Neutral meson mixing and related observables in the $D_{(s)}$ and $B_{(s)}$ meson systems

Justus Tobias Tsang

for the RBC-UKQCD Collaborations

based on **arXiv:1812.08791**

Wuhan, Lattice2019

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THE UNIVERSITY of EDINBURGH

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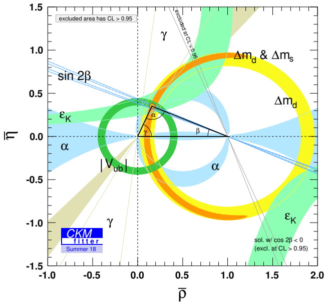
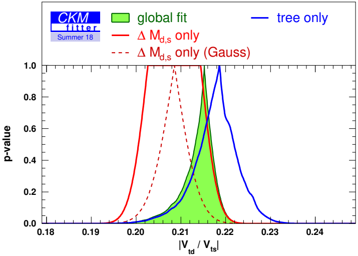
Outline

Motivation for charm and bottom flavour physics

- Huge experimental efforts: LHC, Belle II, BES III, . . .
- Constrain CKM unitarity by combining non-perturbative input with experimental data.
- Test CKM matrix by determining the same CKM matrix element from different processes
- Constrain BSM models
- Address lepton flavour universality (violations?)





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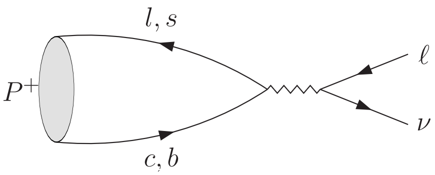
Related RBC/UKQCD talks:

- Mon 15:40 F. Erben: *"An exploratory study of heavy-light semi-leptonics using distillation"*
- Mon 16:50 R. Hill: *"Semi-leptonic B decays with RHQ b quarks"*
- Poster O. Witzel: *"Semi-leptonic form factors for exclusive $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ decays"*

Flavour Physics and CKM: leptonic decay constants

Experiment $\approx$ *CKM* $\times$ Lattice $\times$ (PT+kinematics)



Leptonic decays: $\Gamma(P \to \ell\nu_\ell) \approx |V_{q_2 q_1}|^2 \times f_P^2 \times$ known factors

where $\mathcal{Z}_A \langle 0| \bar{c}\gamma_4\gamma_5 q \,|D_q(0)\rangle = f_{D_q} m_{D_q}, \qquad q = d, s$

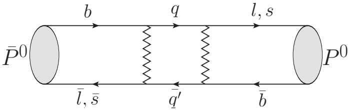
[HFLAV+BESIII] $f_D\,|V_{cd}| = (45.9 \pm 1.1)\,\mathrm{MeV}, \quad f_{D_s}\,|V_{cs}| = (249.1 \pm 3.2)\,\mathrm{MeV}$

Computing f_{D_s}/f_D gives access to V_{cs}/V_{cd}

Neutral meson mixing

Neutral mesons oscillate with their antiparticles:

$\Rightarrow$ Difference between mass eigenstates: $\Delta m^{\rm exp}$ measured to $< 1\%$!



$$\mathcal{A} \propto \left| \sum_{q=u,c,t} \frac{m_q^2}{M_W^2} V_{qb} V_{ql}^* \right|^2 \approx \frac{m_t^4}{M_W^4} \left| V_{tb} V_{tl}^* \right|^2$$

$$\Delta m \propto \underbrace{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}^0_{(s)} \right\rangle}_{\text{Short distance}} + \underbrace{\sum_n \frac{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=1} \middle| n \right\rangle \left\langle n \middle| \mathcal{H}^{\Delta b=1} \middle| \bar{B}^0_{(s)} \right\rangle}{E_n - M_{B_{(s)}}}}_{\text{Long distance}}$$

$m_t^2 V_{tb} V_{tl}^* \gg m_c^2 V_{cb} V_{cl}^* \gg m_u^2 V_{ub} V_{ul}^* \Rightarrow$ **Short distance dominated**.

Operator Product Expansion

Two scale problem: $\Lambda_{\mathrm{QCD}} \sim 1\,\mathrm{GeV} \ll m_{EW} \sim 100\,\mathrm{GeV}$:

$\Rightarrow$ Factorise via OPE

$$\Delta m \propto \sum_i C_i(\mu) \left\langle B^0_{(s)} \middle| \mathcal{O}_i^{\Delta b=2}(\mu) \middle| \bar{B}^0_{(s)} \right\rangle$$

- Perturbative model-dependent Wilson coefficients $C_i(\mu)$
- Non-perturbative model-independent matrix elements of $\mathcal{O}_i^{\Delta b=2}(\mu)$
- 5 independent (parity even) operators $\mathcal{O}_i$.

