$\Delta b = 2$ mixings and ξ

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for the RBC-UKQCD Collaborations

Based on arXiv:1812.08791

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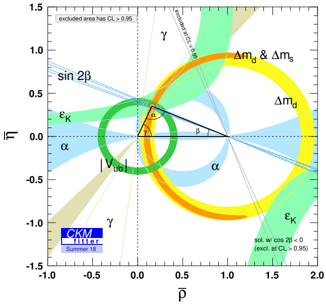
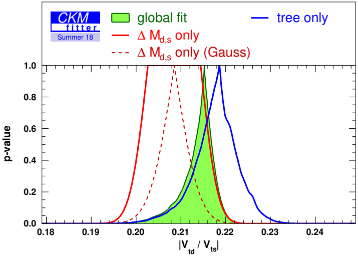
THE UNIVERSITY *of* EDINBURGH

Outline

Motivation for charm and bottom flavour physics

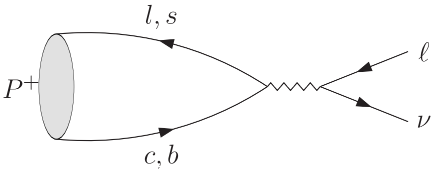
- Huge experimental efforts: LHC, Belle II, BES III, ...
- Constrain CKM unitarity by combining non-perturbative input with experimental data.
- Test CKM matrix by determining the same CKM matrix element from different processes
- Constrain BSM models
- Address lepton flavour universality (violations?)





Flavour Physics and CKM: leptonic decay constants

Experiment $\approx$ *CKM* $\times$ Lattice $\times$ (PT+kinematics)



Leptonic decays: $\Gamma(P \to \ell\nu_\ell) \approx |V_{q_2 q_1}|^2 \times f_P^2 \times$ known factors

where $\mathcal{Z}_A \langle 0| \bar{c}\gamma_4\gamma_5 q |D_q(0)\rangle = f_{D_q} m_{D_q}, \qquad q = d, s$

[HFLAV+BESIII] $f_D |V_{cd}| = (45.9 \pm 1.1)\,\mathrm{MeV}, \quad f_{D_s} |V_{cs}| = (249.1 \pm 3.2)\,\mathrm{MeV}$

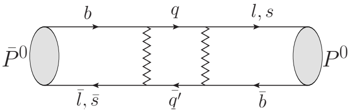
Computing f_{D_s}/f_D gives access to V_{cs}/V_{cd}

Neutral meson mixing

Neutral mesons oscillate with their antiparticles:

$\Rightarrow$ Difference between mass eigenstates: $\Delta m^{\rm exp}$ measured to $< 1\%$!

$$\Delta m \propto \overbrace{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=2} \middle| \bar{B}^0_{(s)} \right\rangle}^{\text{Short distance}} + \overbrace{\sum_n \frac{\left\langle B^0_{(s)} \middle| \mathcal{H}^{\Delta b=1} \middle| n \right\rangle \left\langle n \middle| \mathcal{H}^{\Delta b=1} \middle| \bar{B}^0_{(s)} \right\rangle}{E_n - M_{B_{(s)}}}}^{\text{Long distance}}$$



$$\propto \left| \sum_{q=u,c,t} \frac{m_q^2}{M_W^2} V_{qb} V_{ql}^* \right|^2 \approx \frac{m_t^4}{M_W^4} \left| V_{tb} V_{tl}^* \right|^2$$

SD: Top enhanced: $m_t^2 V_{tb} V_{tl}^* \gg m_c^2 V_{cb} V_{cl}^* \gg m_u^2 V_{ub} V_{ul}^*$

LD: Only m_c, m_u in intermediate states: no top + CKM suppressed

$\Rightarrow$ **Short distance dominated**.

