

# Nonperturbative determination of form factors for semileptonic $B_s$ meson decays

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# RBC- and UKQCD collaborations

## BNL/RBRC

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Tomomi Ishikawa  
Taku Izubuchi  
Luchang Jin  
Chulwoo Jung  
Christoph Lehner  
Meifeng Lin  
Hiroshi Ohki  
Shigemi Ohta (KEK)  
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Norman Christ  
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Christopher Kelly  
Bob Mawhinney  
David Murphy  
Masaaki Tomii  
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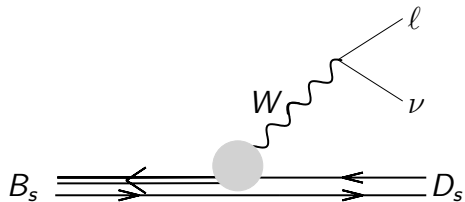
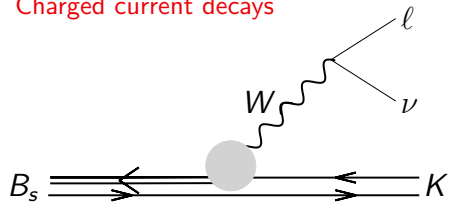
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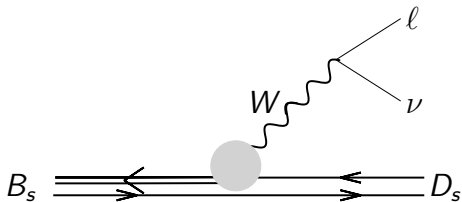
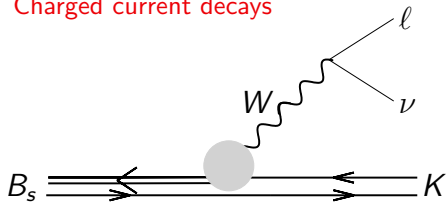
Nicolas Garron

introduction

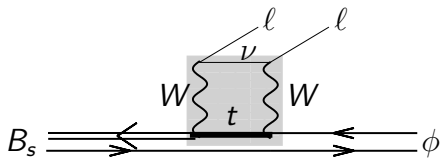
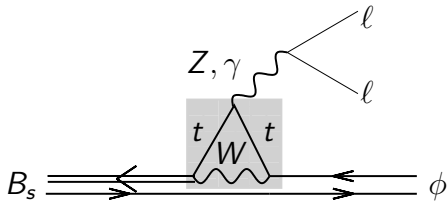
## Charged current decays



## Charged current decays



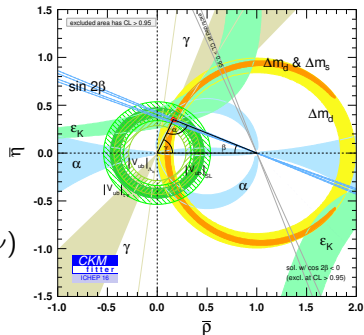
## Flavor changing neutral current decays



## Why $B_s$ meson decays?

- ▶ Alternative, tree-level determination of  $|V_{cb}|$  and  $|V_{ub}|$  from  $B_s \rightarrow D\ell\nu$  and  $B_s \rightarrow K\ell\nu$
- ▶ Commonly used  $B \rightarrow \pi\ell\nu$  and  $B \rightarrow D^{(*)}\ell\nu$
- ▶ Longstanding  $2 - 3\sigma$  discrepancy between exclusive ( $B \rightarrow \pi\ell\nu$ ) and inclusive ( $B \rightarrow X_u\ell\nu$ )
- ▶  $B \rightarrow \tau\nu$  has larger error
- ▶ Alternative, exclusive ( $\Lambda_b \rightarrow p\ell\nu$ ) determination

[Detmold, Lehner, Meinel, PRD92 (2015) 034503]



[<http://ckmfitter.in2p3.fr>]

