LATTICE 2015

The 33rd International Symposium on Lattice Field Theory
Kobe International Conference Center, Kobe, Japan
Tuesday, July 14 — Saturday, July 18, 2015

Improving our determinations of the decay constant f_B and the $B \to \pi l \nu$ semi-leptonic form factors using physical light quarks

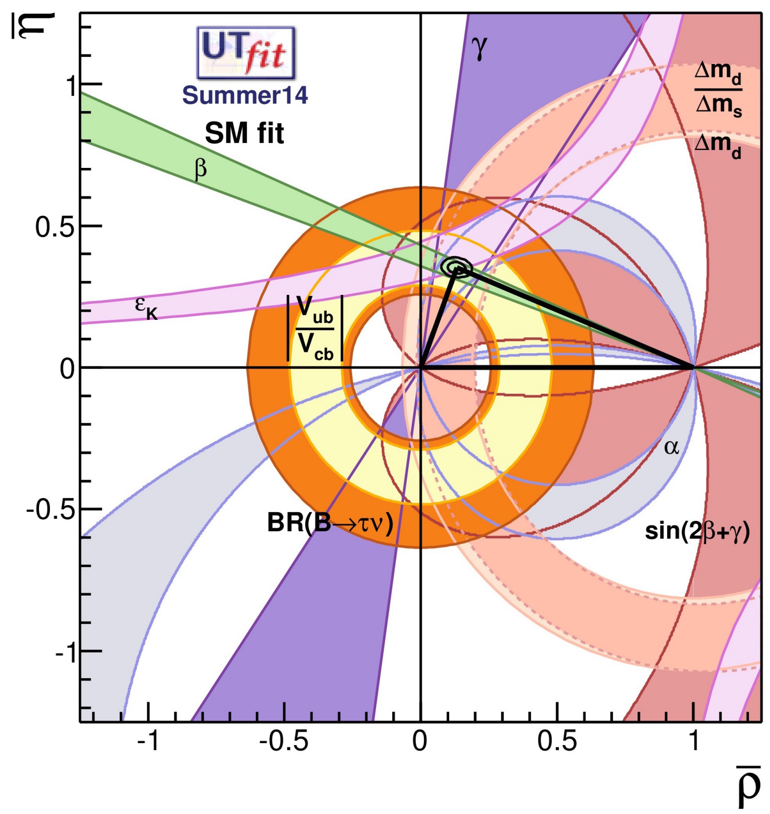
Taichi Kawanai (Jülich Supercomputing Centre)
in collaboration with RBC-UKQCD

JÜLICH FORSCHUNGSZENTRUM

Motivation

B-physics calculations on the lattice are of great phenomenological importance.

- Constraints on the apex $(\bar{\rho},\ \bar{\eta})$ of the CKM triangle will strengthen tests of the Standard Model in the quark-flavor sector.
  - V_{ub} from $B \to \pi l \nu$ (yellow ring)
    from $B \to \tau \nu$ (orange ring)
  - B^0-$\overline{B^0}$ mixing matrix elements (pink ring)
- B-physics allows us to identify new physics in rare B-decays.



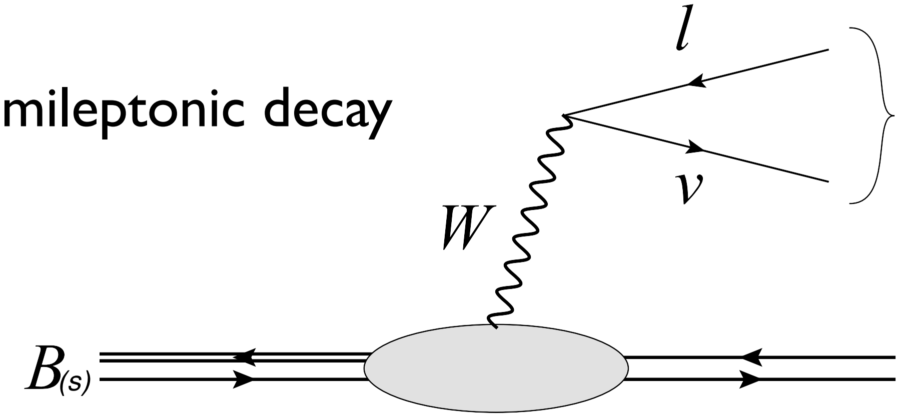
- Both experimental results and calculating hadronic contribution are needed.
- The hadronic contribution must be computed nonperturbatively via lattice QCD.

Experiment + Lattice → CKM matrix element

Exclusive determination of $|V_{ub}|$

$f_+(q^2)$ is crucial for the determination of the CKM matrix element $|V_{ub}|$.

B meson semileptonic decay



$$q^2 = m^2_{B_{(s)}} + m^2_P - 2m_{B_{(s)}}E_P$$

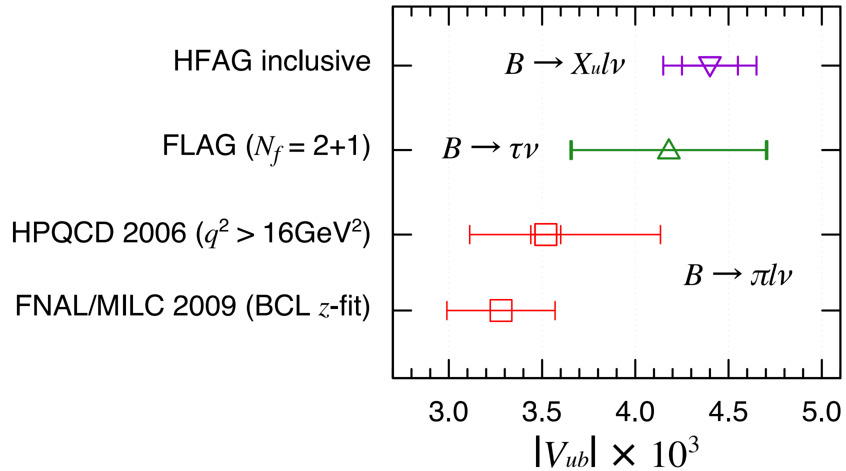
- The exclusive $B \to \pi l \nu$ semileptonic decay allows the determination of $|V_{ub}|$ via:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 m^3_{B_{(s)}}} \left[(m^2_{B_{(s)}} + m^2_P - q^2)^2 - 4m^2_{B_{(s)}} m^2_P \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|$$

Experiment — Known factor — Hadronic part — CKM matrix

- There has been a long standing puzzle in the determination of $|V_{ub}|$.

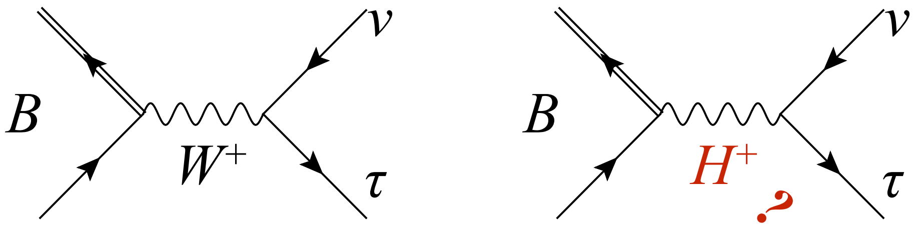
  $\sim 3\sigma$ discrepancy between exclusive ($B \to \pi l \nu$) and inclusive ($B \to X_u l \nu$) determination.



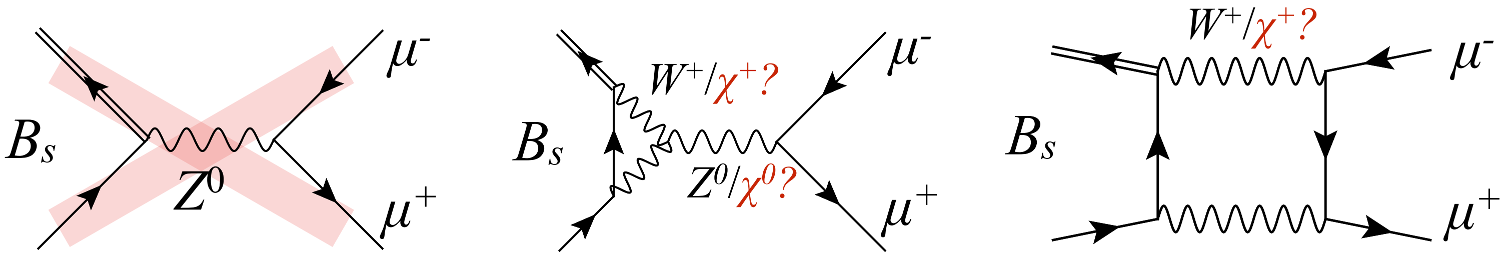
New Physics in rare B-decays?

f_B and f_{Bs} are important to identify new physics in Rare B decays.

- $B \to \tau\nu$ decay
  - f_B is needed for the Standard-Model prediction of BR($B\to\tau\nu$)
  - Potentially sensitive to charged-Higgs exchange due to large τ mass



- $B_s \to \mu^+\mu^-$ decay
  - f_{Bs} is needed for the Standard-Model prediction of BR($B_s \to \mu^+\mu^-$)
  - Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the Standard-Model.