$\Rightarrow$ SM: $\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$
$+$ 4 BSM operators: $\mathcal{O}_2 - \mathcal{O}_5$

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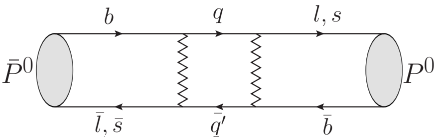
RBC/UKQCD's $K - \bar{K}$ BSM mixing calculation

P. Boyle, N. Garron, J. Hudspith, A. Jüttner, **J. Kettle**, A. Khamseh, C. Lehner, A. Soni, JTT [1812.04981 PoS Lat'18, in preparation]

Flavour Physics and CKM: neutral meson mixing

$$\Delta m_P = \left| V^*_{tq_2} V_{tq_1} \right| \times f_P^2 m_P \hat{B}_P \times \text{known factors}$$



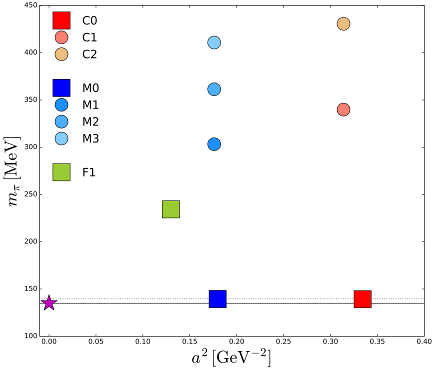
[HFLAV]

$$\Delta m_d = 0.5064 \pm 0.0019\,\mathrm{ps}^{-1}$$
$$\Delta m_s = 17.757 \pm 0.021\,\mathrm{ps}^{-1}$$

Computing ξ gives access to ratio V_{td}/V_{ts}:

$$\xi^2 = \frac{f_{B_s}^2 B_{B_s}}{f_B^2 B_B} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

RBC/UKQCD $N_f = 2+1$ ensembles



- Iwasaki gauge action
- Domain Wall Fermion action
  $\Rightarrow N_f = 2+1$ flavours in the sea
  $\Rightarrow M_5 = 1.8$ for light and strange
- **2 ensembles with physical pion masses** [PRD **93** (2016) 074505]
- 3 Lattice spacings [JHEP **12** (2017) 008]
- Heavier m_π ensembles guide small chiral extrapolation of F1*

Chiral Fermions:

$\Rightarrow$ $O(a)$ improved

$\Rightarrow$ Multiplicative renormalisation

* F1 properties under investigation but expect only minor effects

Lattice set-up I

Light and strange

- Unitary light quark mass
- Physical strange quark mass
- DWF parameters same between sea and valence
- Gaussian source (sink) smearing for better overlap with ground state

Heavy (charm and beyond)

- Möbius DWF
- $M_5 = 1.0,\ L_s = 12$
- Stout smeared (3 hits, $\rho = 0.1$)
- Range of quark masses from below charm to $\sim m_b/2$ on finest ensemble

$\Rightarrow$ **All DWF** mixed action set-up

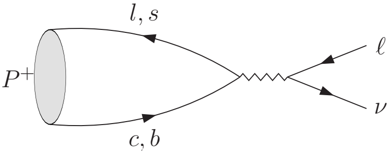
$\Rightarrow$ $\mathbb{Z}_2$-noise sources (volume average) on every 2nd time slice

$\Rightarrow$ Increased heavy quark reach compared to [JHEP **04** (2016) 037,JHEP **12** (2017) 008]

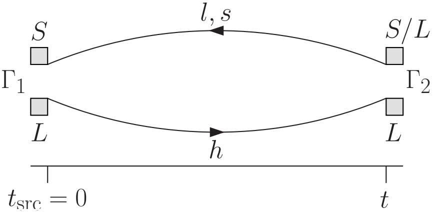
$\rightarrow$ extrapolation towards b

Lattice setup II

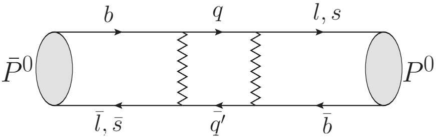
Leptonic decays:



