# Semileptonic $B$ decays with RHQ $b$ quarks

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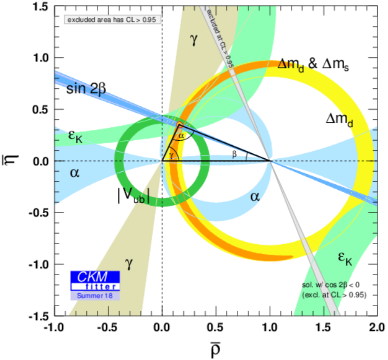
# Talk Outline

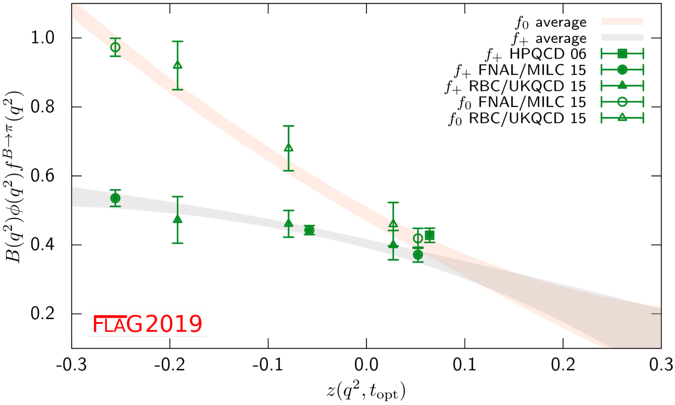
# Motivation

- Test unitarity of CKM matrix
- $B \to \pi \ell \nu$ constrains $|V_{ub}|$
- $B \to D \ell \nu$ constrains $|V_{cb}|$
- 2-3$\sigma$ discrepancy between exclusive ($B \to \pi \ell \nu$) and inclusive ($B \to X_u \ell \nu$)
- Measurements of $R(D)$, $R(D^*)$, $R(\pi)$




CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

# Motivation




[Flavour Lattice Averaging Group: http://flag.unibe.ch/2019/MainPage]

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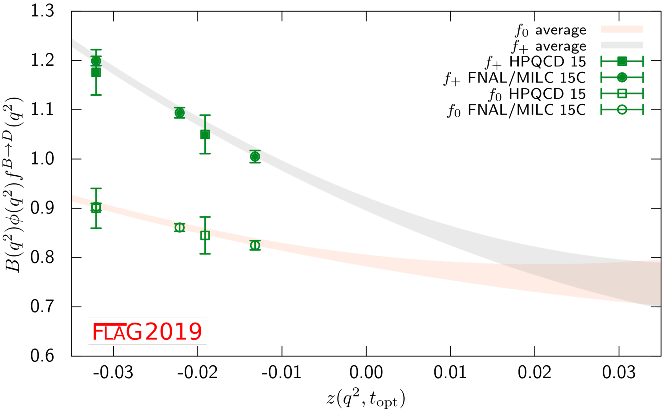


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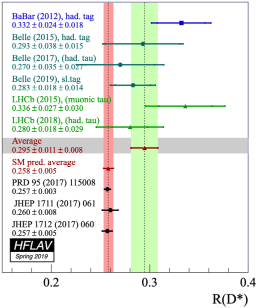
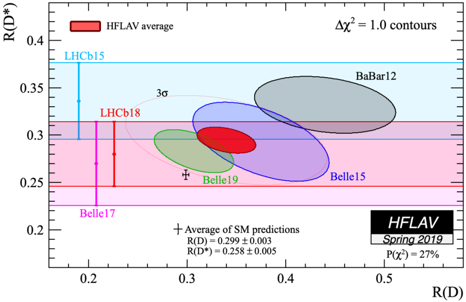



[Heavy Flavour Averaging Group: https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/main.shtml]

# Goal

- Differential $B \to \pi\ell\nu$ decay rate

$$\underbrace{\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2}}_{\text{Experiment}} = \underbrace{\frac{G_F^2}{192\pi^3 M_B^3}\lambda(q^2)^{3/2}}_{\text{Known}} \underbrace{f_+(q^2)}_{\text{Lattice}} \underbrace{|V_{ub}|}_{\text{Target}}{}^2$$

$$\left[\lambda(q^2) = (M_B^2 + M_\pi^2 - q^2)^2 - 4M_B^2 M_\pi^2\right]$$

$q^2$ — momentum transfer to $\ell\nu$

- So seek to compute vector form factor $f_+(q^2)$ on the lattice

# Goal

- Compute the hadronic matrix element for the flavour-changing vector currents $\langle P|\mathcal{V}^\mu|B\rangle$
- Standard parameterisation in terms of the scalar and vector form factors $f_0$ and $f_+$:

$$\langle P|\mathcal{V}^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_P^\mu - \frac{M_B^2 - M_P^2}{q^2} q^\mu\right) + f_0(q^2)\left(\frac{M_B^2 - M_P^2}{q^2} q^\mu\right)$$