## Why $B_s$ meson decays?

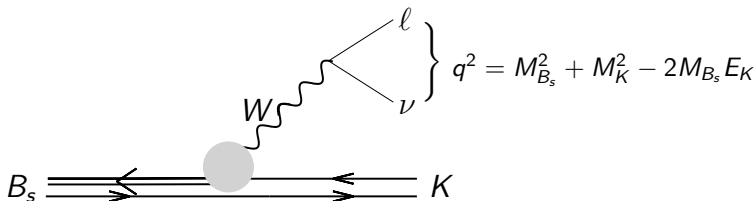
- ▶ Not (yet) experimentally measured with sufficient precision
- ▶  $B$ -factories typically run at the  $\Upsilon(4s)$  threshold
  - $B$  but no  $B_s$  mesons are produced
- ▶ At the LHC energies are large enough to produce sufficient  $B_s$  mesons
- ▶ LHCb is working on the analysis
  - Absolute normalization is challenging; ratios are preferred
  - Determine  $|V_{cb}|/|V_{ub}|$
- ▶ strange-quarks are easier on the lattice



# flavor changing charged currents

(tree-level in the Standard Model)

# $|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K \ell \nu$ decay



► Conventionally parametrized by

$$\frac{d\Gamma(B_s \rightarrow K \ell \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3} \left[ (M_{B_s}^2 + M_K^2 - q^2)^2 - 4M_{B_s}^2 M_K^2 \right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

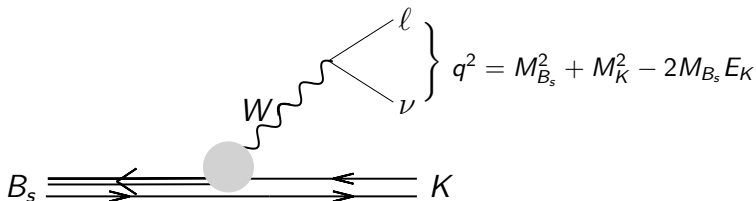
experiment

known

nonperturbative input

CKM

# $|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K\ell\nu$ decay



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experiment

known

nonperturbative input

CKM

For  $|V_{cb}|$  from  $B_s \rightarrow D_s\ell\nu$  replace  $K$  with  $D_s$

## 2+1 Flavor Domain-Wall Iwasaki ensembles

L	$a^{-1}(\text{GeV})$	$am_l$	$am_s$	$M_\pi(\text{MeV})$	# configs.	#sources	
24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	81/1*	[PRD 93 (2016) 074505]
64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
48	2.774	0.002144	0.02144	234	70	24	[arXiv:1701.02644]

\* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

► Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]

►  $a$ :  $\sim 0.11$  fm,  $\sim 0.08$  fm,  $\sim 0.07$  fm

## Up, down, and strange quarks

- ▶ Domain-wall fermions with same parameters as in the sea-sector (domain-wall height  $M_5$ , extension of 5<sup>th</sup> dimension  $L_5$ )
- ▶ Unitary and partially quenched quark masses
- ▶ Strange quarks at/near physical the physical value

## Charm quarks

- ▶ Möbius DWF optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
- ▶  $M_5 = 1.6$ ,  $L_5 = 12$
- ▶ Discretization errors well under control for  $am_c < 0.45$ 
  - On coarse ( $a^{-1} = 1.784$  GeV) ensembles we simulate just below  $m_c^{\text{phys}}$
  - Simulate 3 or 2 charm-like masses and then extrapolate/interpolate
  - Linear extrapolation is small and benign; interpolation is safe

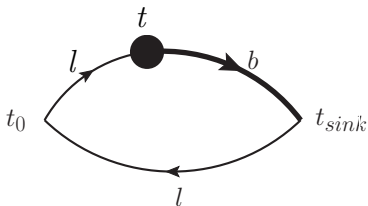
## Bottom quarks

- ▶ Relativistic Heavy Quark action developed by Christ, Li, and Lin  
[Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
- ▶ Allows to tune the three parameters ( $m_0 a$ ,  $c_P$ ,  $\zeta$ ) nonperturbatively  
[PRD 86 (2012) 116003]
- ▶ Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]  
by tuning all parameters of the clover action non-perturbatively;  
close relation to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]
- ▶ Heavy quark mass is treated to all orders in  $(m_b a)^n$
- ▶ Expand in powers of the spatial momentum through  $O(\vec{p}a)$ 
  - ▶ Resulting errors will be of  $O(\vec{p}^2 a^2)$
  - ▶ Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- ▶ Applies for all values of the quark mass
- ▶ Has a smooth continuum limit
- ▶ Recently re-tuned to account for updated values of  $a^{-1}$