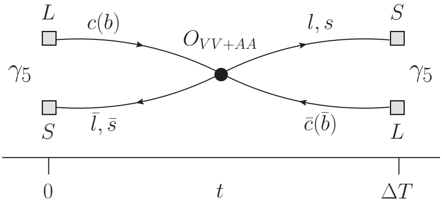
$$\mathcal{Z}_A \langle 0 | \overline{c}\gamma_4\gamma_5 q \, | D_q(0) \rangle = f_{D_q} m_{D_q}$$



$P^0 - \bar{P}^0$-mixing

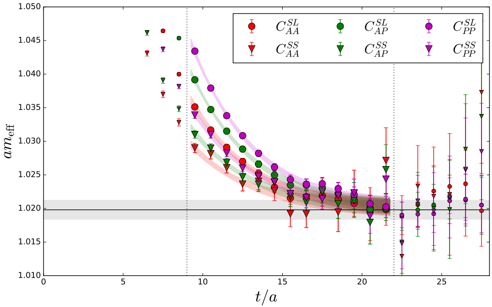


$$B_P = \frac{\langle \bar{P}^0 | O_{VV+AA} | P^0 \rangle}{8/3 f_P^2 m_P^2}$$

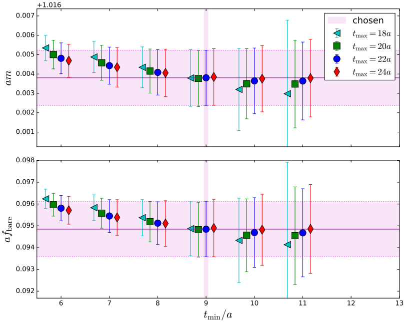


Many source-sink separations ΔT for 4-quark operator

Simultaneous two-exponential fit of 6 channels to extract masses and matrix elements of interest

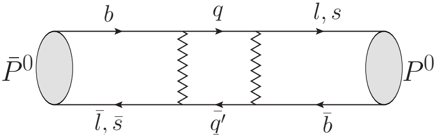


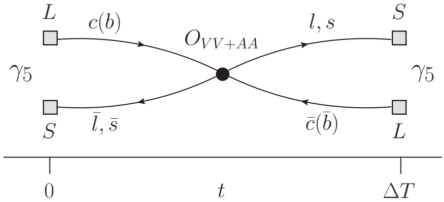
Example fit of worst case:
heavy-light meson with $am_h = 0.68$ on M0



Stability

Correlator Fitting of 4-quark operators I





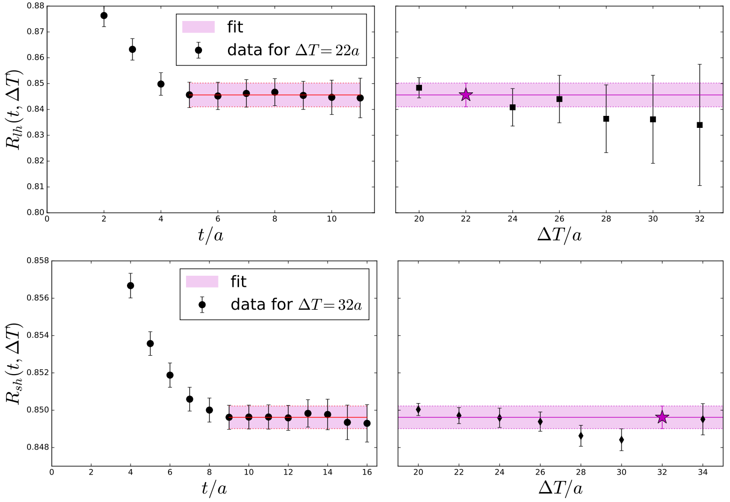
$$C_3(t, \Delta T) \equiv \left\langle P(\Delta T) O_{VV+AA}(t) \bar{P}^\dagger(0) \right\rangle$$

$$R(t, \Delta T) = \frac{C_3(t, \Delta T)}{8/3 C_{PA}(\Delta T - t) C_{AP}(t)} \to B_P \quad \text{for} \quad t, \Delta T \gg 0$$