# Goal

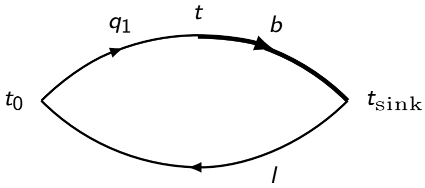
- Parallel and perpendicular form factors $f_\|$ and $f_\perp$ are easier to relate to lattice data in the rest frame of the $B$-meson:

$$\langle P|\mathcal{V}^\mu|B\rangle = \sqrt{2M_B}\left[v^\mu f_\|(E_P) + p_\perp^\mu f_\perp(E_P)\right]$$

with
$v^\mu$ — $B$-meson 4-velocity
$p_\perp^\mu$ — $p_P^\mu - (p_P \cdot v)v^\mu$
$p_P^\mu$ — momentum of pseudoscalar particle

$$f_\| = \frac{\langle P|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}} \qquad f_\perp = \frac{\langle P|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}}\frac{1}{p^i}$$

# Strategy




- Insert momentum at $t_0$
- Quark propagator $(t_0 \xrightarrow{q_1} t)$
- Sequential propagator $(t_0 \xrightarrow{l} t_{\rm sink} \xrightarrow{b} t)$
- Insert current at $t$
- $q_1$: $l$ or $c$

# Light and Strange Quark Action

- RBC-UKQCD's 2+1 Domain-Wall Fermion and Iwasaki gauge action ensembles
  [PRD 78 (2008) 114509] [PRD 83 (2011) 074508] [PRD 93 (2016) 074505] [JHEP 1712 (2017) 008]
  - Unphysical $5^{\mathrm{th}}$ dimension with extent $L_s$, physical fields live on the 4-D boundary
  - Preserves chiral symmetry in limit $L_s \to \infty$
  - Compute with finite $L_s$, gives residual mass $m_{\mathrm{res}}$ from controllable chiral symmetry breaking

# Heavy Quark Action

- RHQ Action for $b$ quarks, Columbia interpretation
  [Christ et al. PRD 76 (2007) 074505] [Lin and Christ PRD 76 (2007) 074506]
  - Builds on original Fermilab action [El-Khadra et al. PRD 55 (1997) 3933]
  - Related to Tsukuba interpretation [S. Aoki et al. PTP 109 (2003) 383]
  - Clover action with anisotropic clover term
  - Uses 3 parameters $(m_0 a, c_p, \zeta)$ that can be non-pertubatively tuned to remove $\mathcal{O}((m_0 a)^n)$, $\mathcal{O}(\vec{p}a)$, $\mathcal{O}((\vec{p}a)(m_0 a)^n)$ errors [PRD 86 (2012) 116003]
  - Use current improvement terms to get $\mathcal{O}(a)$ improved discretisation errors, in line with DWF fermions

# Ensembles

| | $L^3 \times T \ / \ a^4$ | $a^{-1}$ / GeV | $m_\pi$ / MeV |
|---|---|---|---|
| C1 | $24^3 \times 64$ | 1.78 | 340 |
| C2 | $24^3 \times 64$ | 1.78 | 430 |
| M1 | $32^3 \times 64$ | 2.38 | 300 |
| M2 | $32^3 \times 64$ | 2.38 | 360 |
| M3 | $32^3 \times 64$ | 2.38 | 410 |
| F1 | $48^3 \times 96$ | 2.77 | 230 |

- 2+1f ensembles: Degenerate light quark
- Sea quarks: Domain-wall fermions
- Future plans to include physical pion mass ensemble C0 to the analysis.

# $B \to \pi$ Analysis

# Form Factors

- Starting point:

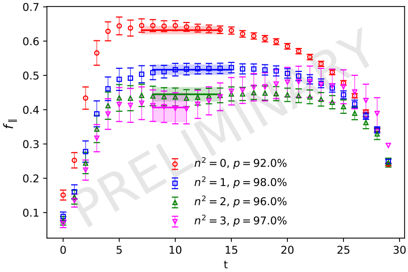
$$\langle \pi | \mathcal{V}^\mu | B \rangle = \sqrt{2M_B}\left[v^\mu f_{\parallel}(E_P) + p_\perp^\mu f_\perp(E_P)\right]$$