## $B_s \rightarrow Kl\nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current  $V^\mu$  in terms of the form factors  $f_+(q^2)$  and  $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2) \left( p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu$$



- ▶ Calculate 3-point function by
  - Inserting a quark source for a “light” propagator at  $t_0$
  - Allow it to propagate to  $t_{sink}$ , turn it into a sequential source for a  $b$  quark
  - Use another “light” quark propagating from  $t_0$  and contract both at  $t$
- ▶ Updating calculation of [PRD 91 (2015) 074510] with new  $a^{-1}$  and RHQ parameters

## Relating form factors $f_+$ and $f_0$ to $f_{\parallel}$ and $f_{\perp}$

- ▶ On the lattice we prefer using the  $B_s$ -meson rest frame and compute

$$f_{\parallel}(E_K) = \langle K | V^0 | B_s \rangle / \sqrt{2M_{B_s}} \quad \text{and} \quad f_{\perp}(E_K) p_K^i = \langle K | V^i | B_s \rangle / \sqrt{2M_{B_s}}$$

- ▶ Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2} [(M_{B_s} - E_K) f_{\parallel}(E_K) + (E_K^2 - M_K^2) f_{\perp}(E_K)]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} [f_{\parallel}(E_K) + (M_{B_s} - E_K) f_{\perp}(E_K)]$$

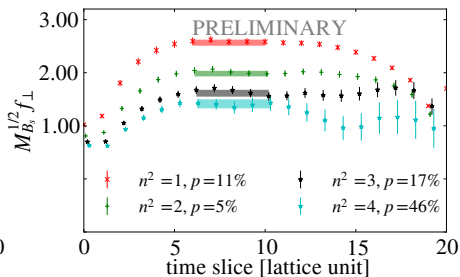
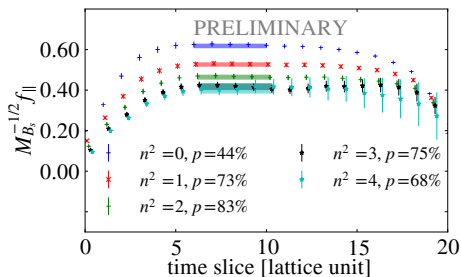


# Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ for $B_s \rightarrow K \ell \nu$

$$f_{\parallel} = \lim_{t, t_{\text{sink}} \rightarrow \infty} R_0^{B_s \rightarrow K}(t, t_{\text{sink}})$$

$$f_{\perp} = \lim_{t, t_{\text{sink}} \rightarrow \infty} \frac{1}{p_i^j} R_i^{B_s \rightarrow K}(t, t_{\text{sink}})$$

$$R_{\mu}^{B_s \rightarrow K}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s \rightarrow K}(t, t_{\text{sink}})}{C_2^K(t) C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4M_{B_s} E_K}{e^{-E_k t} e^{-M_{B_s}(t_{\text{sink}} - t)}}$$

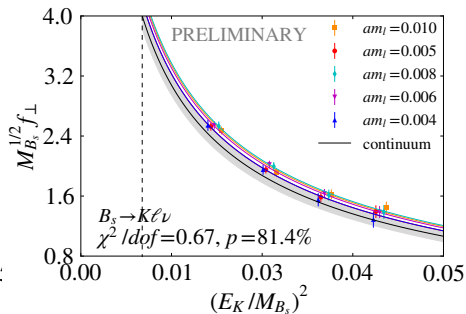
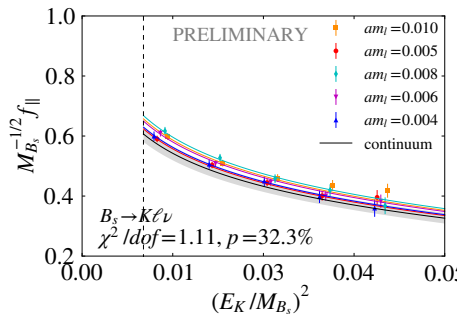