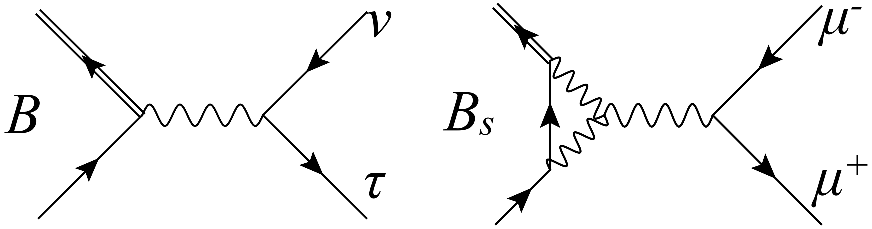


Higher-order flavor changing neutral current processes for the $B_s \to \mu^+\mu^-$ decay allowed in the SM.

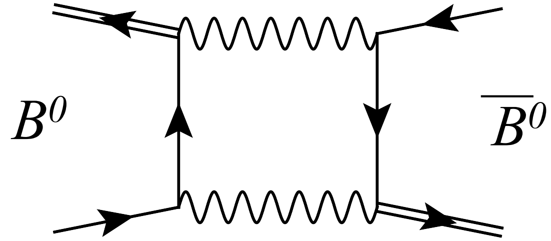
Our B-project

- $g_{B^*B\pi}$ coupling constat J.M. Flynn et al. [arXiv:1506.06413]

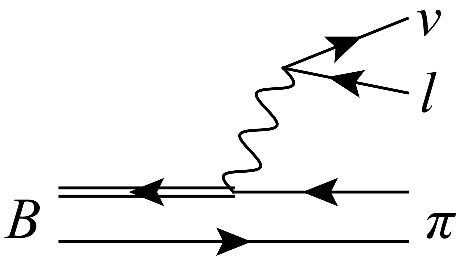
- Decay constant f_B and f_{Bs}
  J.M. Flynn et al. Phys. Rev. D91 (2015) 074510



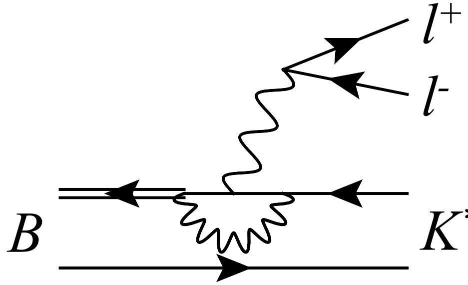
- Neutral B meson mixing



- $B \to \pi l \nu$ semileptonic decay
  N. H. Christ, et al. Phys. Rev. D91 (2015) 054502



- Rear semileptonic decay e.g. $B \to K^* l^+ l^-$



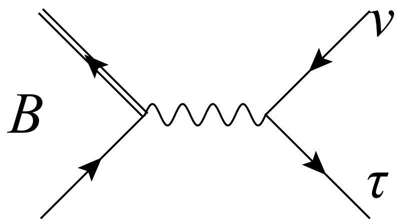
Our B-project

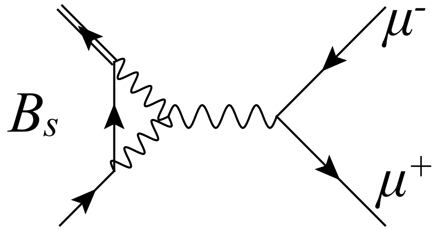
- $g_{B^*B\pi}$ coupling constat J.M. Flynn et al. [arXiv:1506.06413]

This talk by T.K.

- Decay constant f_B and f_{Bs}

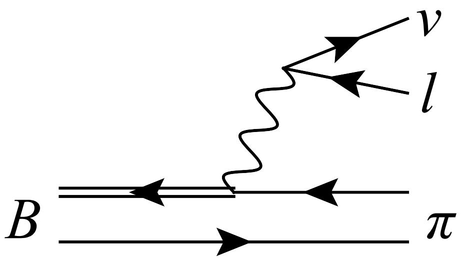
J.M. Flynn et al. Phys. Rev. D91 (2015) 074510





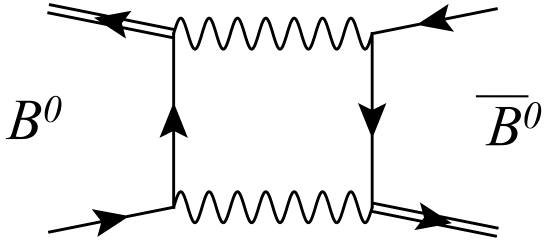
- $B \to \pi l \nu$ semileptonic decay

N. H. Christ, et al. Phys. Rev. D91 (2015) 054502



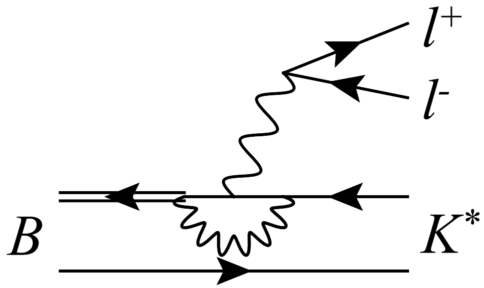
Next to next talk by O. Witzel

- Neutral B meson mixing

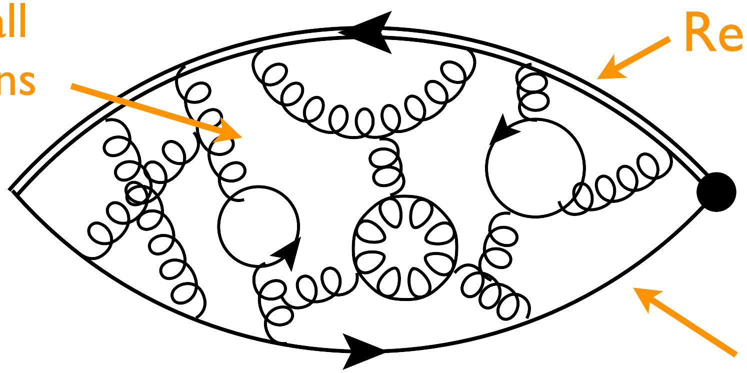


Next talk by E. Lizarazo

- Rear semileptonic decay e.g. $B \to K^* l^+ l^-$



Lattice actions and setup



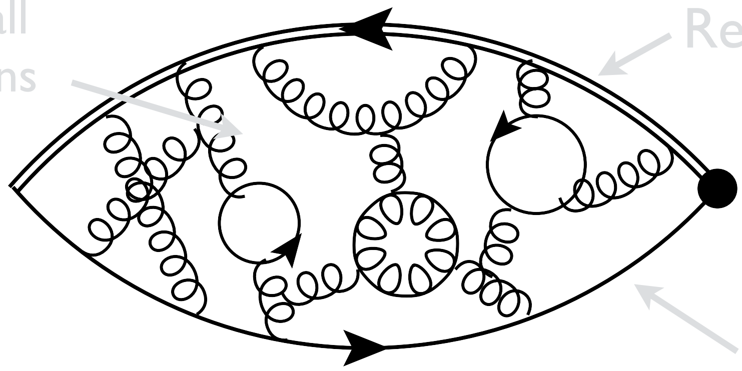
- We use the 2+1 flavor dynamical **domain-wall** fermion gauge field configurations generated by the RBC/UKQCD Collaborations. C. Allton et al., Phys. Rev. D78, 114509 (2008) Y. Aoki et al., Phys.Rev. D83, 074508 (2011)

$L^3 \times T$	a [fm]	m_{ud}	m_s	m_π [MeV]	# of configs.	# of sources
$32^3 \times 64$	≈ 0.08	0.004	0.03	289	628	2
$32^3 \times 64$	≈ 0.08	0.006	0.03	345	445	2
$32^3 \times 64$	≈ 0.08	0.008	0.03	394	544	2
$24^3 \times 64$	≈ 0.11	0.005	0.04	329	1636	1
$24^3 \times 64$	≈ 0.11	0.01	0.04	422	1419	1

- For the b-quark we use the relativistic heavy quark (RHQ) action developed by Li, Lin, and Christ. N. H. Christ et al., Phys.Rev. D76, 074505 (2007), H.-W. Lin et al., Phys.Rev. D76, 074506 (2007)
  - We use the nonperturbatively tuned parameters of the RHQ action.

Y.Aoki et. al Phys. Rev. D 86, 116003 (2012)

Lattice actions and setup



- We use $O(\alpha_s a)$ improved current operator with factors computed by lattice PT.

C. Lehner arXiv:1211.4013

- We calculate the heavy-light current renormalization factor Z_V^{bl} using the mostly nonperturbative method.

A. X. El-Khadra et al. Phys.Rev. D64, 014502 (2001)

$$Z^{bl}_{V_\mu} = \underbrace{\rho^{bl}_{V_\mu}}_{\approx 1} \sqrt{Z^{bb}_V Z^{ll}_V}$$

≈1

compute with 1-loop mean-field improved lattice PT

compute nonperturbatively

- Z_V^{ll} obtained by the RBC/UKQCD collaborations by exploiting the fact $Z_A=Z_V$ for domain-wall fermions.

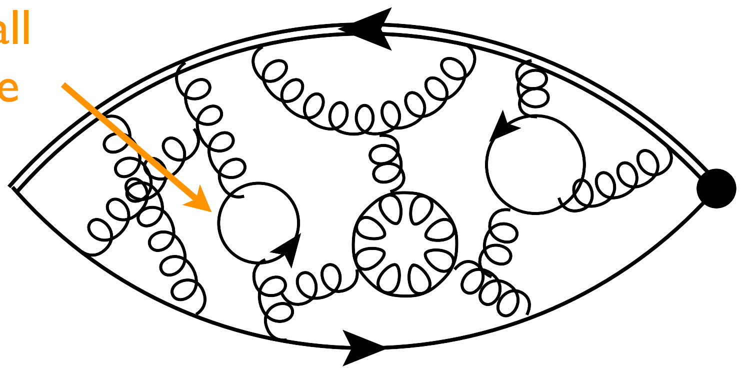
Y. Aoki et al., Phys.Rev. D83, (2911) 074508

- Z_V^{bb} obtained from the matrix element of the $b \to b$ vector current between two Bs mesons.