Operator Product Expansion

Two scale problem: $\Lambda_{\mathrm{QCD}} \sim 1\,\mathrm{GeV} \ll m_{EW} \sim 100\,\mathrm{GeV}$:

$\Rightarrow$ Factorise via OPE

$$\Delta m \propto \sum_i C_i(\mu) \left\langle B^0_{(s)} \middle| \mathcal{O}_i^{\Delta b=2}(\mu) \middle| \bar{B}^0_{(s)} \right\rangle$$

- Perturbative model-dependent Wilson coefficients $C_i(\mu)$
- Non-perturbative model-independent matrix elements of $\mathcal{O}_i^{\Delta b=2}(\mu)$
- 5 independent (parity even) operators $\mathcal{O}_i$.

$\Rightarrow$ SM: $\mathcal{O}_1 = \left(\bar{b}_a \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_a\right) \left(\bar{b}_b \gamma_\mu \left(\mathbb{1} - \gamma_5\right) q_b\right) = \mathcal{O}_{VV+AA}$
$+$ 4 (B)SM operators: $\mathcal{O}_2 - \mathcal{O}_5$

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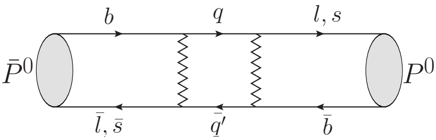
RBC/UKQCD's $K-\bar{K}$ BSM mixing calculation

P. Boyle, N. Garron, J. Hudspith, A. Jüttner, **J. Kettle**, A. Khamseh, C. Lehner, A. Soni, JTT [1812.04981 PoS Lat'18, in preparation]

Flavour Physics and CKM: neutral meson mixing

$$\Delta m_P = \left| V^*_{tq_2} V_{tq_1} \right| \times f_P^2 m_P \hat{B}_P \times \text{known factors}$$



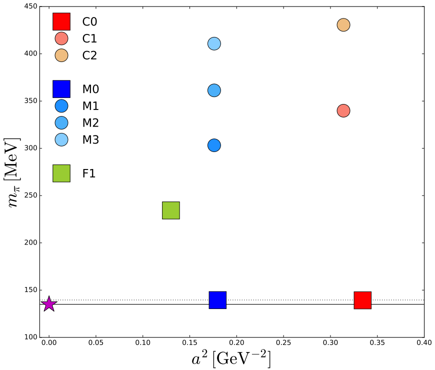
[HFLAV]

$$\Delta m_d = 0.5064 \pm 0.0019\,\mathrm{ps}^{-1}$$
$$\Delta m_s = 17.757 \pm 0.021\,\mathrm{ps}^{-1}$$

Computing ξ gives access to ratio V_{td}/V_{ts}:

$$\xi^2 \equiv \frac{f_{B_s}^2 B_{B_s}}{f_B^2 B_B} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\Delta m_s}{\Delta m_d} \frac{m_B}{m_{B_s}}$$

RBC/UKQCD $N_f = 2+1$ ensembles



- Iwasaki gauge action
- Domain Wall Fermion action
  $\Rightarrow N_f = 2+1$ flavours in the sea
  $\Rightarrow M_5 = 1.8$ for light and strange
- **2 ensembles with physical pion masses** [PRD **93** (2016) 074505]
- 3 Lattice spacings [JHEP **12** (2017) 008]
- Heavier m_π ensembles guide small chiral extrapolation of F1

Chiral Fermions:

$\Rightarrow$ $O(a)$ improved

$\Rightarrow$ Multiplicative renormalisation

Lattice set-up

Light and strange

- Unitary light quark mass
- Physical strange quark mass
- DWF parameters same between sea and valence
- Gaussian source (sink) smearing for better overlap with ground state

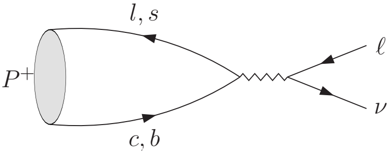
Heavy (charm and beyond)

- Möbius DWF
- $M_5 = 1.0,\ L_s = 12$
- Stout smeared (3 hits, $\rho = 0.1$)
- Range of quark masses from below charm to $\sim m_b/2$ on finest ensemble