# Chiral-continuum extrapolation using SU(2) hard-pion $\chi$ PT

$$f_{\parallel}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_{\parallel}^{(1)} \left[ 1 + \left( \frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_K}{\Lambda} + c_{\parallel}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right) \right]$$

$$f_{\perp}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c_{\perp}^{(1)} \left[ 1 + \left( \frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)} \frac{M_K^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_K}{\Lambda} + c_{\perp}^{(4)} \frac{E_K^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right) \right]$$

with  $\delta f$  non-analytic logs of the kaon mass and hard-kaon limit is taken by  $\frac{M_K}{E_K} \rightarrow 0$

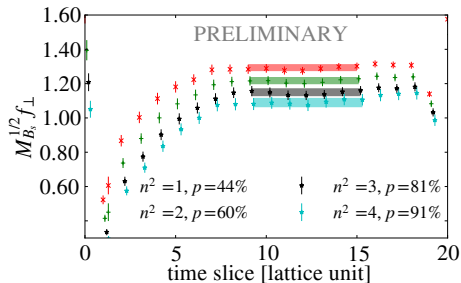
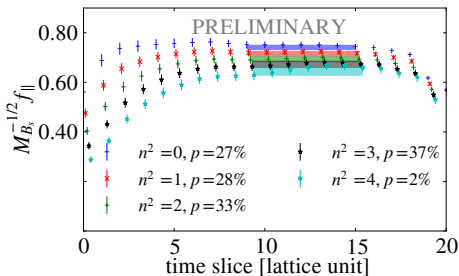


# Lattice results for form factors $f_{\parallel}$ and $f_{\perp}$ for $B_s \rightarrow D_s \ell \nu$

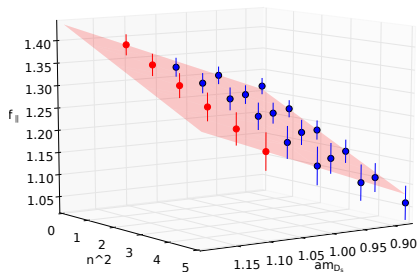
$$f_{\parallel} = \lim_{t, t_{\text{sink}} \rightarrow \infty} R_0^{B_s \rightarrow D_s}(t, t_{\text{sink}})$$

$$f_{\perp} = \lim_{t, t_{\text{sink}} \rightarrow \infty} \frac{1}{p_i^j} R_i^{B_s \rightarrow D_s}(t, t_{\text{sink}})$$

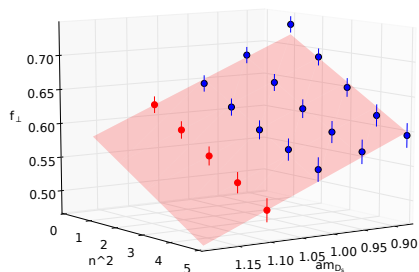
$$R_{\mu}^{B_s \rightarrow D_s}(t, t_{\text{sink}}) = \frac{C_{3,\mu}^{B_s \rightarrow D_s}(t, t_{\text{sink}})}{C_2^{D_s}(t) C_2^{B_s}(t_{\text{sink}} - t)} \sqrt{\frac{4M_{B_s} E_{D_s}}{e^{-E_{D_s} t} e^{-M_{B_s}(t_{\text{sink}} - t)}}$$



