Charmed (and heavier) meson decay constants and heavy neutral meson mixing in the continuum limit using 2+1f of domain wall fermions

Justus Tobias Tsang

RBC-UKQCD Collaborations

Michigan State University

26 July 2018

THE UNIVERSITY of EDINBURGH

Ratio of decay constants f_{D_s}/f_D and neutral meson mixing parameter ξ

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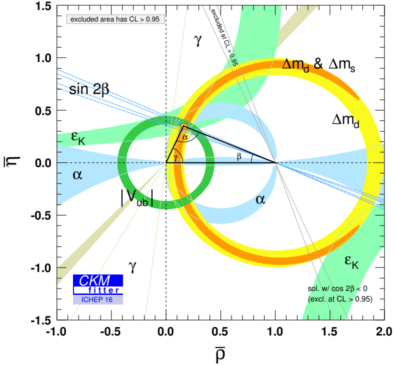
Outline

1. Introduction
2. Set-Up
3. Results
4. Summary

Motivation - Flavour Physics



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Experiment

- Belle, BaBar, CLEO-c
- LHCb, Belle II, BESIII

Theory

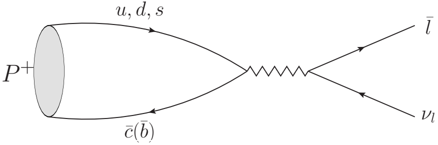
- Compute non-perturbative SM quantities for K, D and B

Combine Ex+Th to **extract CKM matrix elements**

$\Rightarrow$ **Test CKM unitarity and place tight bounds on SM**

Flavour Physics and CKM

Experiment $\approx$ CKM $\times$ Lattice $\times$ (PT+kinematics)



Leptonic decays: $\Gamma(P \to l\nu_l) \approx |V_{q_2 q_1}|^2 \times f_P^2 \times \mathcal{K}_1$

where $\mathcal{Z}_A \langle 0| \bar{c}\gamma_4\gamma_5 q |D_q(0)\rangle = f_{D_q} m_{D_q}$, $q = d, s$

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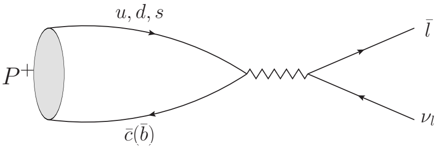

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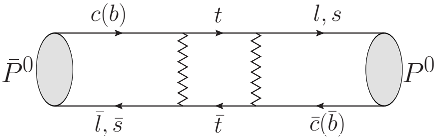
[HFLAV] $f_D \, |V_{cd}| = (45.9 \pm 1.1)\,\mathrm{MeV}, \qquad f_{D_s} \, |V_{cs}| = (250.3 \pm 4.5)\,\mathrm{MeV}$

Computing f_{D_s}/f_D gives access to V_{cs}/V_{cd}

Flavour Physics and CKM

$$\text{Experiment} \approx CKM \times \text{Lattice} \times (\text{PT+kinematics})$$

Neutral meson mixing:



$$\Delta m_P = \left| V^*_{tq_2} V_{tq_1} \right| \times f_P^2 m_P \hat{B}_P \times \frac{G_F^2 m_W^2}{6\pi^2} \mathcal{K}_2$$

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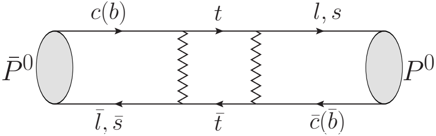

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HFLAV

$$\Delta m_d = 0.5064 \pm 0.0019\,\mathrm{ps}^{-1}$$

$$\Delta m_s = 17.757 \pm 0.021\,\mathrm{ps}^{-1}$$

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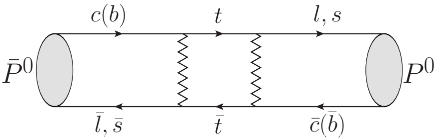

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Computing ξ gives access to

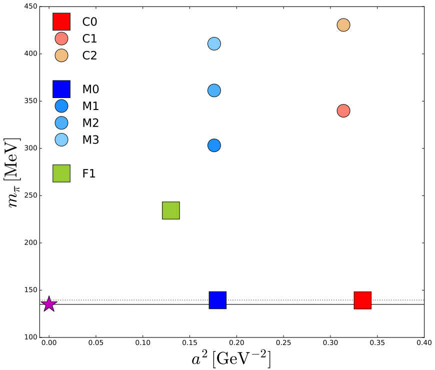
$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_B^2 B_B} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

Ensembles

	$L^3 \times T/a^4$	a^{-1}/GeV	m_π/MeV
C0	$48^3 \times 96$	**1.73**	**139**
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M0	$64^3 \times 128$	**2.36**	**139**
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1	$48^3 \times 96$	**2.77**	230

- Iwasaki gauge action
- Domain Wall Fermion action
  $\Rightarrow$ $N_f = 2+1$ flavours in the sea
  $\Rightarrow$ automatically $O(a)$-improved
  $\Rightarrow$ multiplicative renormalisation
- **2 ensembles with physical pion masses**
- 3 Lattice spacings

Ensembles



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Actions

Light and strange

- Unitary light quark mass
- Physical strange quark mass
- DWF parameters same between sea and valence
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- Möbius DWF
- $M_5 = 1.0$, $L_s = 12$
- Stout smeared (3 hits, $\rho = 0.1$)
- Range of quark masses from below charm to $\sim m_b/2$ on finest ensemble.

