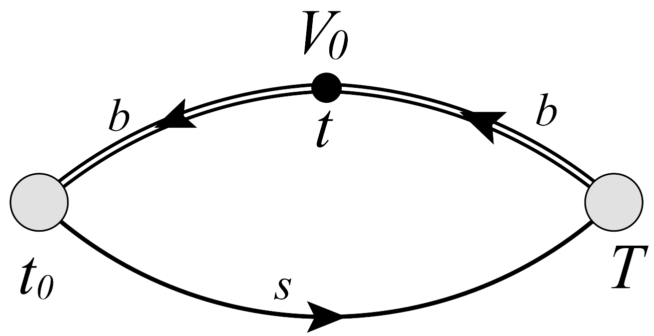
- Errors are of $O(a^2p^2)$.
- Li, Lin, and Christ showed that the parameters $\{m_0, \zeta, c_P\}$ can be tuned nonperturbatively.

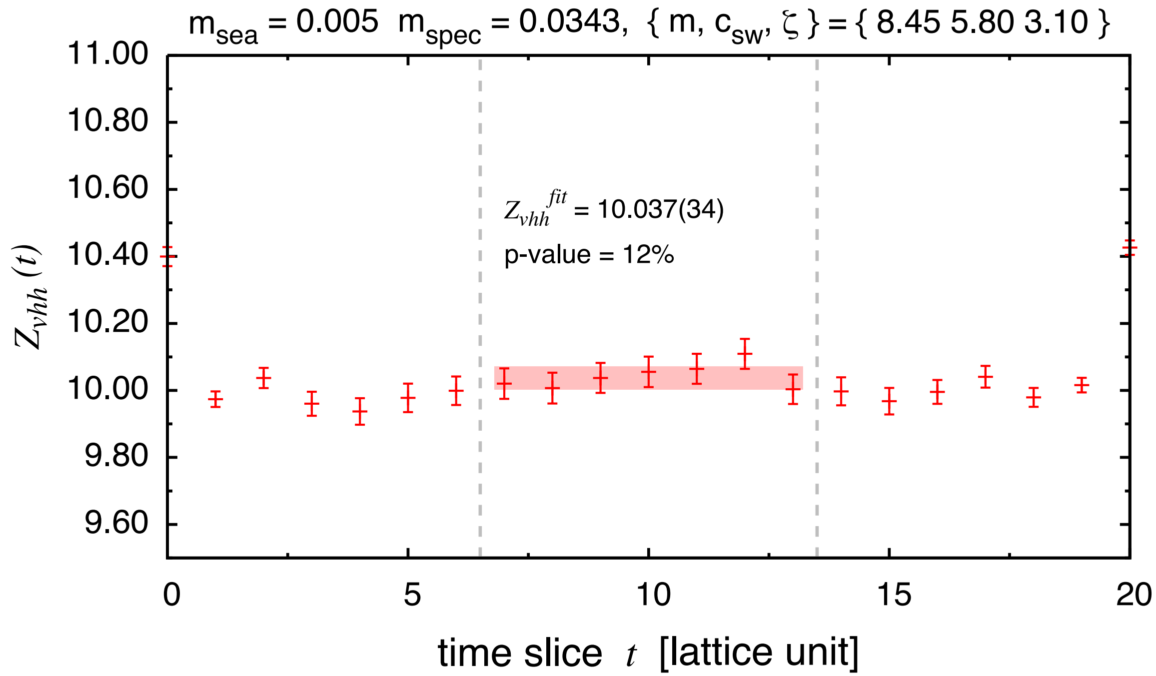
N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

- We use the results for the parameters of the RHQ action obtained for b-quarks in Y. Aoki et. al Phys. Rev. D 86, 116003 (2012)

Renormalization factor $Z_V{}^{bb}$

Norman H.Christ et al., arXiv:1404.4670





$$Z_V^{bb} \times \langle m_0^B | V_0^{bb} | m_0^B \rangle = 2m_B$$

$$\frac{C_2^B(T)}{C_{3,\mu}^{B\to B}(t,T)} \xrightarrow{t,T\to\infty} Z_V^{bb}$$

At tree level, the expression of $Z_V{}^{bb}$ is given by

$$Z_V^{bb} = u_0 \exp(M_1), \quad M_1 = \log[1+\tilde{m}_0], \quad \tilde{m}_0 = \frac{m_0}{u_0} - (1+3\zeta)(1-\frac{1}{u_0})$$

Here $m_0 = 7.80$, $\zeta = 3.20$, $u_0 = 0.8757$.

$$\begin{array}{rcl} \textbf{NP} & : & Z_V^{bb} = 10.037(34) \\ \textbf{tree level} & : & Z_V^{bb} = 9.993 \end{array}$$