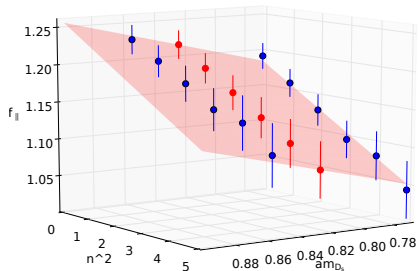
# Charm extra-/interpolation for $B_s \rightarrow D_s l \nu$



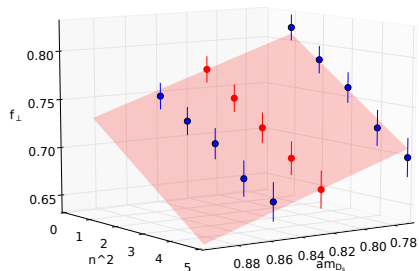
PRELIMINARY



PRELIMINARY



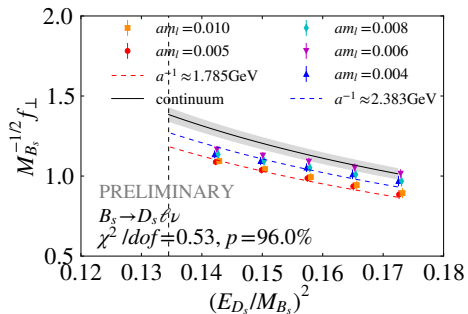
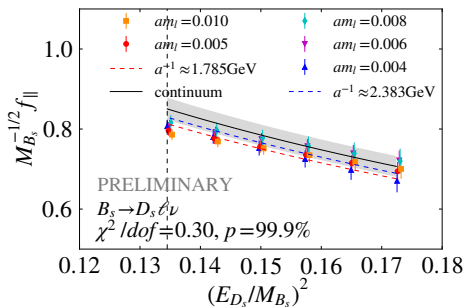
PRELIMINARY



PRELIMINARY

# Chiral-continuum extrapolation for $B_s \rightarrow D_s \ell \nu$

$$f(q, a) = \frac{c_0 + c_1(\Lambda_{\text{QCD}} a)^2}{1 + c_2(q/M_{B_c})^2}$$



## Next steps

- ▶ Estimate full systematic errors for three “synthetic” data points”
- ▶ Perform  $z$ -expansion and polynomial fits
- ▶ Comparison with other result(s) [HPQCD 2017]
- ▶ Explore advantageous ratios

# Conclusion

- ▶ About to complete calculation for  $B_s \rightarrow Kl\nu$  and  $B_s \rightarrow D_s l\nu$   
→ Finalizing systematic error estimates and kinematic extrapolations
- ▶ Not enough time to cover  $B_s \rightarrow \phi l^+ l^-$  (→ see appendix)
- ▶ We have more data for
  - $B \rightarrow \pi l\nu$ ,  $B \rightarrow l^+ l^- \nu$
  - $B \rightarrow K^* l^+ l^-$
  - $B \rightarrow D^{(*)} l\nu$
  - ...

# Appendix

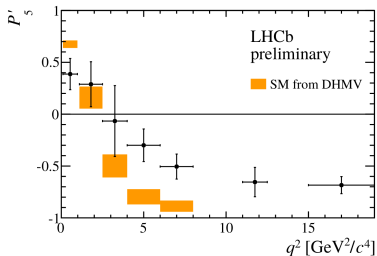


# flavor changing neutral currents

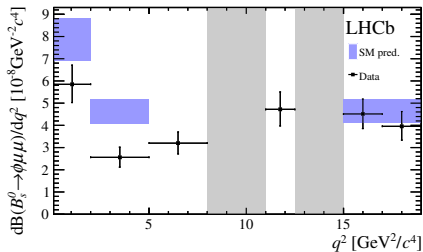
(loop-level in the Standard Model)

# Rare $B$ decays (FCNC)

- ▶ GIM suppressed in the Standard Model  $\Rightarrow$  sensitive to new physics
- ▶ Angular observable  $P'_5$  in  $B \rightarrow K^* \mu^+ \mu^-$  received a lot of attention