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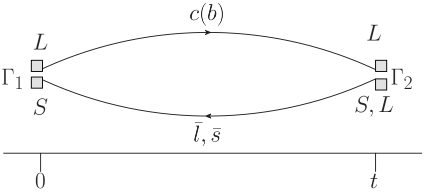
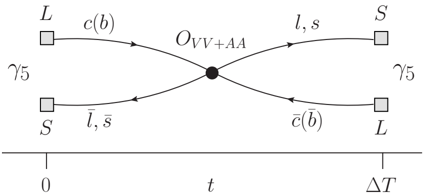
$\Rightarrow$ **All DWF** mixed action set-up
$\Rightarrow$ Increased heavy quark reach compared to [JHEP **04** (2016) 037,JHEP **12** (2017) 008]
$\Rightarrow$ Second continuum limit trajectory for m_c and $a_\mu^{\mathrm{LOHVP},c}$

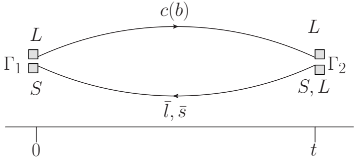
Measurement strategy





- Z_2-Wall sources on every 2nd time-slice
- Light and strange propagators Gaussian smeared sources (L and S sinks)
- Unitary light and physical strange quark masses
- Range of charm (and heavier) quark masses
- Many source-sink separations ΔT for 4-quark operator

Correlator Fitting of two-point functions I



$$C_{ij}(t) = \sum_{n=0}^{\infty} (\psi_n)_i (\psi_n^*)_j e^{-E_n t}$$

with $E_n < E_{n+1}$ and $(\psi_n)_i = \frac{\langle 0|O_i|n\rangle}{\sqrt{2E_n}}$ for $O = \bar{c}_2^L \Gamma q_1^X$ where $X = S, L$.

Consider $\Gamma = \gamma_5$ (**P**seudo scalar) and $\Gamma = \gamma_4\gamma_5$ (**A**xial vector current).

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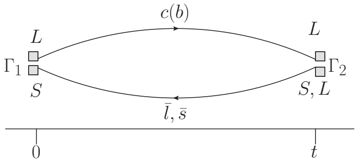

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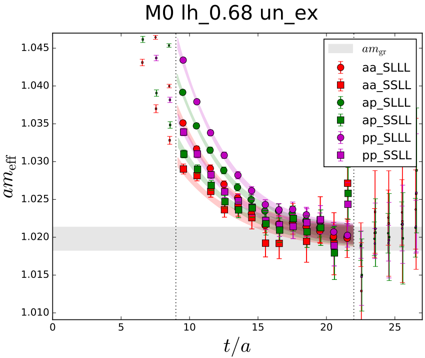
Consider $\Gamma = \gamma_5$ (**P**seudo scalar) and $\Gamma = \gamma_4\gamma_5$ (**A**xial vector current).

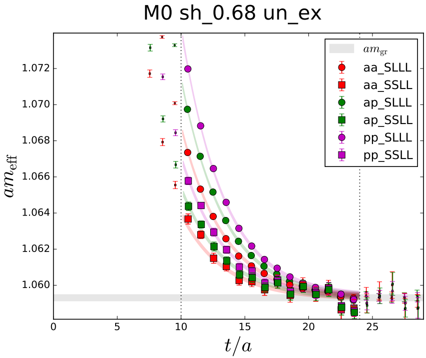
ISSUE: Exponential noise growth i.e. **signal-to-noise problem**

$\Rightarrow$ Simultaneous uncorrelated excited state fits to 6 channels:
$\langle AA\rangle^{SL}$, $\langle AP\rangle^{SL}$, $\langle PP\rangle^{SL}$, $\langle AA\rangle^{SS}$, $\langle AP\rangle^{SS}$ and $\langle PP\rangle^{SS}$

Correlator Fitting - two point functions II



Example fit (heavy-light meson with $am_h = 0.68$ on M0).



Example fit (heavy-strange meson with $am_h = 0.68$ on M0).

Correlator Fitting - checks and improvements

$$C_{AP}^{LS}(t) \approx A_0^L P_0^S e^{-E_0 t} + A_1^L P_1^S e^{-E_1 t}$$

$$C_{AP}^{SS}(t) \approx A_0^S P_0^S e^{-E_0 t} + A_1^S P_1^S e^{-E_1 t}$$

Construct Linear Combination

$$\begin{aligned} C_1^{AP}(t) &\equiv C_{AP}^{LS}(t) X^S - C_{AP}^{SS}(t) X^L \\ &\approx P_0^S \left(A_0^L X^S - A_0^S X^L \right) e^{-E_0 t} \\ &+ P_1^S \left(A_1^L X^S - A_1^S X^L \right) e^{-E_1 t} \end{aligned}$$

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Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.

