B-physics with domain-wall light quarks and nonperturbatively tuned relativistic \( b \)-quarks

RBC and UKQCD collaborations

Oliver Witzel
Center for Computational Science

Lattice 2014, New York, USA
Motivation: CKM unitarity triangle fit

- $B^0 - \bar{B}^0$ mixing enables precise constraint on apex of the CKM unitarity triangle
- Requires nonperturbative determination of the ratio of $B$-meson mixing matrix elements $\xi^2 = f_{B_s}^2 B_{B_s}/f_{B_d}^2 B_{B_d}$
- Lattice still the dominant source of error

Motivation: rare $B$-decays

$B \rightarrow \tau \nu$ [UTfit Phys.Lett. B687 (2010) 61]
- $f_B$ is needed for the Standard-Model prediction of $BR(B \rightarrow \tau \nu)$
- Potentially sensitive to charged-Higgs exchange due to large $\tau$ mass

- $f_{B_s}$ is needed for Standard-Model prediction of $BR(B_s \rightarrow \mu^+\mu^-)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Measured by LHCb with $3.5\sigma$ significance [LHCb Phys.Rev.Lett. 110 (2013) 02180], at EPS2013: combination of LHCb and CMS results gives $> 5\sigma$ significance — in agreement with SM

Both are sensitive to new physics!
Our project

- Use domain-wall light quarks and nonperturbatively tuned relativistic $b$-quarks to compute at few-percent precision
  - $B^0 - \bar{B}^0$ mixing
  - Decay constants $f_B$ and $f_{B_s}$
  - $B \rightarrow \pi \ell \nu$ and $B_s \rightarrow K \ell \nu$ form factors [T. Kawanai, Fri. 16:50]
  - $g_{B^*B\pi}$ coupling constant [PoS(Lattice 2013)408]

- Provides important cross-check of other $N_f = 2 + 1$ determinations using the MILC staggered ensembles
2+1 flavor domain-wall gauge field configurations

- Domain-wall fermions for the light quarks (u, d, s)

- Iwasaki gauge action [Iwasaki UTHEP-118(1983)]

- Configurations generated by RBC and UKQCD collaborations

<table>
<thead>
<tr>
<th>L</th>
<th>a(fm)</th>
<th>m_l</th>
<th>m_s</th>
<th>M_π (MeV)</th>
<th># configs.</th>
<th># time sources</th>
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Relativistic heavy quark action for the $b$-quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin

  by tuning all parameters of the clover action non-perturbatively;
  close relation to the Tsukuba formulation

- Heavy quark mass is treated to all orders in $(m_b a)^n$
  - Applies for all values of the quark mass

- Expand in powers of the spatial momentum through $O(\bar{p} a)$
  - Resulting errors will be of $O(\bar{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors
    of the same size as in light-light quantities

- Tune parameters to physical $b$-quarks i.e. require:
  - Spin-averaged mass and hyperfine splitting of the $B_s$-meson agree
    with experiment
  - $B_s$-meson rest and kinetic masses are equal

- Validated by computing $b\bar{b}$ masses and splittings [Phys.Rev. D86 (2012) 116003]
B-meson decay constant

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- On the lattice we compute $\Phi_{Bq} f_B = \Phi_{Bq}^{\text{ren}} \cdot a_2^{-3/2} \sqrt{M_{Bq}}$
$O(a)$ improvement and operator renormalization

[^arXiv:1404.4670 [hep-lat]]

- Axial current is 1-loop $O(\alpha_S a)$ improved
  - Coefficient computed at one-loop in mean-field improved LPT

- Use mostly-nonperturbative renormalization scheme for $f_B$, $f_{B_s}$ and $B \to \pi \ell \nu$

\[
Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}
\]


- Flavor-diagonal factors $Z_V^{bb}$ and $Z_V^{ll}$ account for the bulk of the renormalization factor

- Use one-loop mean-field improved LPT for small correction $\varrho^{bl}$
  [C. Lehner http://physyhcal.lhnr.de]
Strategy for combined chiral-continuum extrapolation