N. H.Christ et al.,Phys. Rev. D91 (2015) 074510

Lattice actions and setup

Möbius domain-wall
+ Iwasaki ensemble
(M_π ~ 139MeV)



We will show preliminary results with physical pions.

- RBC/UKQCD Möbius domain-wall+ Iwasaki ensemble (M_π ~ 139MeV).
  RBC, UKQCD collaborations [Xiv:1411.7017]
- We generate 1 "exact" and 81 "sloppy" propagators on a each configuration.
- We use the all-mode-averaging (AMA) method E. Shintani [arXiv:1402.0244]

$L^3 \times T$	a [fm]	m_{ud}	m_s	m_π [MeV]	# of configs.
$48^3 \times 96$	≈ 0.11	0.00078	0.0362	139	30

Decay constant

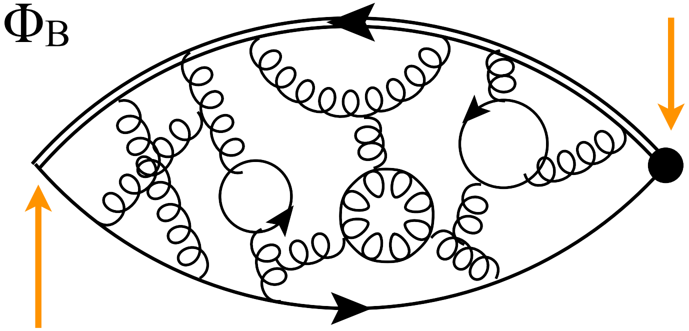
N. H. Christ, et al. Phys. Rev. D91 (2015) 054502

B-meson decay constat

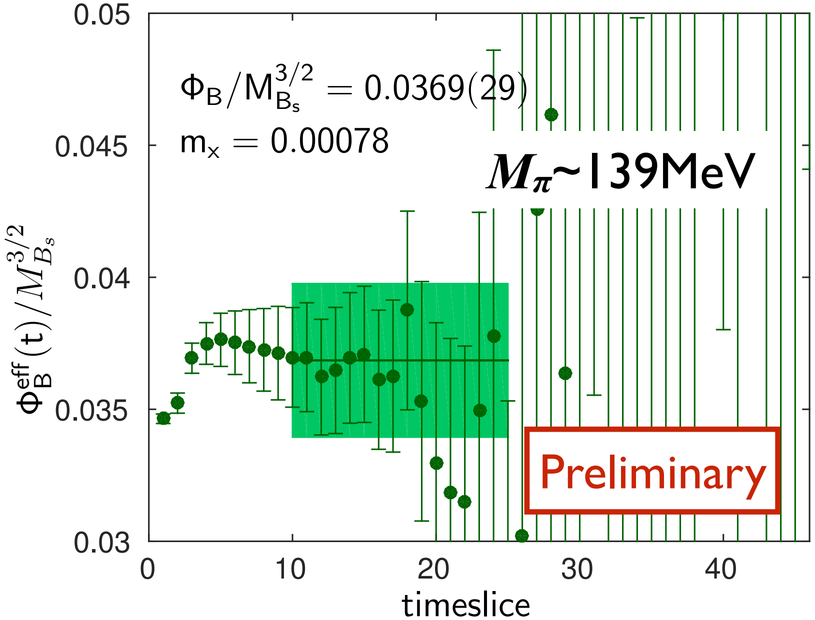
- On lattice, we compute decay amplitude Φ_B

$$\Phi^{\text{eff}}_{B_d} = \sqrt{2} \lim_{t_0 \ll t} \frac{C_{AP}(t,t_0)}{\sqrt{C_{PP}(t,t_0)}}$$

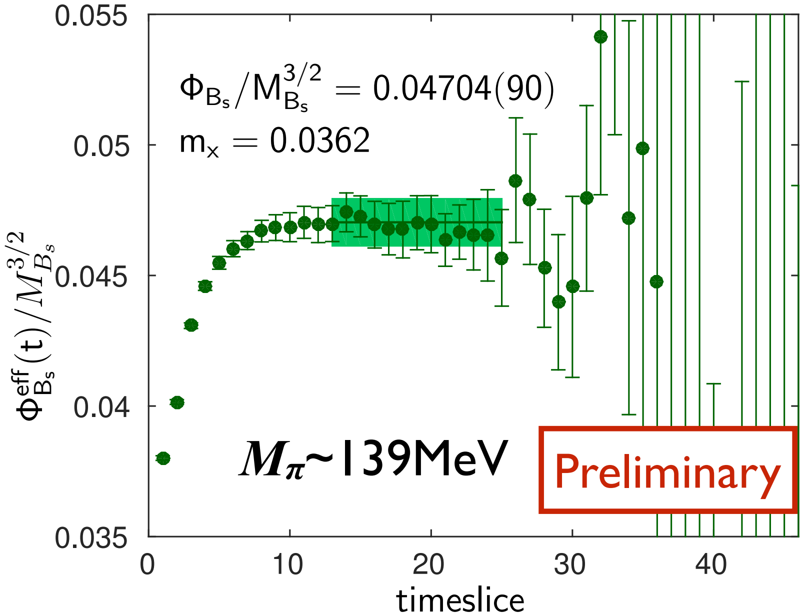
$$f_B = Z_\Phi \Phi^{\text{eff}}_{B_q} a^{-3/2} / \sqrt{M_{B_q}}$$