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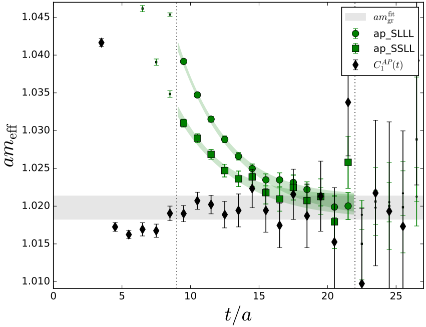


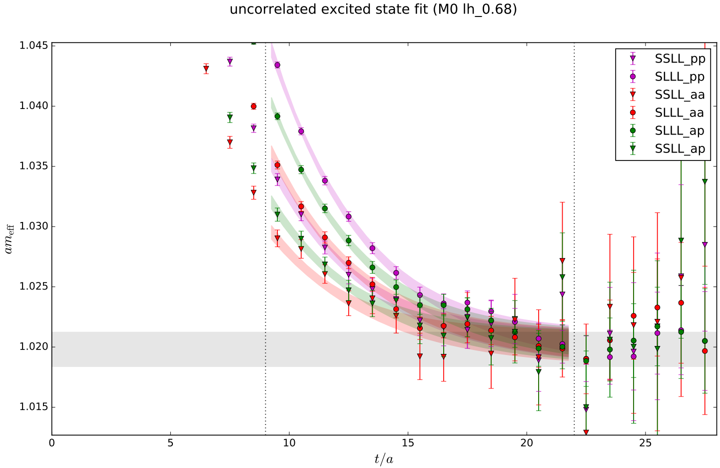
⇒ Removes (most of) excited state

⇒ Strong *a posteriori* check of fitrange

⇒ Possible to *refit* with smaller $t_{\min}$/fewer coefficients.

⇒ Use this as optimised source for 3-point functions.

ASIDE: Re-Fitting



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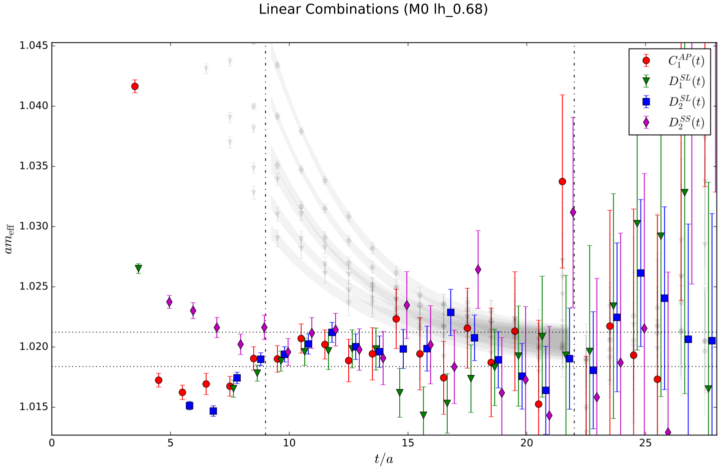

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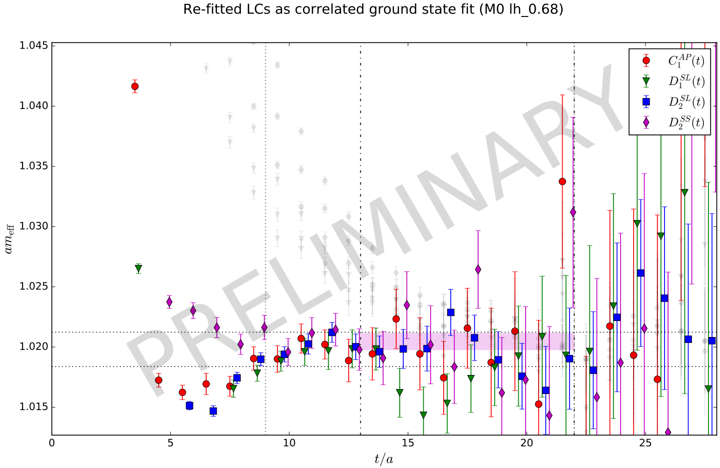

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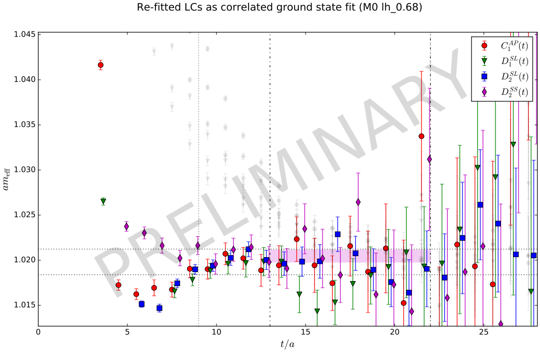
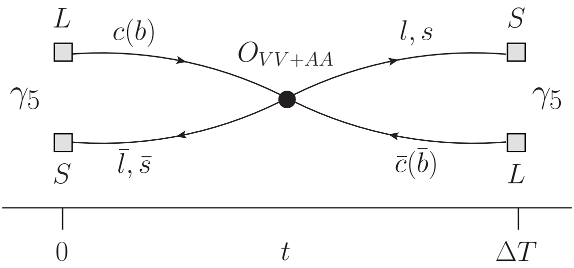