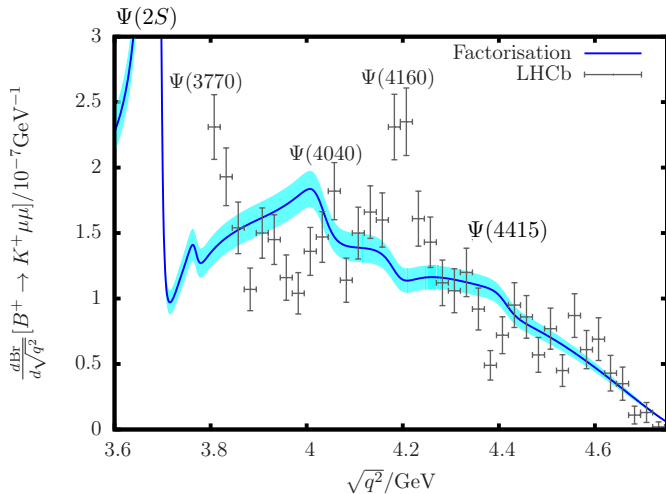
[LHCb-CONF-2015-002]



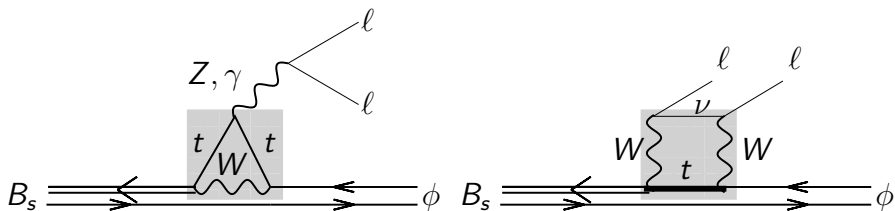
[LHCb JHEP 1509 (2015) 179]

- ▶ Lattice QCD: [Horgan et al. PRD 89 (2013) 094501]

► Charm resonances under control? [Lyon and Zwicky, arXiv:1406.0566]



## Rare $B$ decays: $B_s \rightarrow \phi \ell^+ \ell^-$



- ▶ Pseudoscalar or vector final state (narrow width approximation)
- ▶ Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(l)}$$

- ▶ Leading contributions at short distance

$$O_7^{(l)} = \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$

$$O_9^{(l)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \ell$$

$$O_{10}^{(l)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \gamma^5 \ell$$

## Seven form factors

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu b | B_s(p) \rangle = f_V(q^2) \frac{2i \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_{B_s} + M_\phi}$$

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | B_s(p) \rangle = f_{A_0}(q^2) \frac{2M_\phi \varepsilon^* \cdot q}{q^2} q^\mu$$

$$+ f_{A_1}(q^2) (M_{B_s} + M_\phi) \left[ \varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right]$$

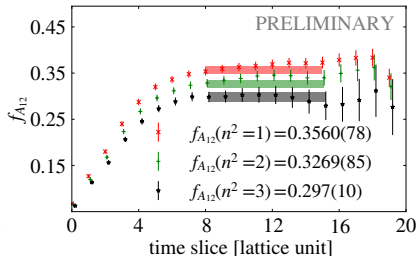
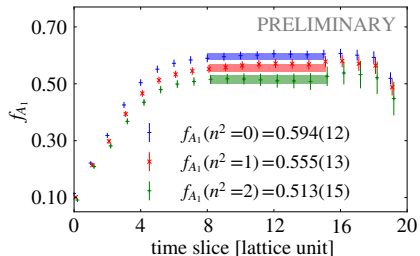
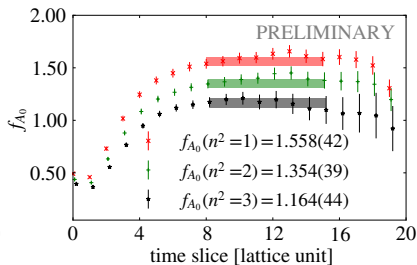
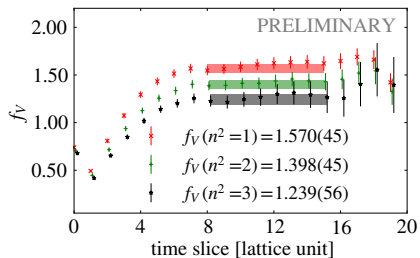
$$- f_{A_2}(q^2) \frac{\varepsilon^* \cdot q}{M_{B_s} + M_\phi} \left[ k^\mu + p^\mu - \frac{M_{B_s}^2 - M_\phi^2}{q^2} q^\mu \right]$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} b | B_s(p) \rangle = 2f_{T_1}(q^2) \epsilon^{\mu\rho\tau\sigma} \varepsilon_\rho^* k_\tau p_\sigma,$$