Decay amplitude Φ_B



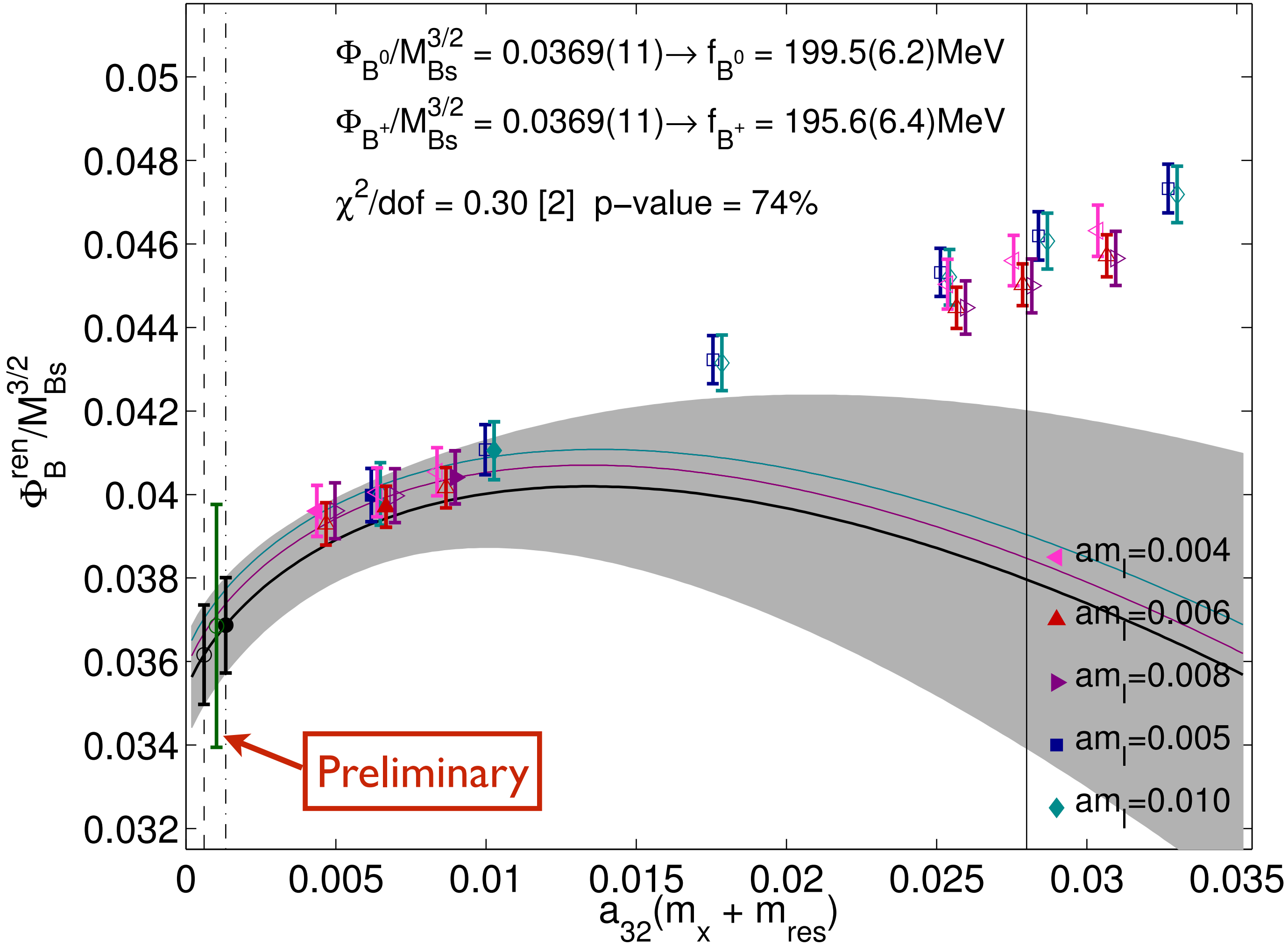
Decay amplitude Φ_{B_s}



Perform analysis in terms of dimensionless ratios over M_{Bs}

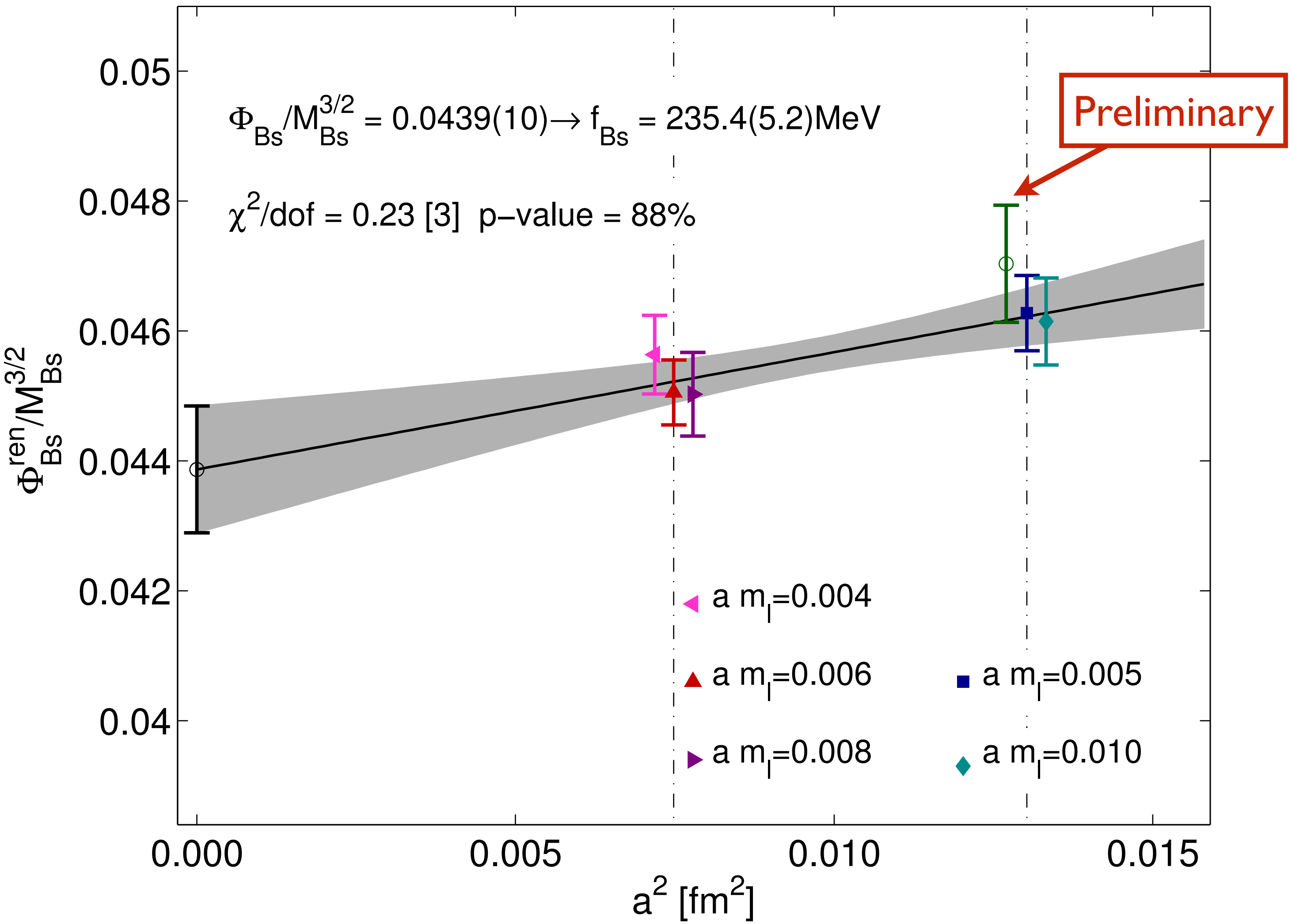
N. H. Christ, et al. Phys. Rev. D91 (2015) 054502

Chiral-continuum extrapolation of f_B



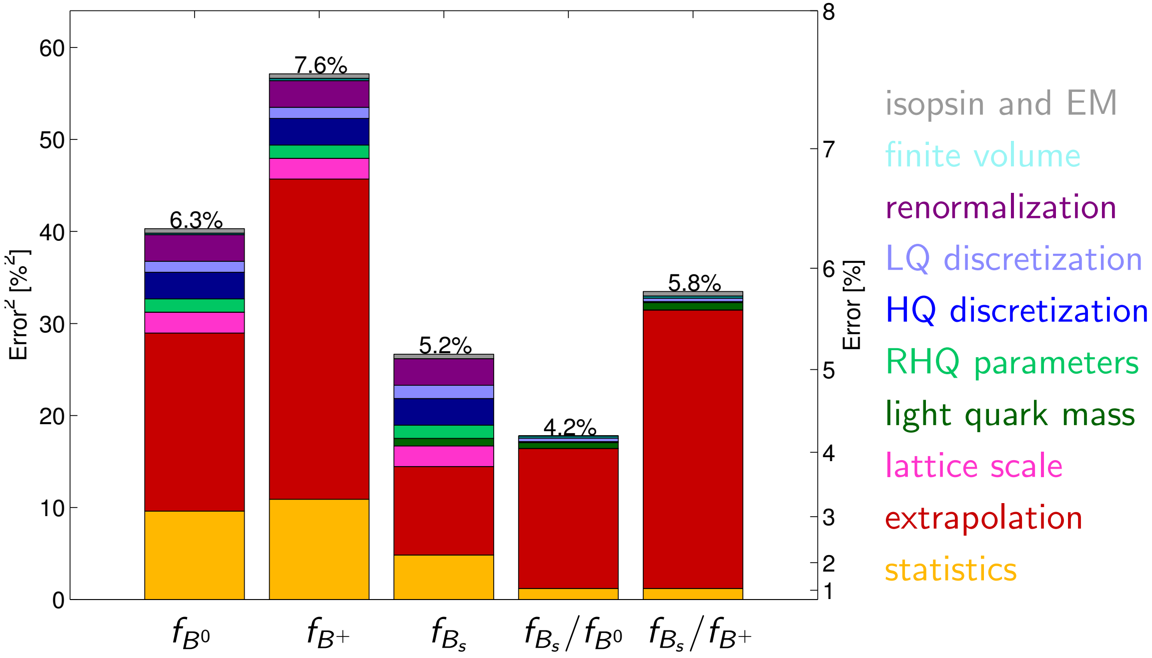
- NLO SU(2) HMχPT to data with unitary M_π
  - $g_{B^*B\pi}$=0.57(8), f_π=130.4MeV, Λ_χ=1GeV
- Only data points with filled symbols included in the fit (M_π<450MeV)
- Statistical errors only

Continuum extrapolation of f_{Bs}

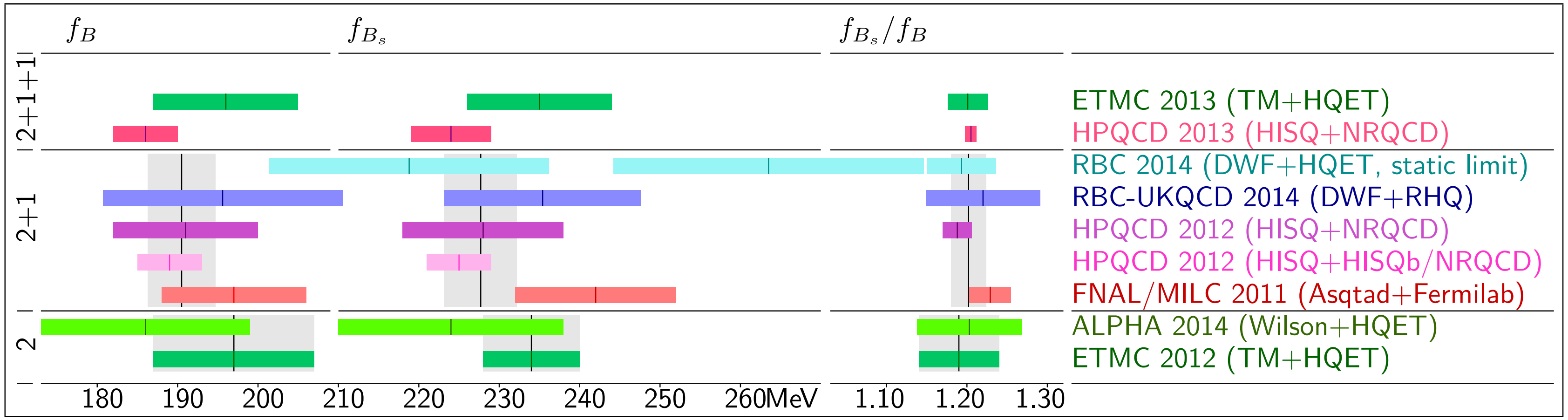


- No sea-quark mass dependence in Φ_{Bs}
- Average data at same lattice spacing
- Statistical errors only

Error budgets and Comparison with other results



- Dominant uncertainties from **statistics** and **chiral extrapolation.**



- Good agreement with other results.