- ⇒ Check $t_{\rm min}$, $t_{\rm max}$ sensititivy
- ⇒ Check sensitivity to choice of X's
- ⇒ Pick optimal LCs (not all simultaneously independent)

Correlator Fitting of 4-quark operators I



$$C_3(t, \Delta T) \equiv \left\langle P(\Delta T) O_{VV+AA}(t) \bar{P}^\dagger(0) \right\rangle$$

$$R(t, \Delta T) = \frac{C_3(t, \Delta T)}{8/3 C_{PA}(\Delta T - t) C_{AP}(t)}$$

$$R(t, \Delta T) \to B_P \quad \text{for} \quad t, \Delta T \gg 0$$

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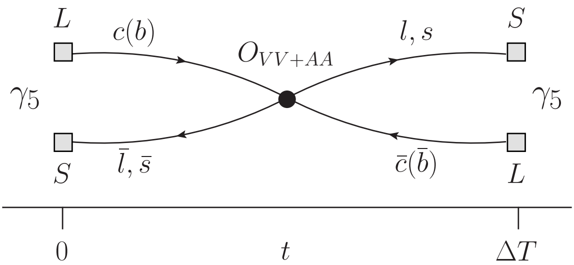

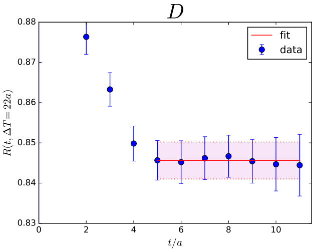

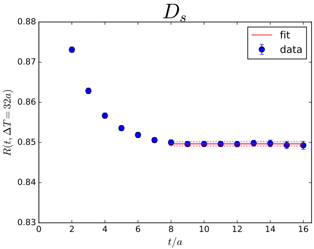

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Ex: $am_h = 0.68$ on M0

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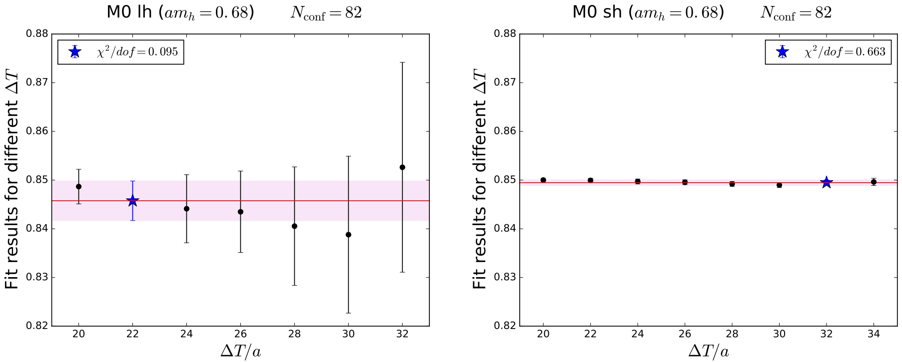

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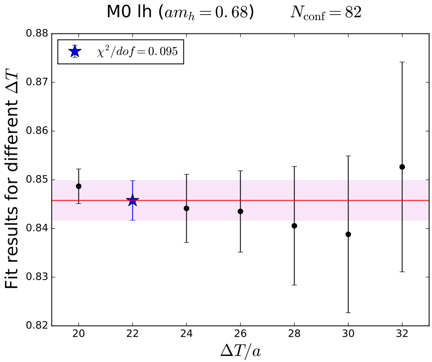
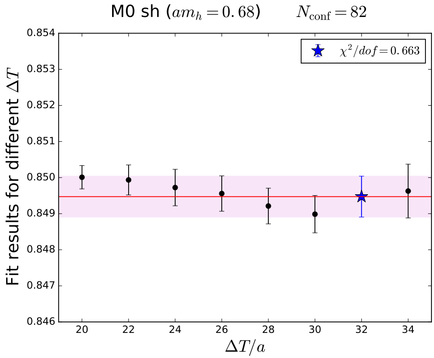
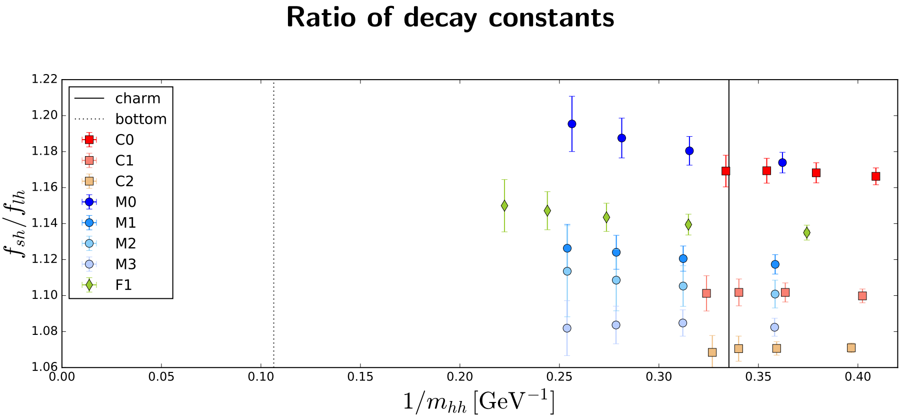




zoom

Results of correlator fits



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⇒ Mild linear behaviour with $1/m_H$ and a^2