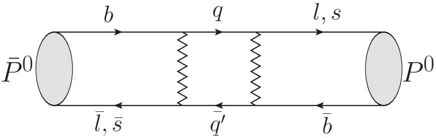
$\Rightarrow$ **All DWF** mixed action set-up
$\Rightarrow$ $\mathbb{Z}_2$-noise sources (volume average) on every 2nd time slice
$\Rightarrow$ Increased heavy quark reach compared to [JHEP **04** (2016) 037,JHEP **12** (2017) 008]
$\rightarrow$ extrapolation towards b

Measurement strategy

Leptonic decays:

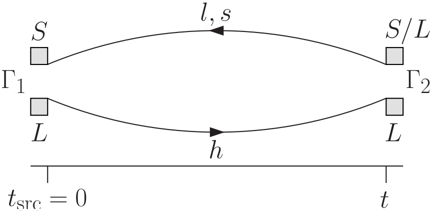


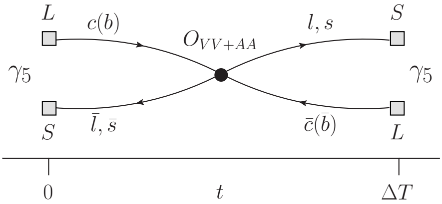
$P^0 - \bar{P}^0$-mixing



$$\mathcal{Z}_A \langle 0 | \, \overline{c}\gamma_4\gamma_5 q \, | D_q(0) \rangle = f_{D_q} m_{D_q}$$

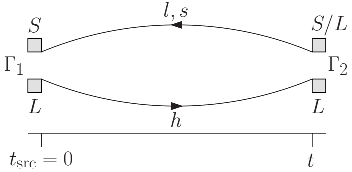
$$B_P = \frac{\langle \bar{P}^0 | O_{VV+AA} | P^0 \rangle}{8/3 f_P^2 m_P^2}$$





Many source-sink separations ΔT for 4-quark operator

Correlator fitting: strategy (2-point functions)



$$C_{ij}(t) = \sum_{n=0}^{\infty} (\psi_n)_i (\psi_n^*)_j e^{-E_n t}$$

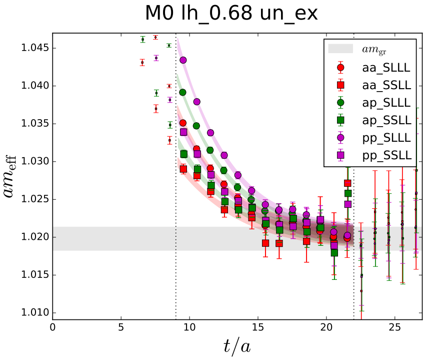
with $E_n < E_{n+1}$ and $(\psi_n)_i = \frac{\langle 0|O_i|n\rangle}{\sqrt{2E_n}}$ for $O = \bar{c}_2^L \Gamma q_1^X$ where $X = S, L$.

Consider $\Gamma = \gamma_5$ (**P**seudo scalar) and $\Gamma = \gamma_4\gamma_5$ (**A**xial vector current).

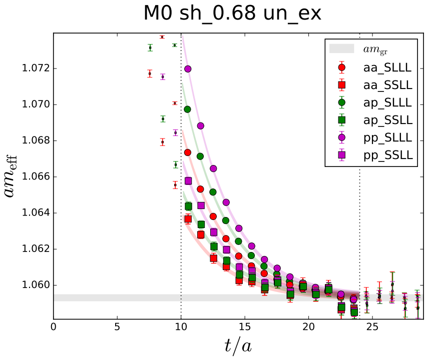
ISSUE: Exponential noise growth i.e. **signal-to-noise problem**

$\Rightarrow$ Simultaneous uncorrelated excited state fits to 6 channels:
$\langle AA\rangle^{SL}$, $\langle AP\rangle^{SL}$, $\langle PP\rangle^{SL}$, $\langle AA\rangle^{SS}$, $\langle AP\rangle^{SS}$ and $\langle PP\rangle^{SS}$

Correlator fitting: fits (2-point functions)



Example fit (heavy-light meson with $am_h = 0.68$ on M0).