Semileptonic decay form factor

J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

Form-factor definitions

- Non-perturbative form factors $f_+(q^2)$ and $f_0(q^2)$ parametrize the hadronic matrix element of the $b \to u$ quark flavor-changing vector current V_μ .

$$\langle P|V_\mu|B_{(s)}\rangle = f_+(q^2)\left(p^\mu_{B_{(s)}} + p^\mu_P - \frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu\right) + f_0(q^2)\frac{m^2_{B_{(s)}} - p^2_P}{q^2}q^\mu$$

- On the lattice, we calculate the form factors $f_{||}$ and $f_\perp$.
  - ▶ Proportional to vector current matrix elements in the $B_{(s)}$ meson rest frame:

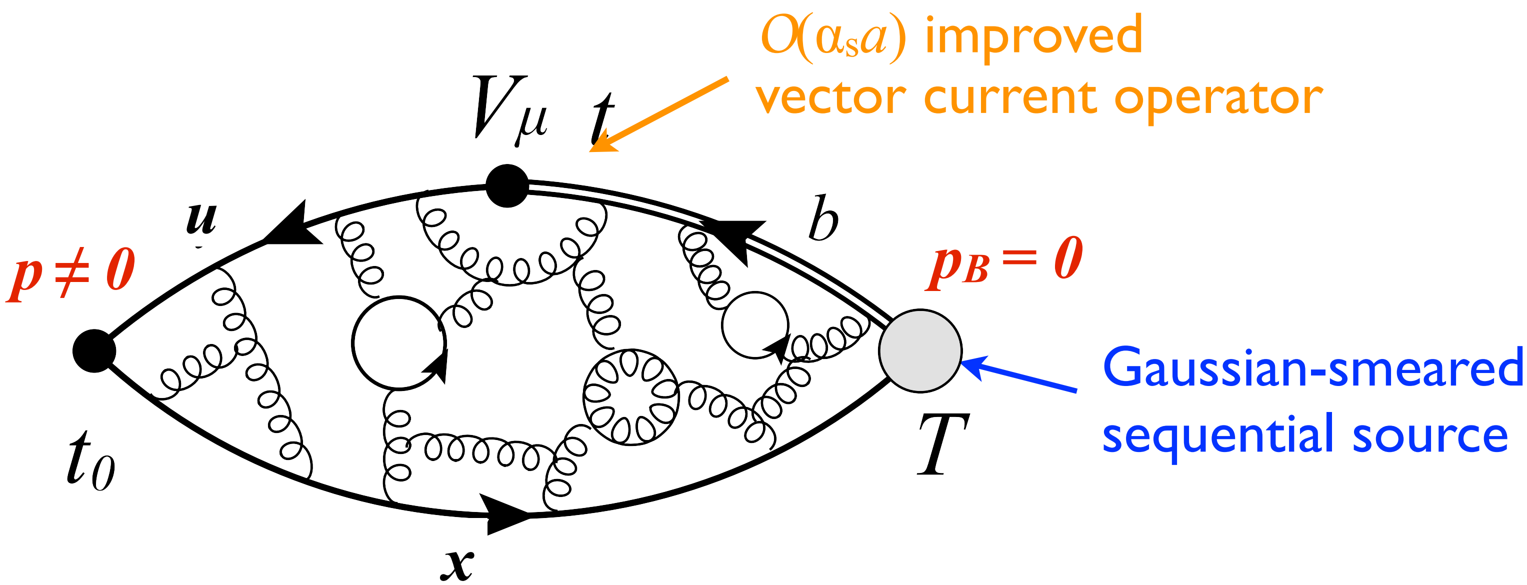
$$\begin{aligned} f_{||}(E_P) &= \langle P|V_0|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \\ f_\perp(E_P)p_i &= \langle P|V_i|B_{(s)}\rangle/\sqrt{2m_{B_{(s)}}} \end{aligned}$$

  - ▶ Easy to relate to the desired form factor $f_+(q^2)$ and $f_0(q^2)$.

$$\begin{aligned} f_0(q^2) &= \frac{\sqrt{2m_{B_{(s)}}}}{m^2_{B_{(s)}} - m^2_P}\left[(m_{B_{(s)}} - E_P)f_{||}(E_P) + (E^2_P - m^2_P)f_\perp(E_P)\right] \\ f_+(q^2) &= \frac{1}{\sqrt{2m_{B_{(s)}}}}\left[f_{||}(E_P) + (m_{B_{(s)}} - E_P)f_\perp(E_P)\right] \end{aligned}$$

J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

Calculation of lattice form factors



- Extract the lattice form factor from the ratio of the 3pt function to 2pt functions:

J. A. Bailey et al. (Fermilab Lattice and MILC), Phys. Rev. D79, 054507 (2009).

$$R_{3,\mu}^{B_{(s)}\to P}(t,T) = \frac{C_{3,\mu}^{B_{(s)}\to P}(t,T)}{\sqrt{C_2^P(t)C_2^{B_{(s)}}(T-t)}}\sqrt{\frac{2E_P}{e^{-E_Pt}e^{-m_{B_{(s)}}(T-t)}}}$$

$$f_\parallel^{\rm lat} = \lim_{t,T\to\infty} R_0^{B_{(s)}\to P}(t,T)$$

$$f_\perp^{\rm lat} = \lim_{t,T\to\infty} \frac{1}{p_P^i} R_i^{B_{(s)}\to P}(t,T)$$

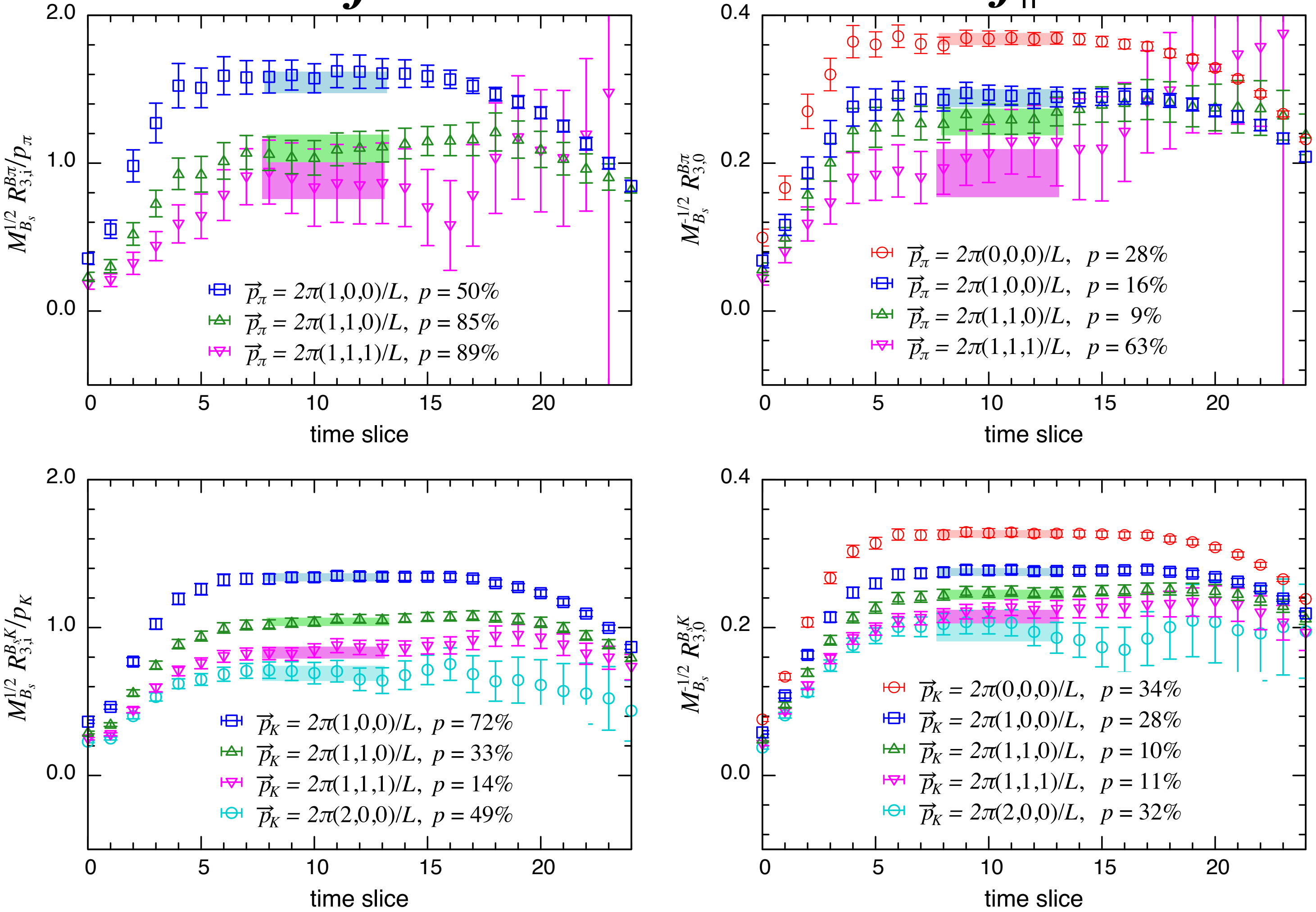
Three-point correlator fits

M_π~289MeV

$f_\perp$ $f_{||}$

$B\to\pi$

$B_s\to K$



- We use the lattice data up to $(1,1,1)$ for $B\to\pi$ and $(2,0,0)$ for $B_s\to K$.
- After a careful study, we fix source-sink separations $T - t_0$
- We fit the ratio to a plateau in the region $0 \ll t \ll T$.