⇒ Stat precision: 0.5-1.5%

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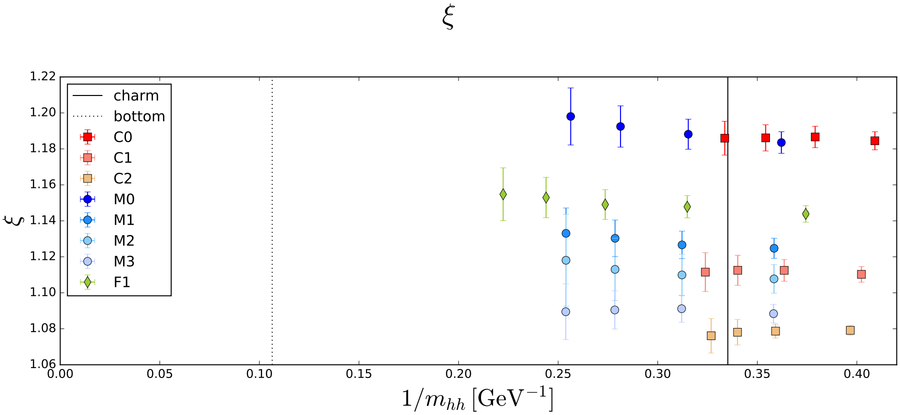

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Global fit form

Base fit

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\mathrm{phys}}, m_H^{\mathrm{phys}}) + C_{CL} a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$

Global fit form

Base fit

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Assess systematic errors by

- varying cuts on pion mass
- using $m_H = m_D$, m_{D_s} and m_{η_c}
- varying inclusion/exclusion of heaviest data points
- varying inclusion/exclusion of fit parameters
- including/estimating higher order terms (a^4, $\left(\Delta m_\pi^2\right)^2$, $\left(\Delta m_H^{-1}\right)^2$)

$\Rightarrow$ All fits are fully correlated.

Global fit results - ratio of decay constants

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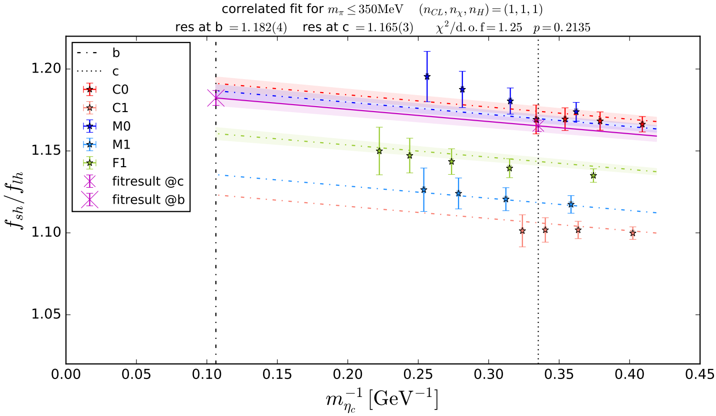

Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

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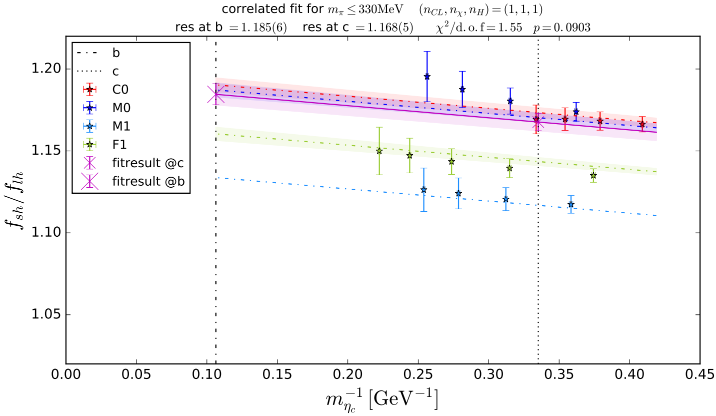

Ratio of decay constants for $m_\pi \leq 330\,\mathrm{MeV}$

Global fit results - ξ

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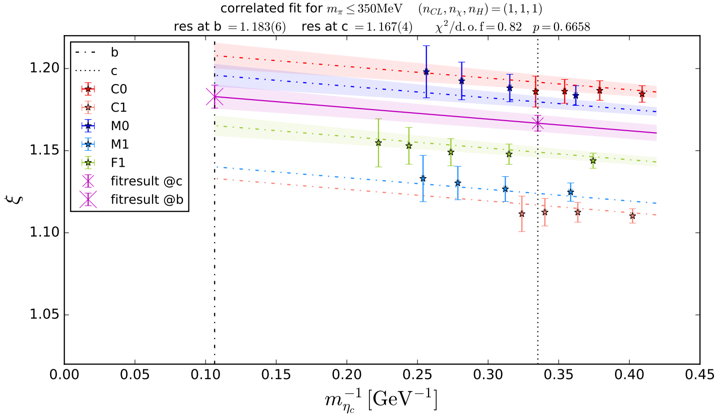