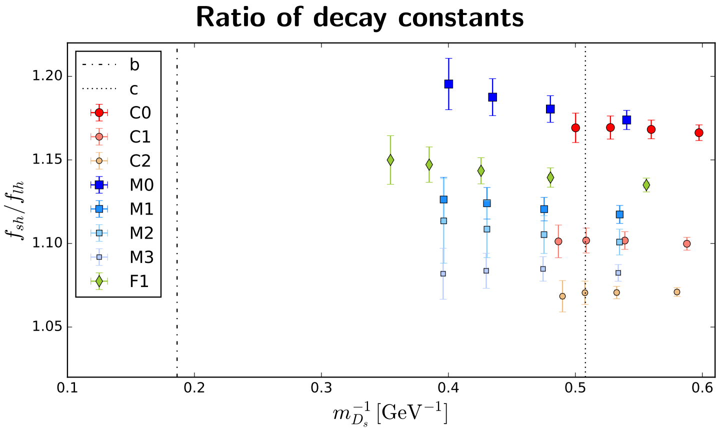
- Expect $R(t, \Delta T)$ to plateau for large t
- Check stability of plateaux value by varying ΔT

Correlator Fitting of 4-quark operators II

Ex: $am_h = 0.68$ on M0



Results of correlator fits



⇒ Renormalisation constants cancel

⇒ Mild linear behaviour with $1/m_H$ and a^2

⇒ Stat precision: 0.4 - 1.0 %

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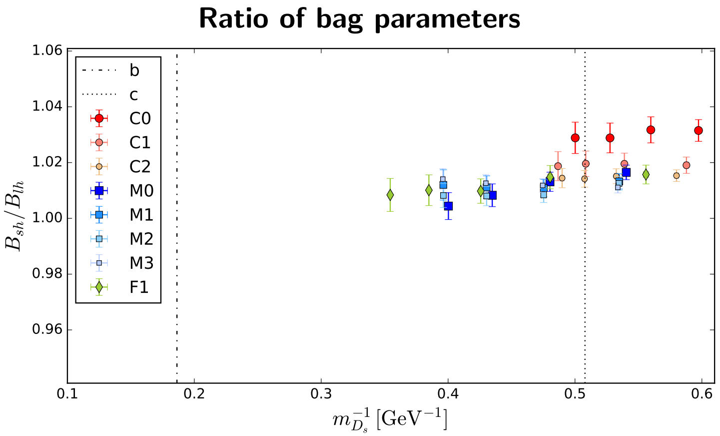

⇒ Renormalisation constants cancel

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Global fit form

Base fit

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\mathrm{phys}}, m_H^{\mathrm{phys}}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$

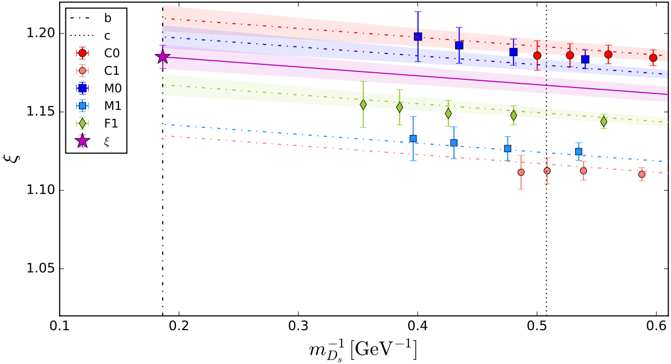
Assess systematic errors by

- varying cuts on pion mass
- using $m_H = m_D$, m_{D_s} and m_{η_c}
- varying inclusion/exclusion of heaviest data points
- varying inclusion/exclusion of fit parameters
- including/estimating higher order terms (a^4, $(\Delta m_\pi^2)^2$, $(\Delta m_H^{-1})^2$)

$\Rightarrow$ Global fits are fully correlated.

Global fit results for ξ

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\rm phys}, m_H^{\rm phys}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

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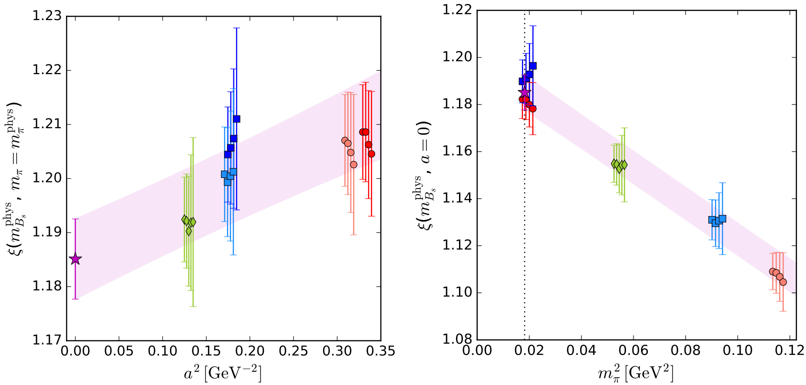
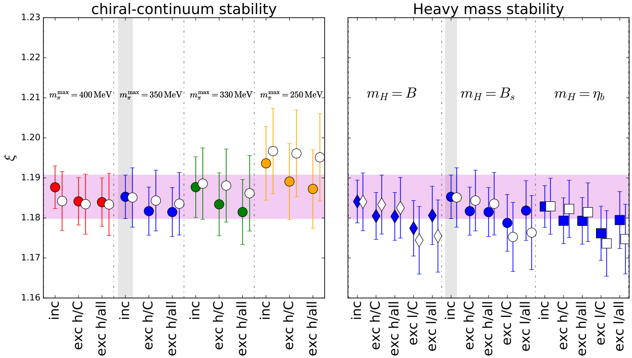


Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

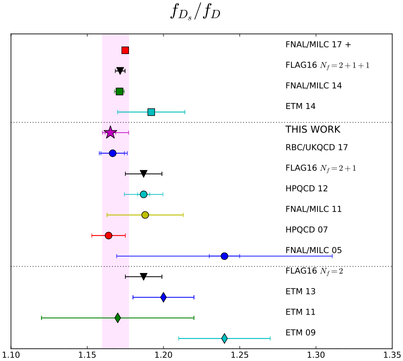
Systematic Errors - variations of cuts to data for ξ

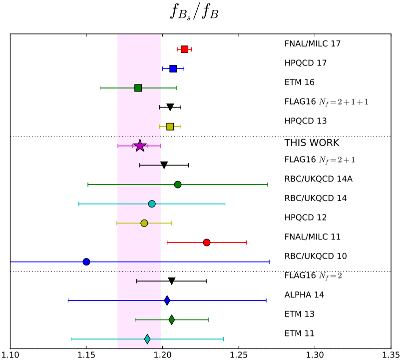
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$\xi = 1.1853(54)_{\mathrm{stat}} \left({}^{+116}_{-156}\right)_{\mathrm{sys}}$$

Comparison to literature - ratio of decay constants



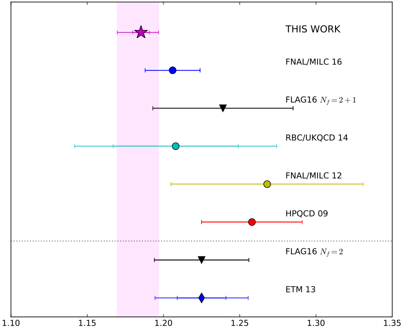


- Self consistent with RBC/UKQCD17: JHEP **12** (2017) 008
- Complimentary to (most) literature - no effective action for *b*.
- One of few results with physical pion masses.

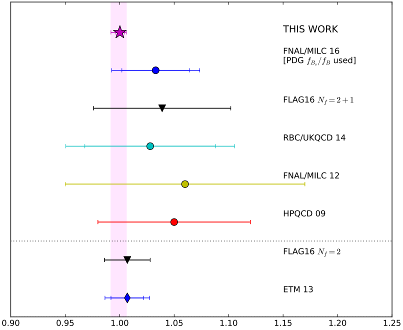
$$|V_{cd}/V_{cs}| = 0.2148(56)_{\mathrm{exp}} \left({}^{+22}_{-10}\right)_{\mathrm{lat}}$$

Comparison to literature - ratio of mixing parameters

ξ



B_{B_s}/B_{B_d}



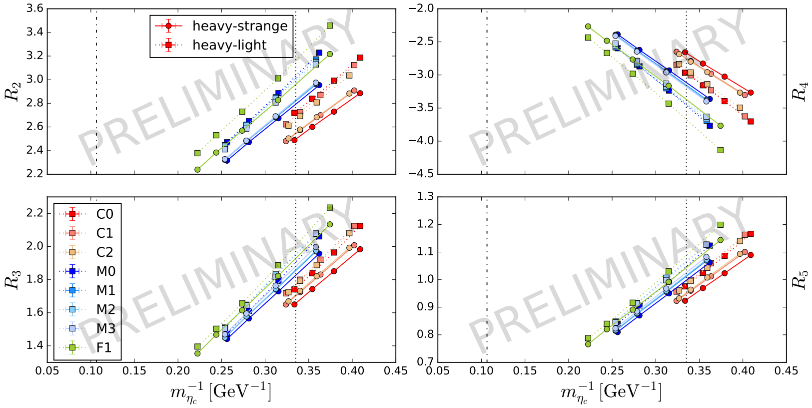
- Complimentary - no effective action needed for *b*
- Complimentary - **no operator mixing**!
- **First time with physical pion masses**

$$|V_{td}/V_{ts}| = 0.2018(\ 4)_{\rm exp} \left({}^{+20}_{-27}\right)_{\rm lat}$$