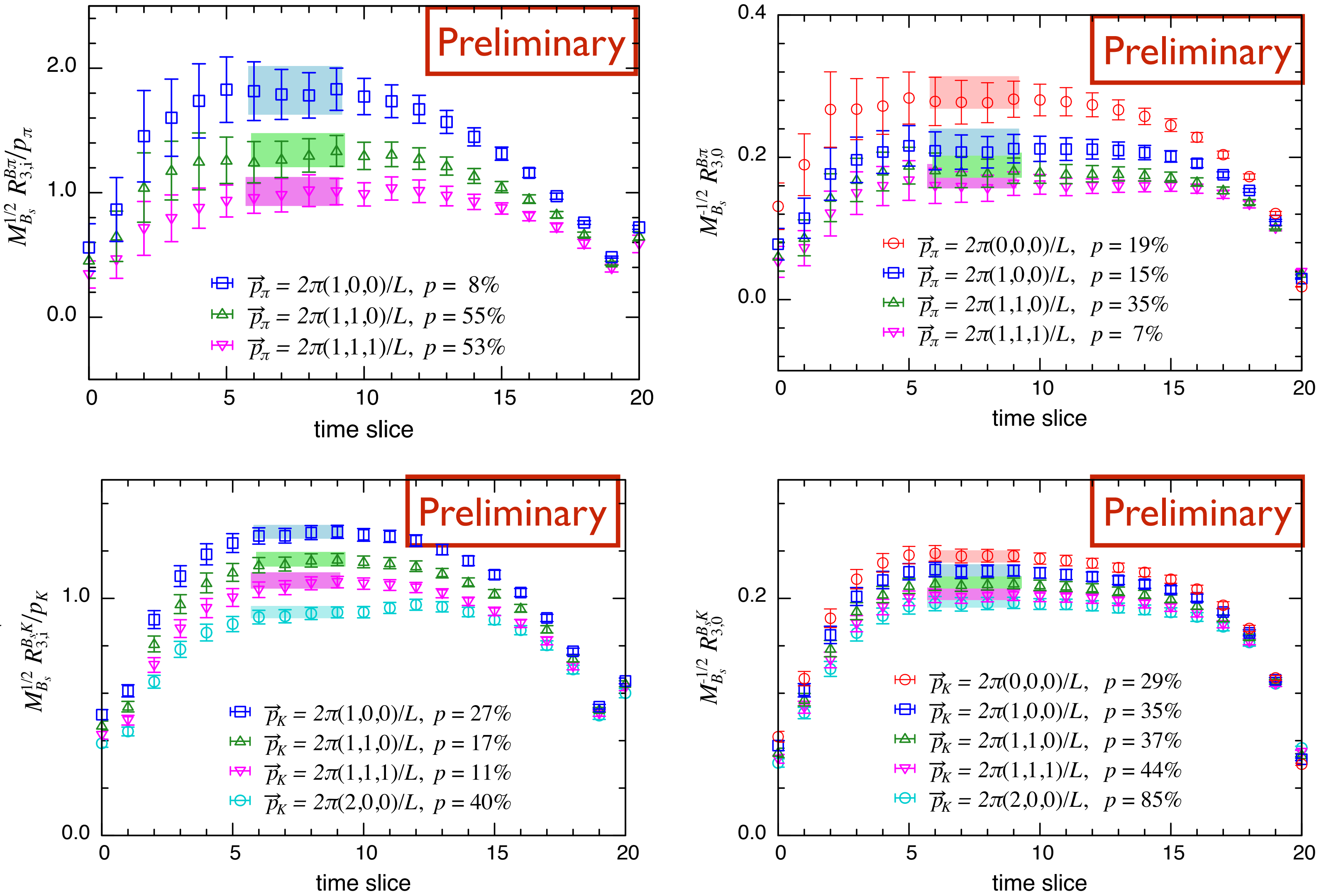
Three-point correlator fits

M_π~139MeV

$f_\perp$ $f_\parallel$

$B \to \pi$

$B_s \to K$

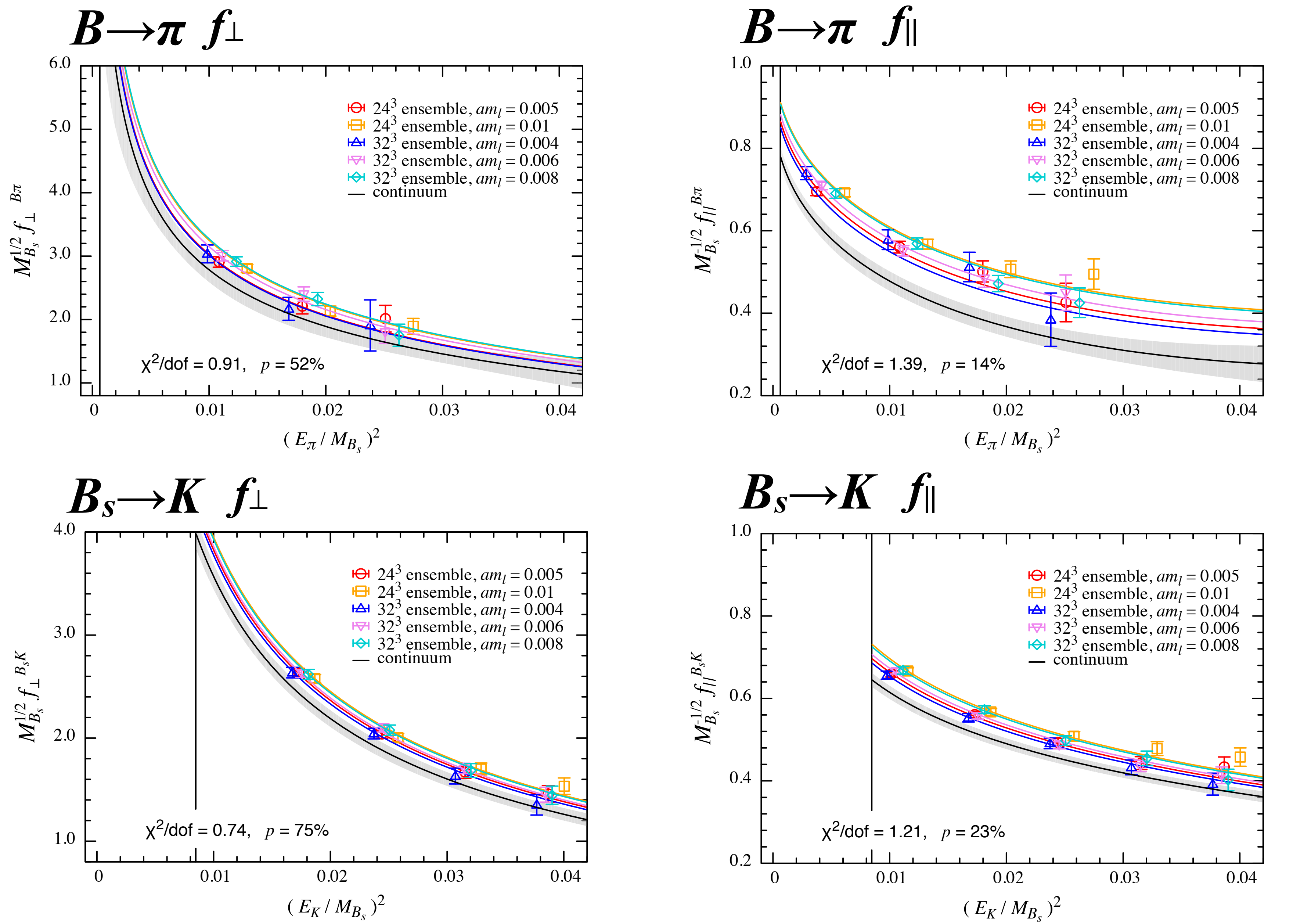


- RBC/UKQCD Möbius domain-wall+ Iwasaki ensemble (M_π ~ 139MeV).

RBC, UKQCD collaborations [Xiv:1411.7017]

J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

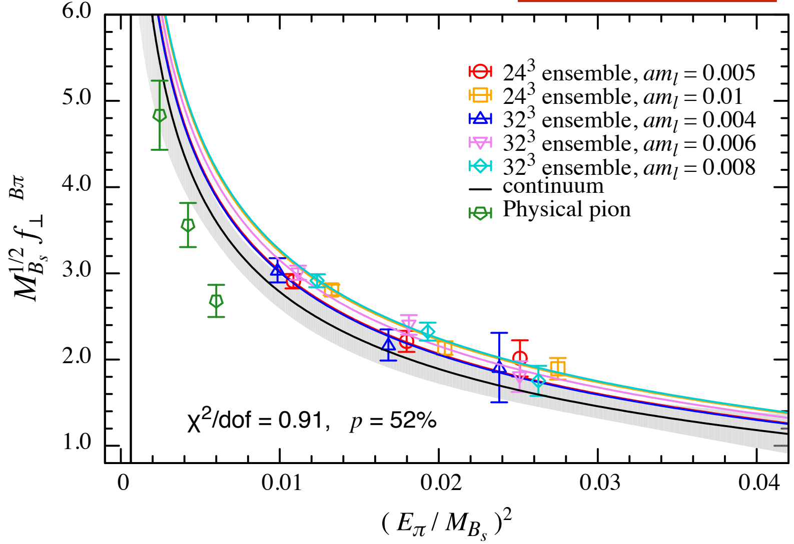


Black curves show chiral-continuum extrapolation using Hard-pion NLO SU(2) χPT.