Example fit (heavy-strange meson with $am_h = 0.68$ on M0).

Correlator fitting: checks I

$$C_{AP}^{LS}(t) \approx A_0^L P_0^S e^{-E_0 t} + A_1^L P_1^S e^{-E_1 t}$$

$$C_{AP}^{SS}(t) \approx A_0^S P_0^S e^{-E_0 t} + A_1^S P_1^S e^{-E_1 t}$$

Construct Linear Combination

$$\begin{aligned} C_1^{AP}(t) &\equiv C_{AP}^{LS}(t) X^S - C_{AP}^{SS}(t) X^L \\ &\approx P_0^S \left(A_0^L X^S - A_0^S X^L \right) e^{-E_0 t} \\ &+ P_1^S \left(A_1^L X^S - A_1^S X^L \right) e^{-E_1 t} \end{aligned}$$

Correlator fitting: checks I

$$C_1^{AP}(t) \approx P_0^S \left(A_0^L X^S - A_0^S X^L\right) e^{-E_0 t} + P_1^S \underbrace{\left(A_1^L X^S - A_1^S X^L\right)}_{\text{small}} e^{-E_1 t}$$

Identify X^S, X^L with **central value** of A_1^S, A_1^L from fit.

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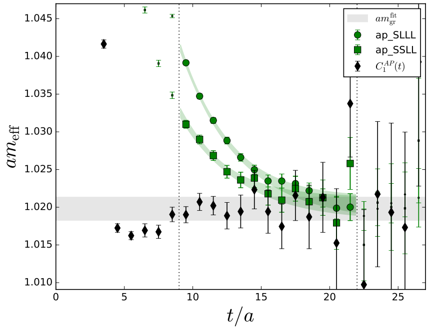
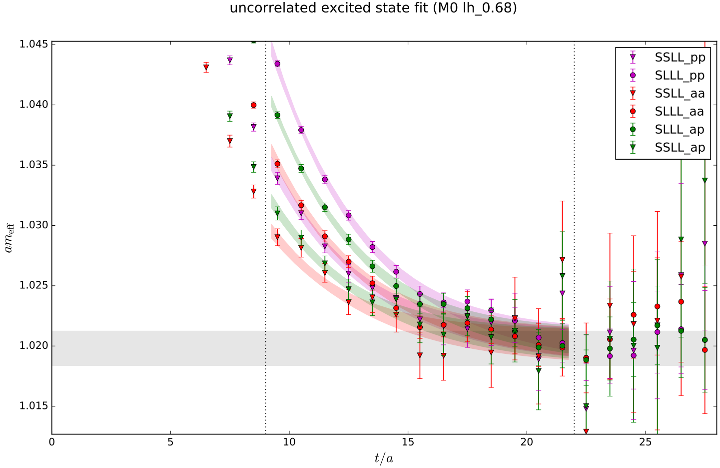
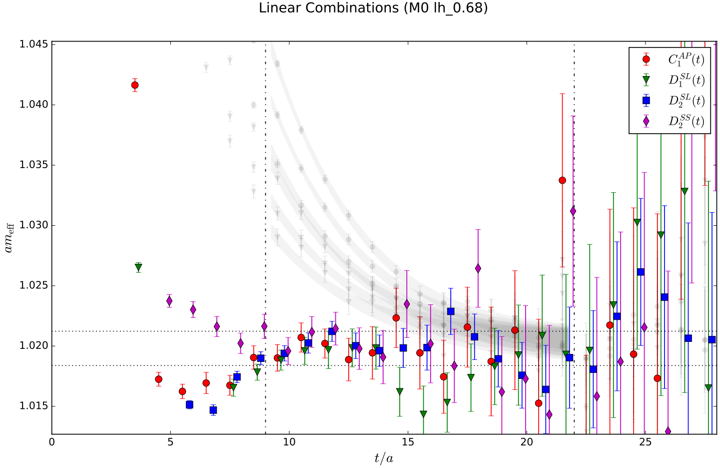