Next steps: Decay constants and bag parameters

- Different choice of (domain wall) action between light/strange and heavy quarks leads to a mixed action
- Mixed action renormalisation constants cancel for appropriate ratios (f_{B_s}/f_B, B_{B_s}/B_B), but are needed for individual decay constants and bag parameters.
- Need to carry out the fully non-perturbative mixed action renormalisation as outlined in JHEP **12** (2017) 008.
- Extend the study to the full BSM operator basis
$\Rightarrow$ analogous to RBC/UKQCD's $K-\bar{K}$ study (1812.04981, in preparation)

$B^0_{(s)} - \bar{B}^0_{(s)}$ and $D^0 - \bar{D}^0$ **PRELIMINARY and BARE**



- "quite linear" in m_H^{-1}
- similar slopes for h-l and h-s $\Rightarrow SU(3)$ breaking rat's?
- renormalisation to be done (mixed action + op mixing)
- analogous analysis to $K - \bar{K}$ paper + m_H dependence

Conclusions and Outlook

$SU(3)$ breaking ratios

- **arXiv:1812.08791**
- f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_B and ξ
- $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$
- 3 lattice spacings, 2 m_π^{phys}
- First result for ξ and B_{B_s}/B_B with m_π^{phys}
- m_h from below m_c to $\sim m_b/2$
  $\Rightarrow$ extrapolation to b for ratios
  $\Rightarrow$ fully relativistic
- Good continuum scaling and self-consistent
- Competitive precision

Ongoing

- Mixed action renormalisation of bilinears and four quark operators underway
- First results look promising

$\Rightarrow$ Determine $f_{B_{(s)}}$, $f_{D_{(s)}}$

$\Rightarrow$ Extend to full mixing operator basis for $B_{(s)}$ and compute short distance part of D.

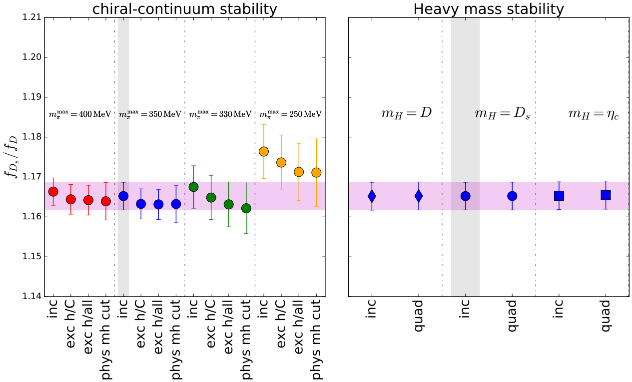
Outlook

- Supplement dataset with very fine JLQCD ensembles
- $a^{-1} = 2.8\,\mathrm{GeV}$, $m_\pi = m_\pi^{\mathrm{phys}}$

ADDITIONAL SLIDES

Systematic Errors - variations of cuts to data for f_{D_s}/f_D

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{D_s}/f_D = 1.1652(35)_{\rm stat} \begin{pmatrix} +120 \\ -\ \ 52 \end{pmatrix}_{\rm sys}$$

Systematic Errors - variations of cuts to data for f_{B_s}/f_B

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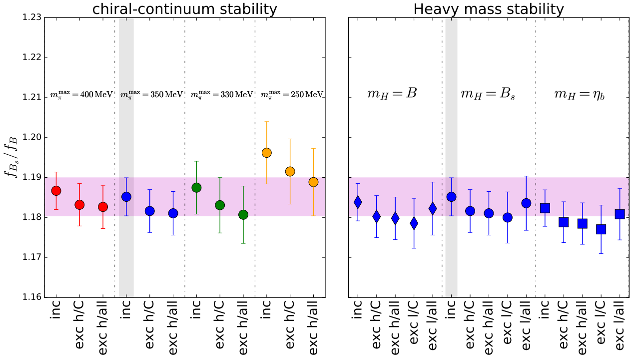

$$f_{B_s}/f_B = 1.1852(48)_{\mathrm{stat}} \left({}^{+134}_{-145}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for B_{B_s}/B_B