[arXiv:1404.4670 [hep-lat]]

- Perform analysis in terms of dimensionless ratios over $M_{B_s}$
- Preferred fit: NLO SU(2) heavy meson chiral perturbation theory
- Alternative fit to estimate systematic error: linear fit
- Restrict to data points with $M^\text{val}_\pi < 420$ MeV

- $g_{B^*B\pi} = 0.57(8)$ [PoS(Lattice 2013)408]
- $f_\pi = 130.4$ MeV [PDG]
- $\Lambda_\chi = 1$ GeV
Chiral-continuum extrapolation of $f_B$ and $f_{B_s}/f_B$

[arXiv:1404.4670 [hep-lat]]

- Only data points with filled symbols included in the fit ($M_\pi < 420$ MeV)
- Statistical errors only
Continuum extrapolation of $f_{B_s}$

[arXiv:1404.4670 [hep-lat]]

- Data for $\Phi_{B_s}$ show no sea-quark mass dependence
- Average data at same lattice spacing
- Assume $a^2$ scaling to remove LQ and gluon discretization errors
- Statistical errors only
## Error budget

[arXiv:1404.4670 [hep-lat]]

<table>
<thead>
<tr>
<th>Source</th>
<th>( f_{B^0}(%) )</th>
<th>( f_{B^+}(%) )</th>
<th>( f_{B_s}(%) )</th>
<th>( f_{B_s}/f_{B^0}(%) )</th>
<th>( f_{B_s}/f_{B^+}(%) )</th>
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<td><strong>5.2</strong></td>
<td><strong>4.9</strong></td>
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Comparison with other results [arXiv:1404.4670 [hep-lat]]

<table>
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<tr>
<th>$f_B$</th>
<th>$f_{B_s}$</th>
<th>$f_{B_s}/f_B$</th>
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<tr>
<td>1.10</td>
<td>1.20</td>
<td>1.30</td>
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ETMC 2013 (TM+HQET) [1]
HPQCD 2013 (HISQ+NRQCD) [2]
HPQCD 2012 (HISQ+NRQCD) [3]
HPQCD 2012 (HISQ+HISQb/NRQCD) [3,4]
FNAL/MILC 2011 (Asqtad+Fermilab) [5]
RBC-UKQCD 2010 (DWF+HQET) [6]
ALPHA 2014 (Wilson+HQET) [7]
ETMC 2012 (TM+HQET) [8]

▶ Good agreement with other results

Future plan: physical pions

<table>
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<tr>
<th></th>
<th>$f_{B^0}(%)$</th>
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- To reduce errors we have to shorten or eliminate the chiral-extrapolation to the physical point
- This will also decrease the statistical uncertainties
Work in progress

Physical pion masses with Möbius DWF

- $48^3 \times 96, \, L_s = 24$ at $a^{-1} \approx 1.73$ GeV
- MDWF light quark propagators are generated
- Improving statistics using all-mode averaging
  
- Tuning of RHQ parameters is in progress
- First result for $f_{B_s}$ using 80% of the desired number of measurements

Finer lattice $a^{-1} \approx 3.1$ GeV

- $32^3 \times 64, \, L_s = 12, \, M_\pi \approx 360$ MeV
$B^0 - B^0$ mixing matrix element calculation

- Location of four-quark operator is fixed
- Location of $B$-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating from operator location
- Project out zero-momentum component using a Gaussian sink
- Tree-level $O(a)$-improvement of operators via HQ field rotation
- Finally we have a first set of data ready to be analyzed!
Conclusion

- Results for $f_B$, $f_{B_s}$ and $f_{B_s}/f_B$ are posted and submitted to PRD
  - Good agreement with results in the literature
  - Errors dominated by chiral-continuum extrapolation
- Already improving our errors by simulating with physical pion mass
- Another ensemble with finer lattice spacing will be added

- Measurements for $B - \overline{B}$ mixing are in progress

- Friday: $B \rightarrow \pi \ell \nu$ and $B_s \rightarrow K \ell \nu$ form factors