⇒ Removes (most of) excited state
⇒ Strong *a posteriori* check of fit range

Correlator fitting: checks II

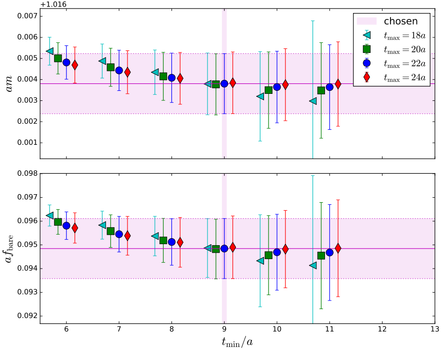


Correlator fitting: checks II

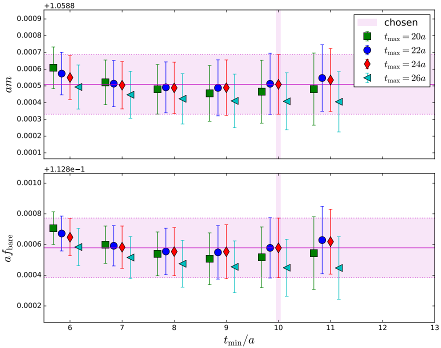


Correlator fitting: stability (2-point functions)

heavy-light on M0 ($am_h = 0.68$)

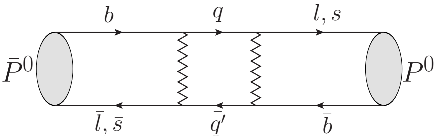


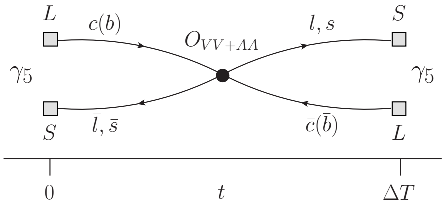
heavy-strange on M0 ($am_h = 0.68$)



Stability under variation of fit ranges

Correlator fitting of 4-quark operators: strategy





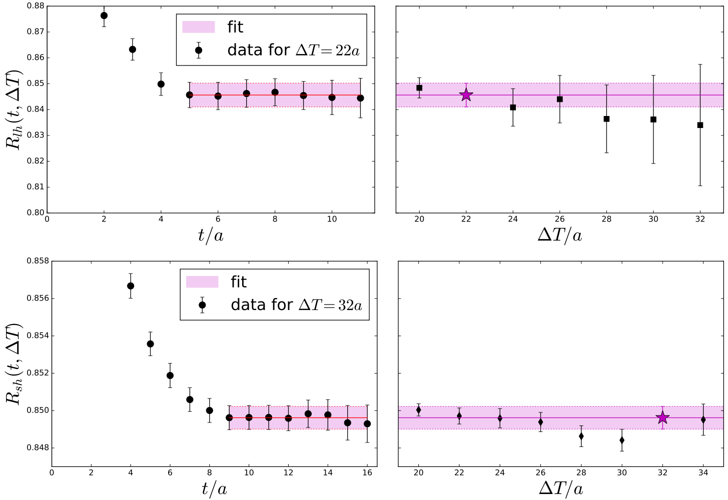
$$C_3(t, \Delta T) \equiv \left\langle P(\Delta T) O_{VV+AA}(t) \bar{P}^\dagger(0) \right\rangle$$

$$R(t, \Delta T) = \frac{C_3(t, \Delta T)}{8/3 C_{PA}(\Delta T - t) C_{AP}(t)} \to B_P \quad \text{for} \quad t, \Delta T \gg 0$$