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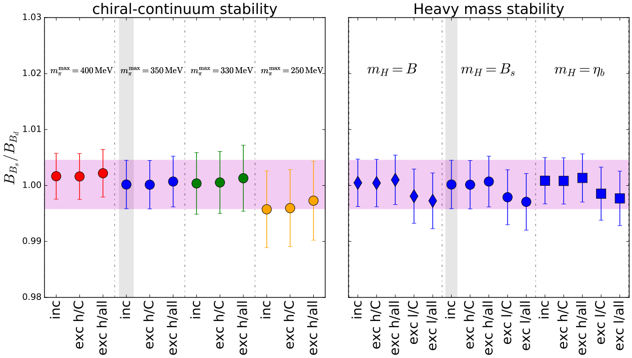

$$B_{B_s}/B_B = 1.0002(43)_{\text{stat}} \left(^{+60}_{-82}\right)_{\text{sys}}$$

Systematic Errors - variations of cuts to data for ξ

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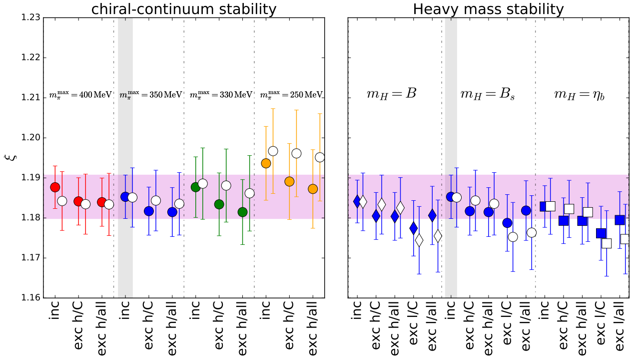
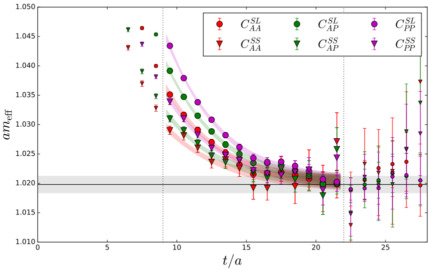


$$\xi = 1.1853(54)_{\mathrm{stat}} \binom{+116}{-156}_{\mathrm{sys}}$$

Cross checks of correlator fits I

$$C^{LS}_{AP}(t) \approx A^L_0 P^S_0 e^{-E_0 t} + A^L_1 P^S_1 e^{-E_1 t}$$

$$C^{SS}_{AP}(t) \approx A^S_0 P^S_0 e^{-E_0 t} + A^S_1 P^S_1 e^{-E_1 t}$$



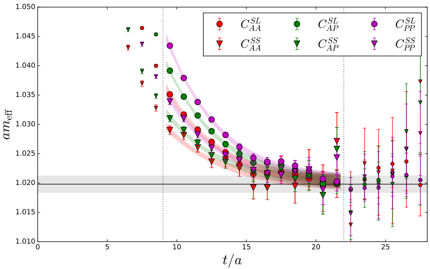
Construct Linear Combination

$$\begin{aligned} C^{AP}_1(t) &\equiv C^{LS}_{AP}(t) X^S - C^{SS}_{AP}(t) X^L \\ &\approx P^S_0 \left(A^L_0 X^S - A^S_0 X^L \right) e^{-E_0 t} \\ &+ P^S_1 \left(A^L_1 X^S - A^S_1 X^L \right) e^{-E_1 t} \end{aligned}$$

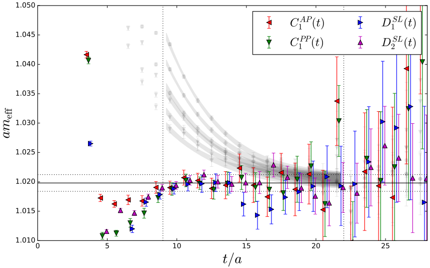
Cross checks of correlator fits I

$$C_1^{AP}(t) \approx P_0^S \left(A_0^L X^S - A_0^S X^L\right) e^{-E_0 t} + \underbrace{P_1^S \left(A_1^L X^S - A_1^S X^L\right)}_{\text{small}} e^{-E_1 t}$$

Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.



$$C_1^{AP}(t) \approx P_0^S \left(A_0^L X^S - A_0^S X^L\right) e^{-E_0 t} + P_1^S \underbrace{\left(A_1^L X^S - A_1^S X^L\right)}_{\text{small}} e^{-E_1 t}$$