- Calculate $f_{\parallel}$ and $f_\perp$ from lattice data

$$f_{\parallel} = \lim_{t_0 \ll t \ll t_{\text{snk}}} R_0^{B\to\pi}(t, t_{\text{snk}})$$

$$f_\perp = \lim_{t_0 \ll t \ll t_{\text{snk}}} \frac{1}{p^i} R_i^{B\to\pi}(t, t_{\text{snk}})$$

$$R_\mu^{B\to\pi} = \frac{C_{3,\mu}^{B\to\pi}(t, t_{\text{snk}})}{\sqrt{C_2^\pi(t) C_2^B(t - t_{\text{snk}})}} \sqrt{\frac{2E_\pi}{e^{-E_\pi t} e^{-M_B(t - t_{\text{snk}})}}}$$



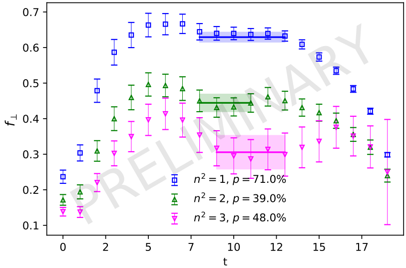

$f_{\parallel}$ on F1 ensemble

# Form Factors

- Calculate $f_\parallel$ and $f_\perp$ from lattice data

$$f_\parallel = \lim_{t_0 \ll t \ll t_{\text{snk}}} R_0^{B\to\pi}(t, t_{\text{snk}})$$

$$f_\perp = \lim_{t_0 \ll t \ll t_{\text{snk}}} \frac{1}{p^i} R_i^{B\to\pi}(t, t_{\text{snk}})$$




$f_\perp$ on C1 ensemble

- Use this to find $f_0$ and $f_+$:

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 + E_P^2}\left[(M_B - E_P)f_\parallel(q^2) + (E_P^2 - M_P^2)f_\perp(q^2)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(q^2) + (M_B - E_P)f_\perp(q^2)\right]$$

# Kinematic extrapolation strategy

- Separate chiral-continuum extrapolation from the $q^2$ extrapolation
  - Use results in the continuum
  - Use a model-independent extrapolation from this point on: $z$-expansions
  - Involves a change of variables $q^2 \to z$
  - By using continuum results, z-expansion coefficients are not lattice-dependent

# Kinematic extrapolation strategy

- Fit to synthetic data points in order to propagate both statistical and systematic errors to final result
  - Interpolate continuum results for the form factors to reference $q^2$ values equally spaced in $z$
  - Build up an aggregate correlation matrix of statistical and systematic errors
  - Create synthetic bootstrap samples at reference $q^2$ by taking a fake Monte-Carlo time series correlated by aggregate correlation matrix
  - Fit the $z$-expansion to synthetic data points that now combine both systematics and statistics

# Chiral Continuum Fits

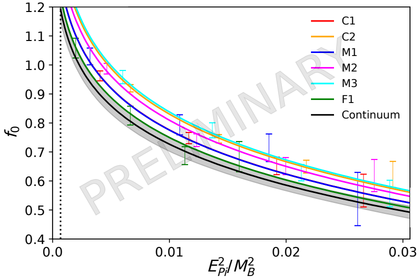
- Extrapolate to physical pion mass and zero lattice spacing simultaneously
- Use NLO hard-pion SU(2) HM$\chi$PT [PRD 67 (2003) 054010]

$$f(M_\pi, E_\pi, a) = \frac{c_1}{\Delta + E_\pi}\left(1 + \frac{\delta f}{(4\pi f_\pi)^2} + c_2\frac{M_\pi^2}{\Lambda^2} + c_3\frac{E_\pi}{\Lambda} + c_4\left(\frac{E_\pi}{\Lambda}\right)^2 + c_5\left(\frac{a}{\Lambda}\right)^2\right)$$

- $\Lambda = 1$ GeV
- $\Delta_0 = 0.263$ GeV
- $\delta f^{B\to\pi} = -\frac{3}{4}(3g_b^2 + 1)M_\pi^2 \log\left(\frac{M_\pi^2}{\Lambda^2}\right)$
- $f_\pi = 0.1304$ GeV
- $\Delta_+ = -0.0416$ GeV
- $g_b = 0.57$