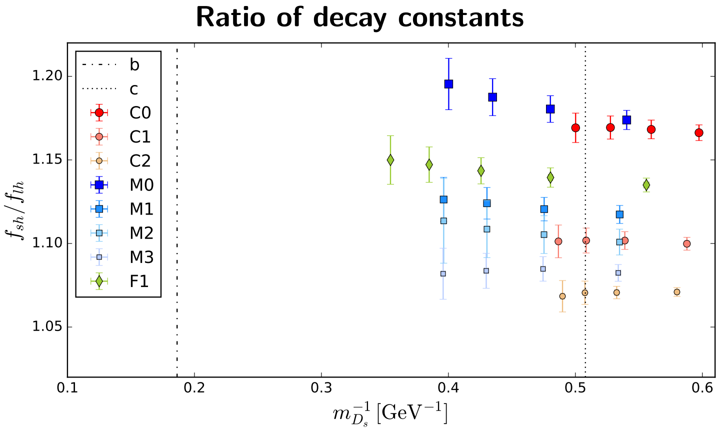
- Expect $R(t, \Delta T)$ to plateau for large t
- Check stability of plateaux value by varying ΔT

Correlator Fitting of 4-quark operators II

Ex: $am_h = 0.68$ on M0



Results of correlator fits



⇒ Renormalisation constants cancel

⇒ Mild linear behaviour with $1/m_H$ and a^2

⇒ Stat precision: 0.4 - 1.0 %

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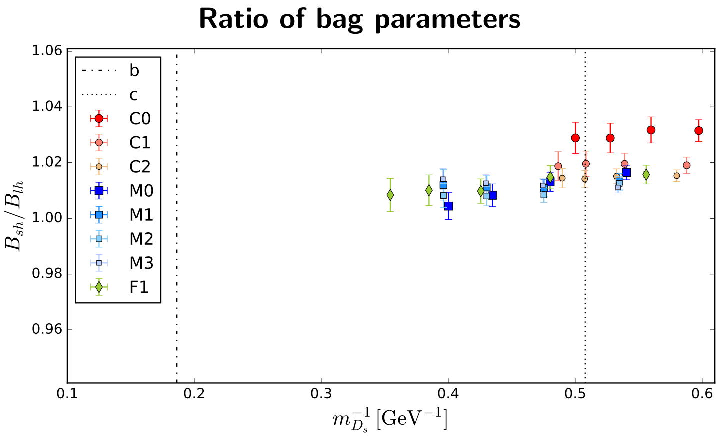

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Global fit: ansatz

Base fit

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\mathrm{phys}}, m_H^{\mathrm{phys}}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$

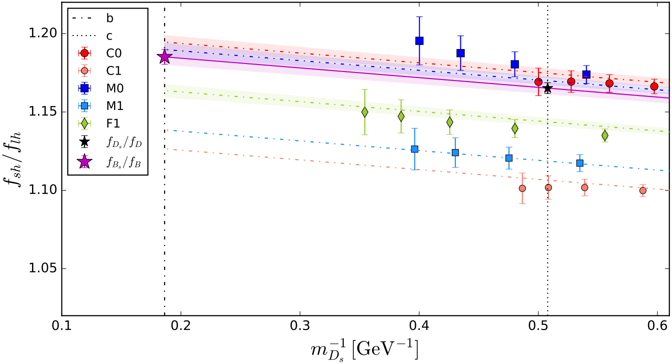
Assess systematic errors by

- varying cuts on pion mass
- using $m_H = m_D$, m_{D_s} and m_{η_c}
- varying inclusion/exclusion of heaviest data points
- varying inclusion/exclusion of fit parameters
- including/estimating higher order terms (a^4, $\left(\Delta m_\pi^2\right)^2$, $\left(\Delta m_H^{-1}\right)^2$)

$\Rightarrow$ Global fits are fully correlated.