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

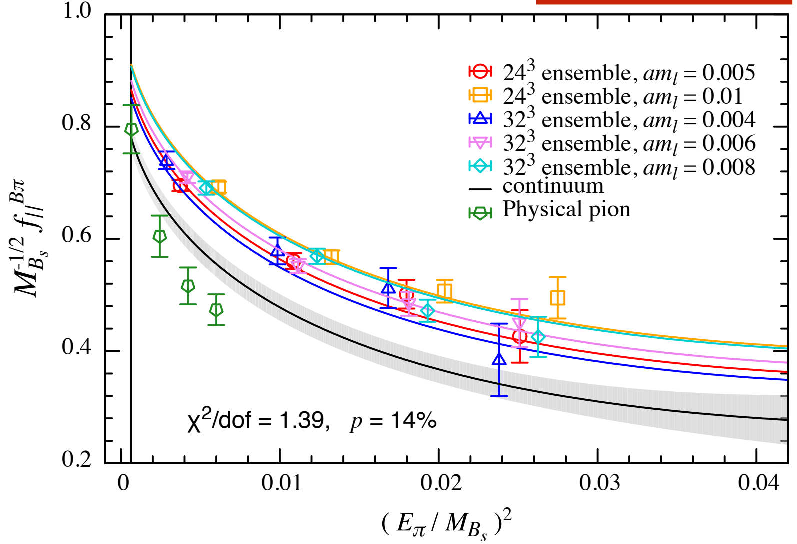
$B \to \pi$ $f_{\perp}$

Preliminary



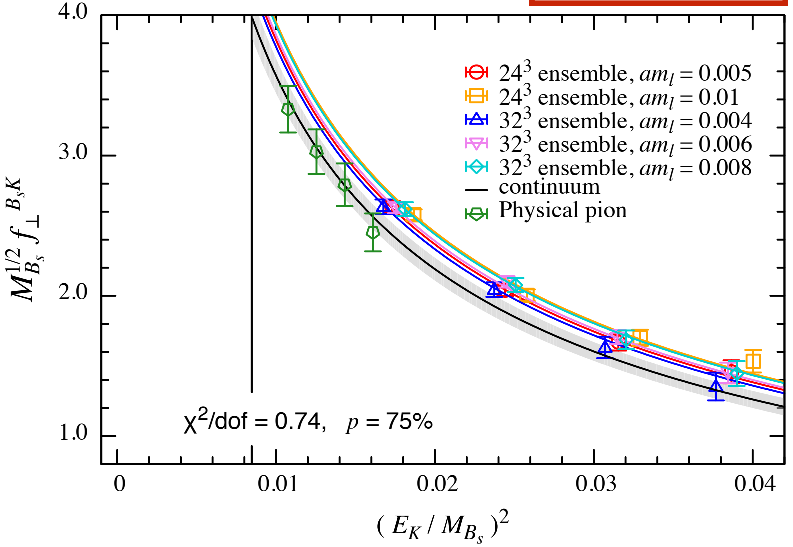
$B \to \pi$ $f_{||}$

Preliminary



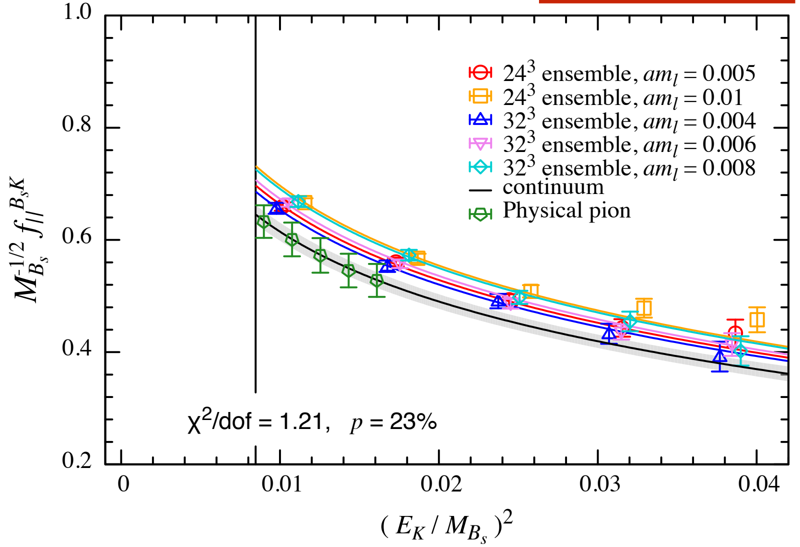
$B_s \to K$ $f_{\perp}$

Preliminary



$B_s \to K$ $f_{||}$

Preliminary



Black curves show chiral-continuum extrapolation using Hard-pion NLO SU(2) χPT.

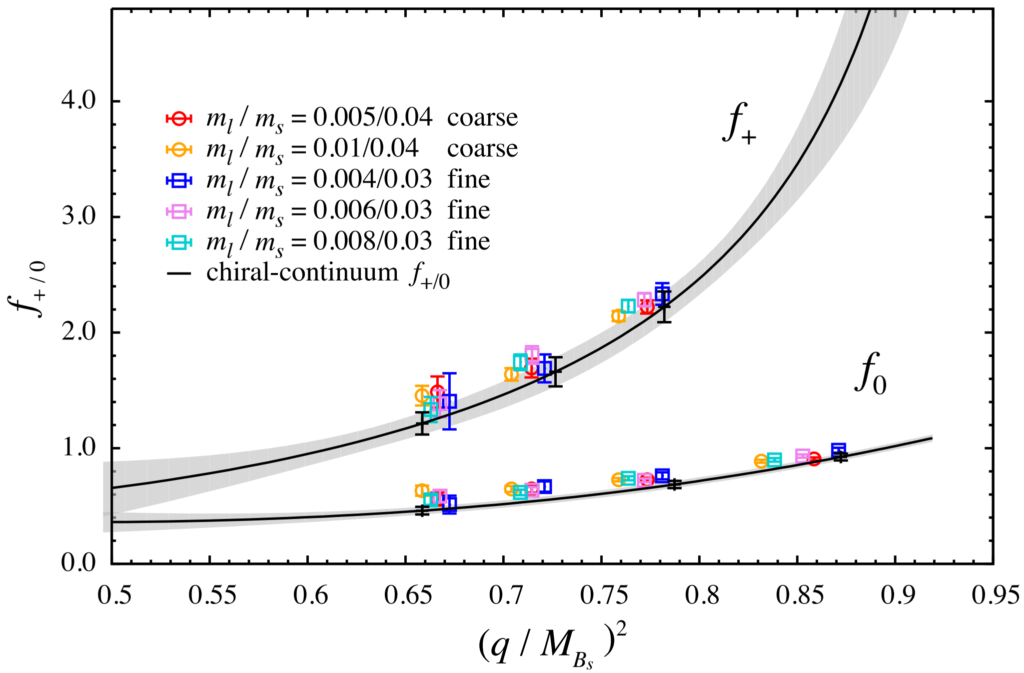
J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

f_+ and f_0

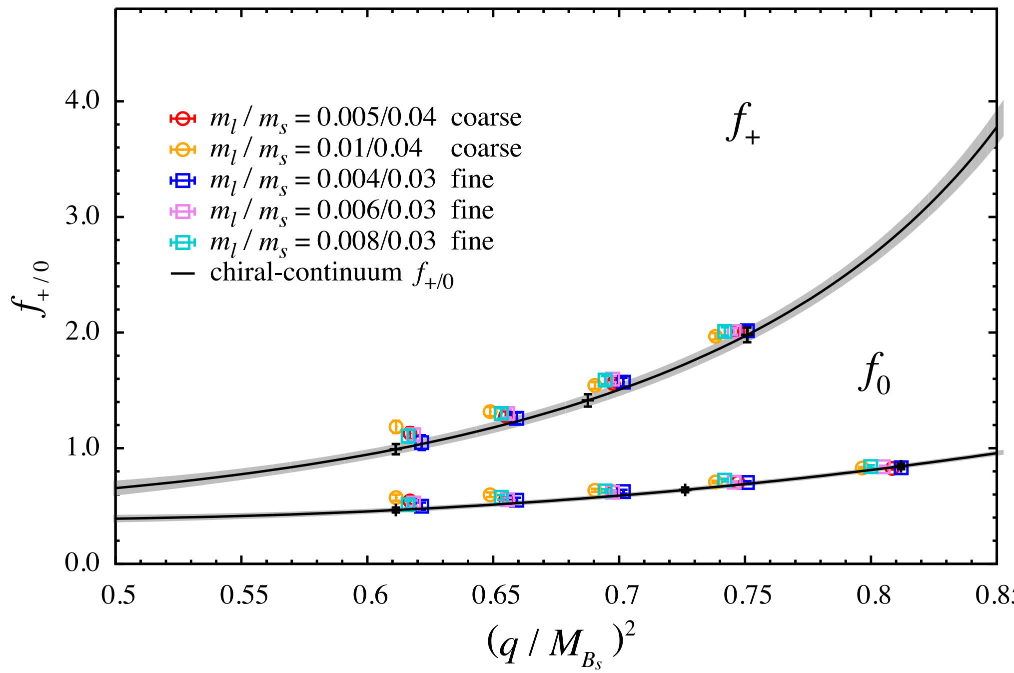
$$f_0(q^2) = \frac{\sqrt{2m_{B_{(s)}}}}{m^2_{B_{(s)}} - m^2_P}\left[(m_{B_{(s)}} - E_P)f_\parallel(E_P) + (E_P^2 - m_P^2)f_\perp(E_P)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2m_{B_{(s)}}}}\left[f_\parallel(E_P) + (m_{B_{(s)}} - E_P)f_\perp(E_P)\right]$$