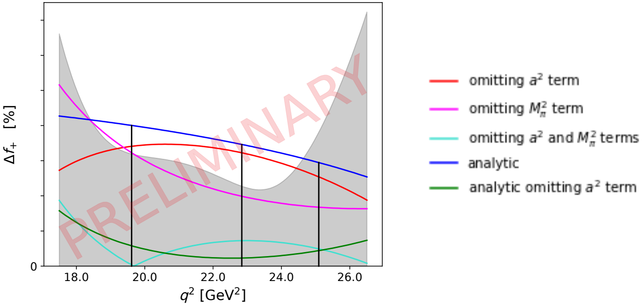
# Chiral Continuum Fits

- Three values of $a$
- Three/four values of $E_\pi$ per ensemble
- Six ensembles/pion masses with three lattice spacings
- Simultaneously fit coefficients $c_{1-5}$ over all data
- Continuum form factor given by $f(M_\pi^{\mathrm{phys}}, E_\pi, 0)$



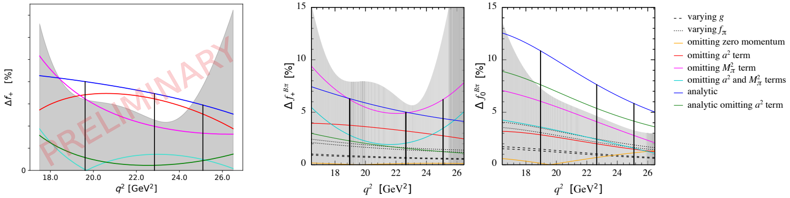
# Systematic Error Analysis

- Sources of systematic error can be estimated by varying fit ansatz, parameters, propagating known uncertainties
- Estimate error due to chiral continuum fit, RHQ parameters uncertainty, quark mass uncertainty...



# Systematic Error Analysis - F1 included/excluded

- Preliminary error analysis suggests a significant reduction in statistcal and systematic errors when the F1 ensemble is included (new!)
- Efforts currently ongoing



## z-expansion

- Change variables from $q^2$ to $z$ with

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

$$t_+ = (M_B + M_\pi)^2$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Allows the form factors to be expanded as a power series in $z$
- Specifics determined by choice of form and parameters: BCL expansion used in this study [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

# z-expansion

- Express $f_+$ as convergent power series

$$f_+(q^2) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[ z^k - (-1)^{k-K} \frac{k}{K} z^k \right]$$

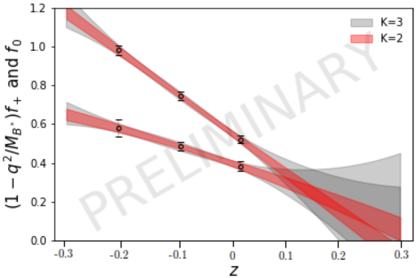
- Express $f_0$ as functional form

$$f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$$

- $|z| < 1$ so convergence is assured

# z-expansion

- Generate synthetic data points using correlation matrices given by error analysis
- Extrapolate $z$-expansion fits over full kinematic range
- Can constrain the fit by enforcing $f_+ = f_0$ at $q^2 = 0$
- Can combine with experimental data to determine $|V_{ub}|$
- Efforts ongoing



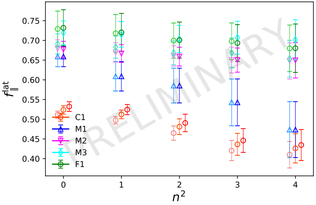
# $B \to D$ Analysis

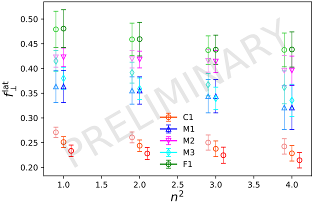
# Form Factors

- Charm quarks simulated with Möbius Domain-Wall Fermions for heavy quarks

  [JHEP 1604 (2016) 037] [JHEP 1712 (2017) 008]

- Simulate charm quarks at 2/3 different masses in order to inter/extrapolate to the physical value of the charm quark mass.



# Analysis

- Extrapolate to physical masses and continuum
- Proceed hereafter as with $B \to \pi$
- Alter fit ansatz etc. in order to estimate systematics
- Create synthetic data at reference $q^2$ values to take into account both systematics and statistics
- Perform z-expansions to obtain form factors over full kinematic range

# $B \to D$ Status

- Analysis in early stages
- Part of the RBC/UKQCD RHQ project and will form a part of upcoming publication

# Summary

- Updates to RHC/UKQCD 2015 results in the pipeline
- More precisely determined lattice spacing and inclusion of finer F1 ensemble reduces errors
- Future plans to include physical pion mass ensemble to strongly constrain chiral continuum extrapolation
- Finalising CL-extrapolation, error budget, z-fits, on $B \to \pi \ell \nu$, $B \to D \ell \nu$
- Will be comparing to other z-expansions
- $B_s \to K \ell \nu$ and $B_s \to D_s \ell \nu$: Oliver Witzel, 14:00 today