Global fit: ratio of decay constants

$$O(a, m_\pi, m_H) = \frac{f_{B_s}}{f_B} + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

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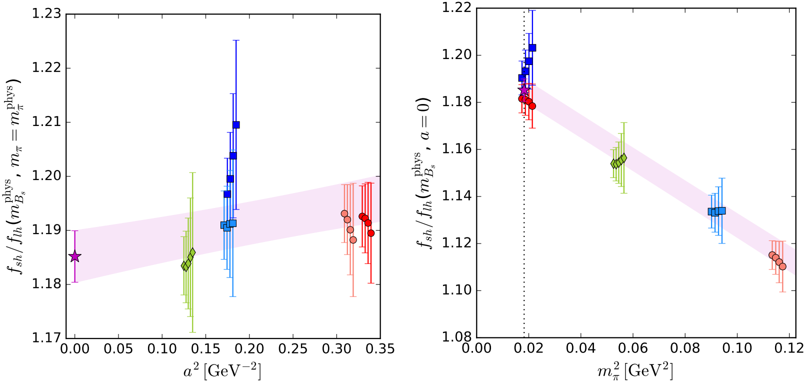

Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

Global fit: ratio of bag parameters

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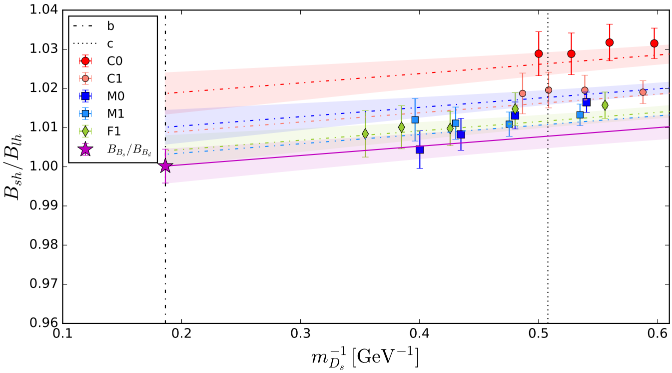

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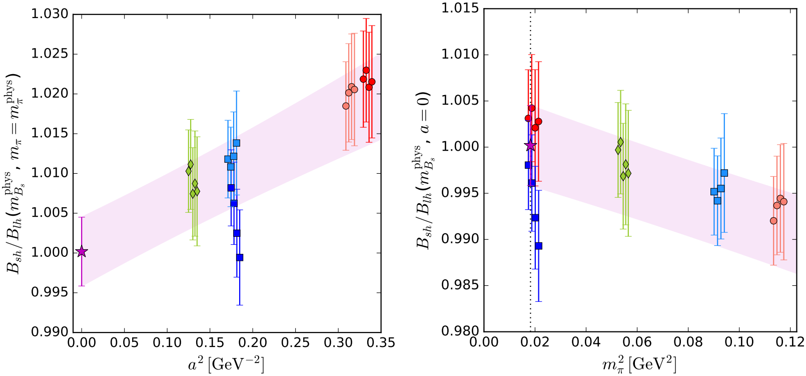


Ratio of bag parameters for $m_\pi \leq 350\,\mathrm{MeV}$

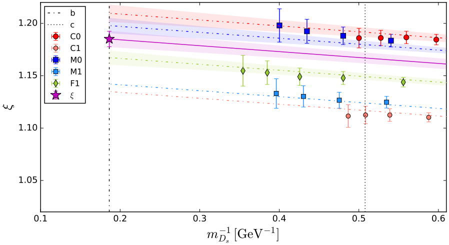
Global fit results - ratio of bag parameters and ξ

Recall:

$$\xi \equiv f_{B_s}/f_B \times \sqrt{B_{B_s}/B_B}$$

1. chiral-CL of product of ratios
2. product of chiral-CL of ratios.

$\xi(a, m_\pi, m_H)$



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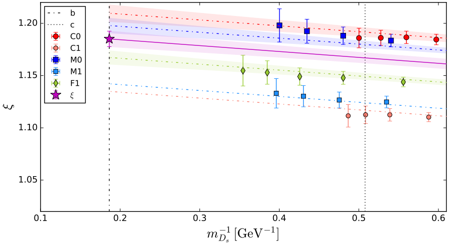