$B \to \pi$



$B_s \to K$



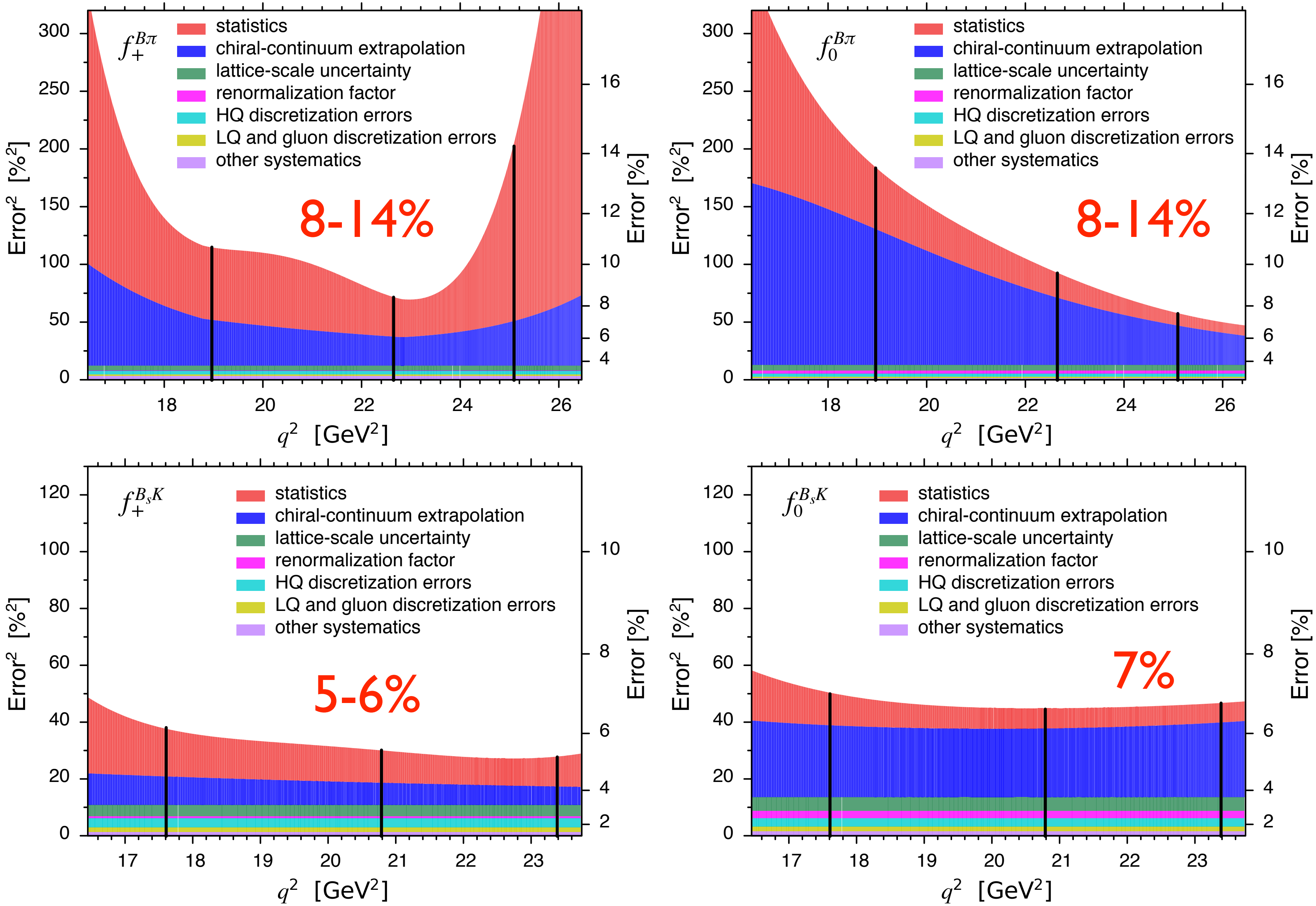
J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

Error budgets

f_+ | f_0

$B \to \pi$

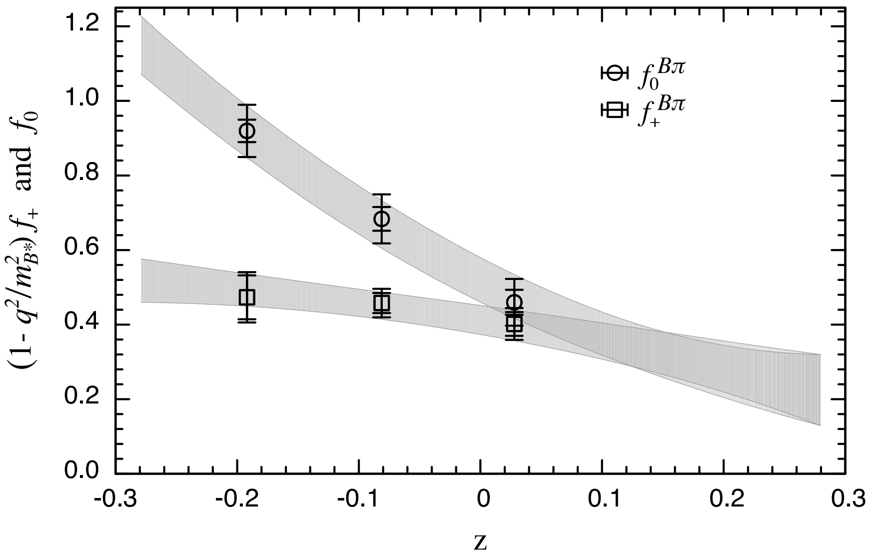
$B_s \to K$



- Dominant uncertainties from **statistics** and **chiral extrapolation.**

z-expansion fit and Determination of $|V_{ub}|$

We use the BCL z-expansion fit to extrapolate lattice results to full kinematic range.

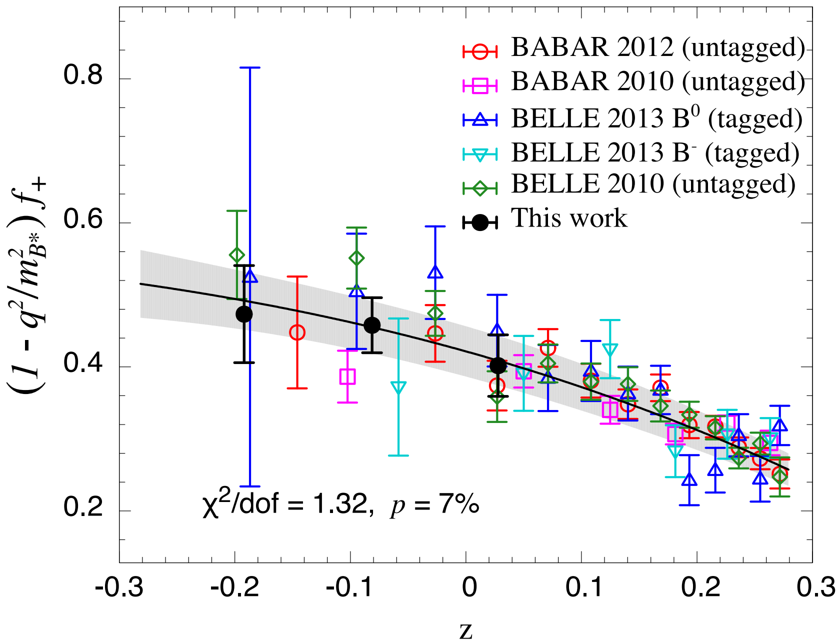


$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

- Kinematic constraint: $f_+(0) = f_0(0)$
- heavy-quark power-counting: $\sum_{k=0}^{N} \left(a_+^{(k)}\right)^2 \sim \left(\frac{\Lambda}{m_b}\right)^3$

Now add experimental data to z-fit to obtain $|V_{ub}|$.



- q^2 dependence of lattice form factor agrees well with experiment.
- Error on normalization (and hence $|V_{ub}|$) saturates with 3-parameter z-fit.

$$|V_{ub}| = 3.61(32) \times 10^{-3}$$

Conclusions and future prospects

- We have calculated the B (B_s) meson decay constant and $B \to \pi$ ($B_s \to K$) form factors using 2+1 flavor dynamical domain-wall fermion gauge field configurations with relativistic heavy quark action.

N. H. Christ, et al. Phys. Rev. D91 (2015) 054502
J.M. Flynn et al. Phys. Rev. D91 (2015) 074510

- We show the preliminary results using RBC/UKQCD Möbius domain-wall + Iwasaki ensemble ($M_\pi \sim$ 139MeV).

- $|V_{ub}|$ is determined by combined z-fit with experimental data from Babar and Belle to about 9% precision.

Future prospect

- We are improving and checking our results using physical light quarks in order to reduce our chiral extrapolation error.
  - Work is in progress to increase statistics.
  - Include new data point in ChPT fit.
  - A new a^{-1} = 2.8 GeV ensemble is in production and we look forward to improve our continuum extrapolation.