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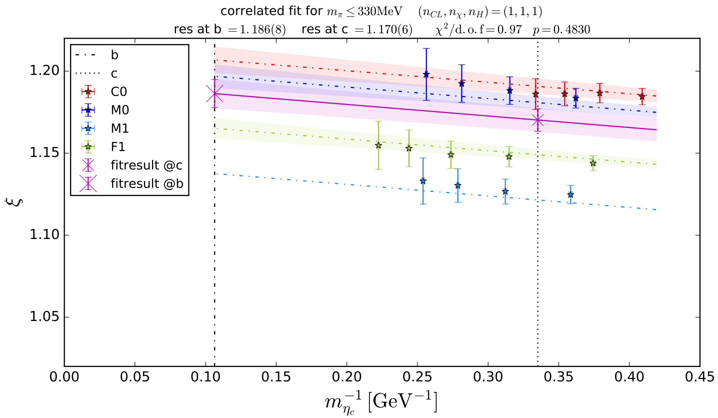
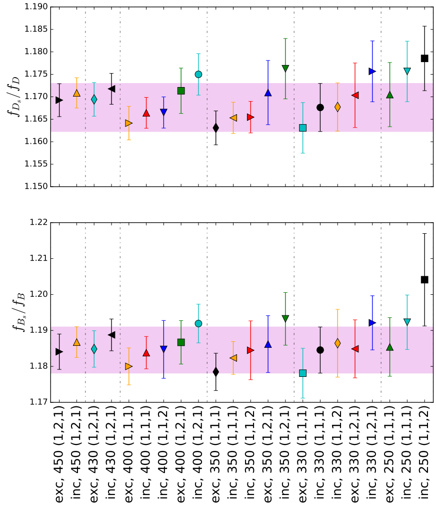
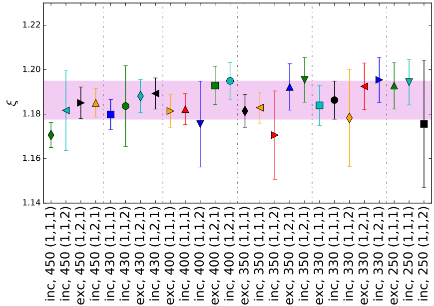


ξ for $m_\pi \leq 330\,\text{MeV}$

Global fit results - fit systematics

PRELIMINARY

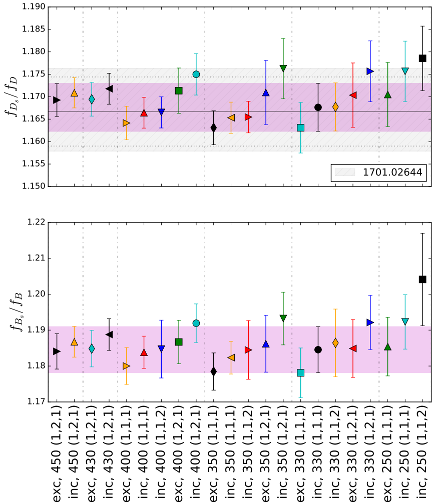


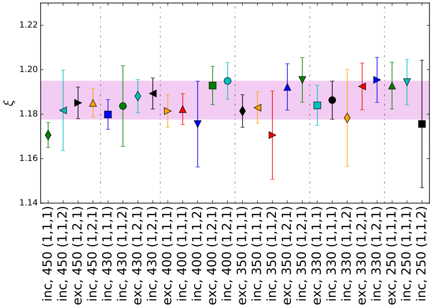


- *p*-values satisfy ≥ 0.05
- Stat error only

Global fit results - fit systematics

PRELIMINARY

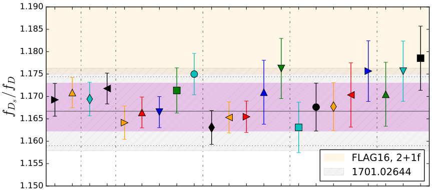


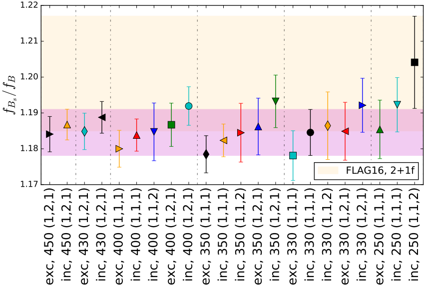


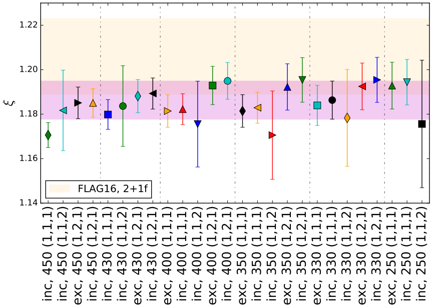
- p-values satisfy ≥ 0.05
- Stat error only