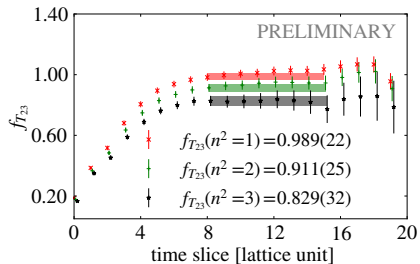
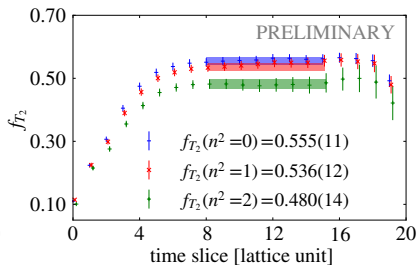
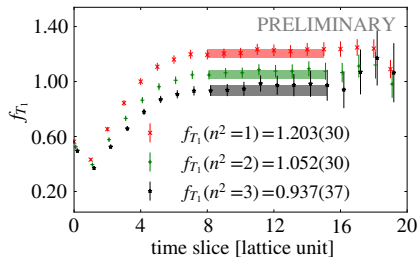
$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} \gamma^5 b | B_s(p) \rangle = if_{T_2}(q^2) \left[ \varepsilon^{*\mu} (M_{B_s}^2 - M_\phi^2) - (\varepsilon^* \cdot q)(p + k)^\mu \right]$$

$$+ if_{T_3}(q^2) (\varepsilon^* \cdot q) \left[ q^\mu - \frac{q^2}{M_{B_s}^2 - M_\phi^2} (p + k)^\mu \right]$$

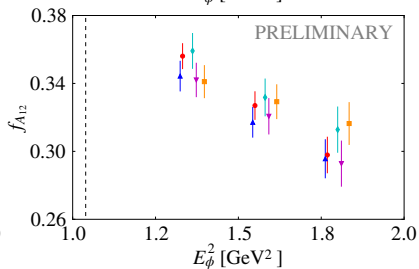
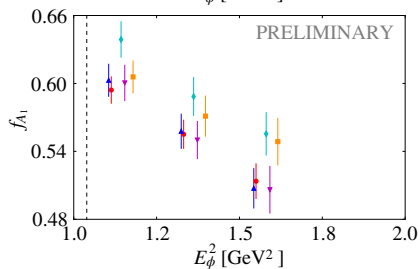
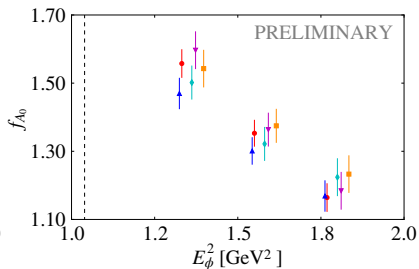
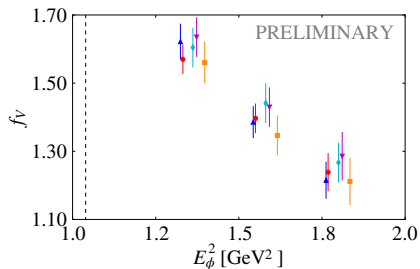
# Seven form factors



# Seven form factors



# Seven form factors vs. $q^2$





# Seven form factors vs. $q^2$