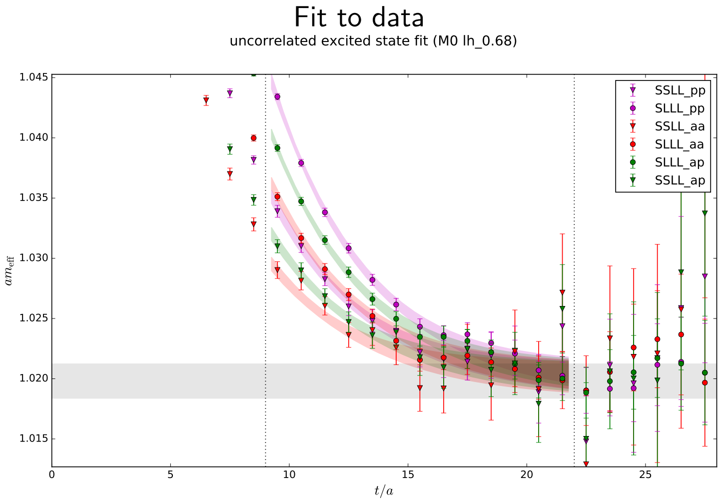


Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.

$\Rightarrow$ Removes (most of) excited state

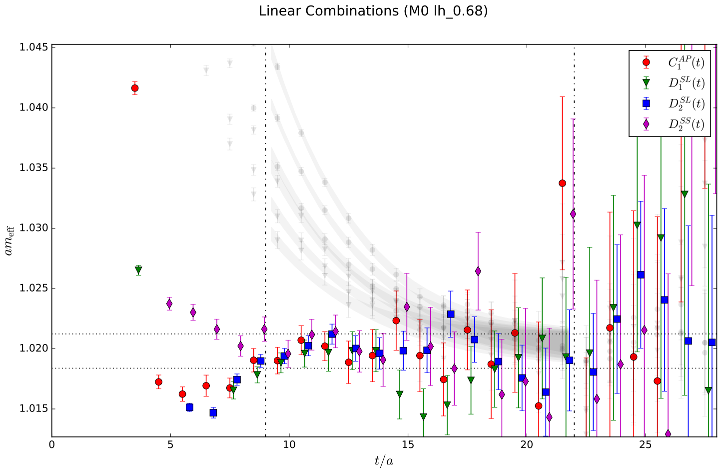
$\Rightarrow$ Strong *a posteriori* check of fit range

Cross checks of correlator fits II



Cross checks of correlator fits II

LCs using the central value of the fit result



LCs plateau in fitrange region. $\Rightarrow$ Excited state contamination removed.

Non-Perturbative Renormalisation of mixed action

SMOM ren. conds. relates amputated vertex functions to Z factors.

$$\begin{aligned} 1 &= \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q\cdot\Lambda_A^{\mathrm{ren}}\right)\gamma_5 \not{q}\right]\Big|_{\mathrm{sym}} \\ &= \frac{Z_A}{Z_q} \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q\cdot\Lambda_A^{\mathrm{bare}}\right)\gamma_5 \not{q}\right]\Big|_{\mathrm{sym}} \\ &\equiv \frac{Z_A}{Z_q}\mathcal{P}[\Lambda_A^{\mathrm{bare}}] \end{aligned}$$

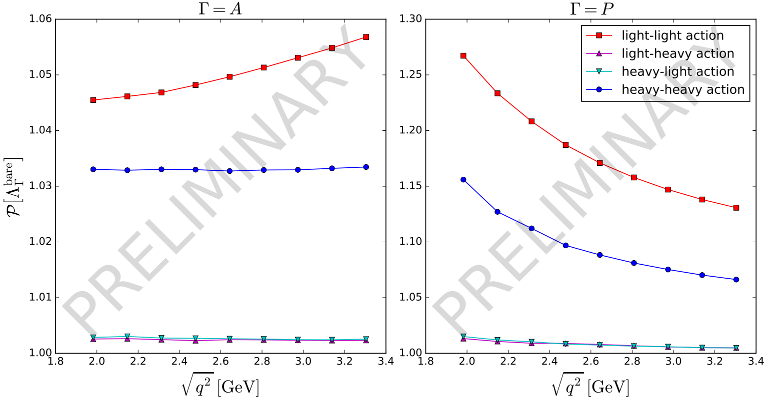
So for actions i,j

$$\frac{\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ii}\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{jj}}{\left(\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ij}\right)^2} = \frac{(Z_A^{ij})^2}{Z_A^{ii}Z_A^{jj}}$$

But for non-mixed actions we can determine Z_A^{ii} from conserved current.

Preliminary mixed action renormalisation

First study on single configuration



$\Rightarrow$ mixed NPR is feasible

$\Rightarrow$ need to compute Z_A^{hh} from conserved current to obtain Z_A^{hl}