Global fit results - fit systematics

PRELIMINARY







- p-values satisfy ≥ 0.05
- Stat error only
- Competitive result (stat only)

Conclusions and Outlook

DONE

Published

- $N_f = 2+1$: f_D, f_{D_s}, f_{D_s}/f_D: JHEP **12** (2017) 008

f_{D_s}/f_D, f_{B_s}/f_B and ξ

- 3 lattice spacings, 2 physical pion masses
- m_h from below m_c to $\sim m_b/2$
  $\Rightarrow$ extrapolation to b for ratios
  $\Rightarrow$ fully relativistic
- Good continuum scaling and self-consistent
- Competitive precision

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ONGOING

Analysis underway

- Finalise sys error on ratios
  $\Rightarrow$ draft in process
- Renormalisation of mixed action: $f_{D_{(s)}}$, bag parameters, BSM mixing
- Improved correlator fitting
- Second approach to the CL: m_c and $a_\mu^{\mathrm{LOHVP},c}$

Conclusions and Outlook

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Data Production

- semi-leptonics
  $D \to \pi$, $D \to K$, $D_s \to K$

ADDITIONAL SLIDES

Linear Combinations I

Possible for any two correlation functions of equal energies which share one matrix element:

$$C_1^{AA}(t) \equiv C_{AA}^{SL}(t)X^S - C_{AA}^{SS}(t)X^L \approx A_0^S\left(A_0^L X^S - A_0^S X^L\right)e^{-E_0 t} + A_1^S\left(A_1^L X^S - A_1^S X^L\right)e^{-E_1 t}$$

$$C_1^{AP}(t) \equiv C_{AP}^{SL}(t)X^S - C_{AP}^{SL}(t)X^L \approx P_0^S\left(A_0^L X^S - A_0^S X^L\right)e^{-E_0 t} + P_1^S\left(A_1^L X^S - A_1^S X^L\right)e^{-E_1 t}$$

$$C_1^{PP}(t) \equiv C_{PP}^{SL}(t)Y^S - C_{PP}^{SL}(t)Y^L \approx P_0^S\left(P_0^L Y^S - P_0^S Y^L\right)e^{-E_0 t} + P_1^S\left(P_1^L Y^S - P_1^S Y^L\right)e^{-E_1 t}$$

$$D_1^{SL}(t) \equiv C_{AA}^{SL}(t)Y^S - C_{AP}^{SL}(t)X^S \approx A_0^L\left(A_0^S Y^S - P_0^S X^S\right)e^{-E_0 t} + A_1^L\left(A_1^S Y^S - P_1^S X^S\right)e^{-E_1 t}$$

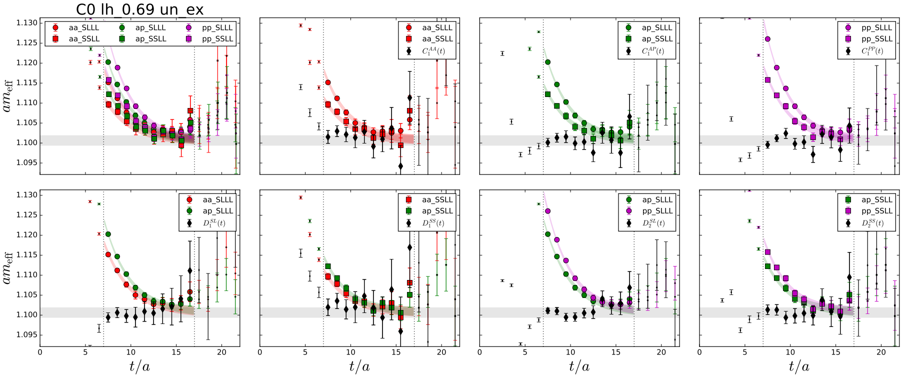
$$D_1^{SS}(t) \equiv C_{AA}^{SS}(t)Y^S - C_{AP}^{SS}(t)X^S \approx A_0^S\left(A_0^S Y^S - P_0^S X^S\right)e^{-E_0 t} + A_1^S\left(A_1^S Y^S - P_1^S X^S\right)e^{-E_1 t}$$

$$D_2^{SL}(t) \equiv C_{AP}^{SL}(t)Y^L - C_{PP}^{SL}(t)X^L \approx P_0^S\left(A_0^L Y^L - P_0^L X^L\right)e^{-E_0 t} + P_1^S\left(A_1^L Y^L - P_1^L X^L\right)e^{-E_1 t}$$

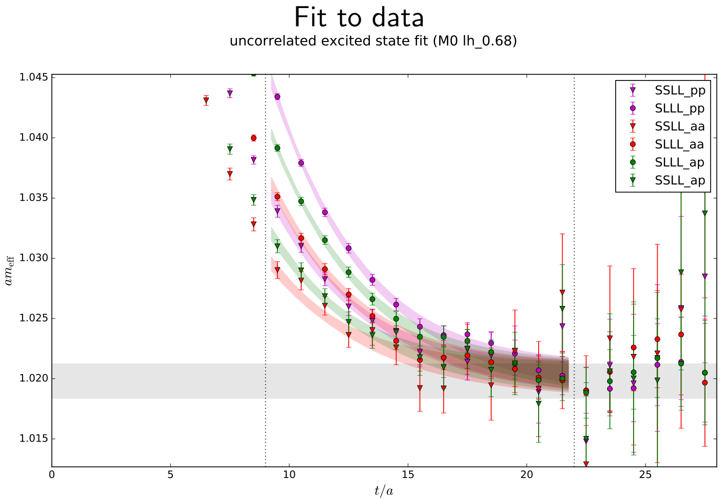
$$D_2^{SS}(t) \equiv C_{AP}^{SS}(t)Y^S - C_{PP}^{SS}(t)X^S \approx P_0^S\left(A_0^S Y^S - P_0^S X^S\right)e^{-E_0 t} + P_1^S\left(A_1^S Y^S - P_1^S X^S\right)e^{-E_1 t}$$

Identify: $X^L \equiv A_1^L$, $X^S \equiv A_1^S$, $Y^L \equiv P_1^L$ and $Y^S \equiv P_1^S$.

Example for the the 7 aforementioned linear combinations

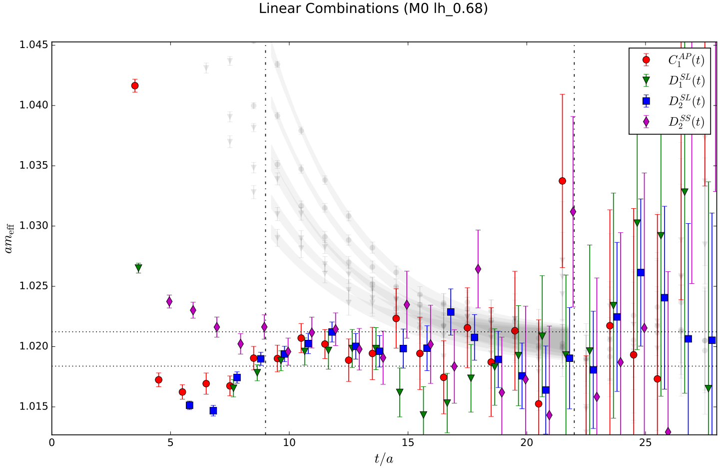


Linear Combinations III



Linear Combinations III

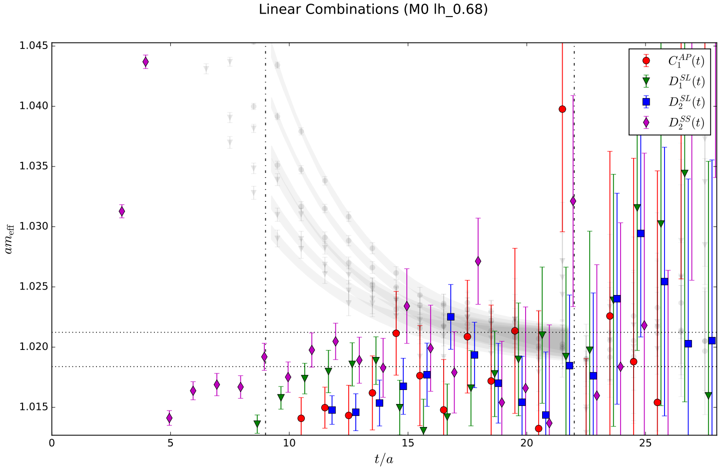
LCs using the central value of the fit result



LCs plateau in fitrange region. $\Rightarrow$ Excited state contamination removed.

Linear Combinations III

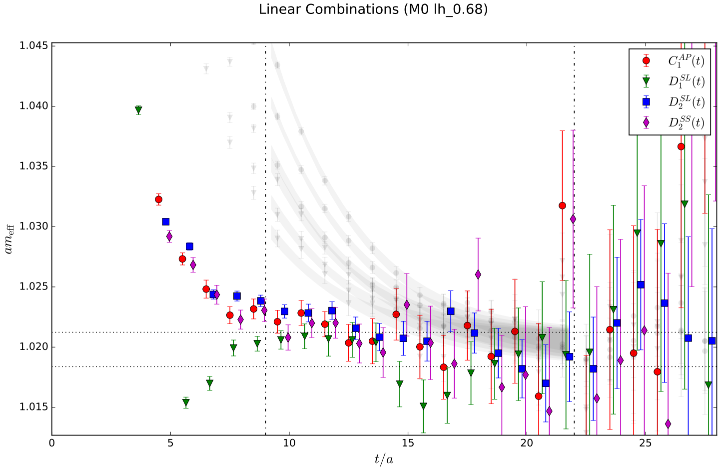
LCs from fit results $X - 3\delta X$



LCs do not plateau. $\Rightarrow$ Excited states still present.

Linear Combinations III

LCs from fit results $X + 3\delta X$



LCs do not plateau. $\Rightarrow$ Excited states still present.