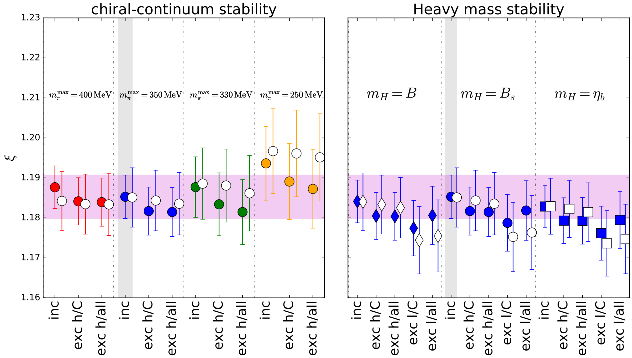
$$\lim_{a\to 0; m_q \to \mathrm{phys}} \left[f_{hs}/f_{hl} \sqrt{B_{hs}/B_{hl}} \right] (a, m_\pi, m_H) = 1.1851(74)_{\mathrm{stat}}$$

$$[f_{B_s}/f_B]_{\mathrm{phys}} \times \sqrt{[B_{B_s}/B_B]_{\mathrm{phys}}} = 1.1853(54)_{\mathrm{stat}}$$

chiral continuum limit of individual ratios gives better signal

Systematic Errors - variations of cuts to data for ξ

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$\xi = 1.1853(54)_{\mathrm{stat}} \binom{+116}{-156}_{\mathrm{sys}}$$

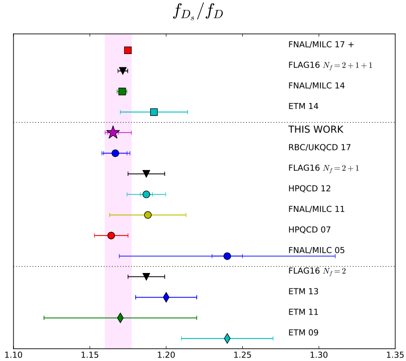
Limitations and "ultimate precision"

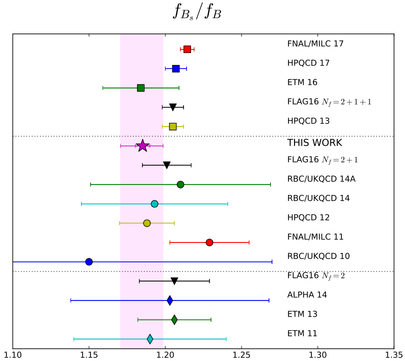
Experimental precision on $\Delta m_s \sim 0.1\%$ and $\Delta m_d \sim 0.4\%$.
Theoretical precision on $\xi \sim 1.3\%$

	f_{D_s}/f_D absolute	f_{D_s}/f_D relative	f_{B_s}/f_B absolute	f_{B_s}/f_B relative	ξ absolute	ξ relative	B_{B_s}/B_{B_d} absolute	B_{B_s}/B_{B_d} relative
central	1.1652		1.1852		1.1853		1.0002	
stat	0.0035	0.30%	0.0048	0.40%	0.0054	0.46%	0.0043	0.43%
fit chiral-CL	$^{+0.0112}_{-0.0031}$	$^{+0.96}_{-0.26}\%$	$^{+0.0110}_{-0.0045}$	$^{+0.93}_{-0.38}\%$	$^{+0.0084}_{-0.0038}$	$^{+0.71}_{-0.32}\%$	$^{+0.0020}_{-0.0044}$	$^{+0.20}_{-0.44}\%$
fit heavy mass	$^{+0.0003}_{-0.0000}$	$^{+0.02}_{-0.00}\%$	$^{+0.0000}_{-0.0081}$	$^{+0.00}_{-0.69}\%$	$^{+0.0000}_{-0.0091}$	$^{+0.00}_{-0.77}\%$	$^{+0.0012}_{-0.0031}$	$^{+0.12}_{-0.31}\%$
H.O. heavy	0.0000	0.00%	0.0054	0.45%	0.0049	0.41%	0.0021	0.21%
H.O. disc.	0.0009	0.07%	0.0009	0.07%	0.0021	0.18%	0.0016	0.16%
$m_u \neq m_d$	0.0009	0.08%	0.0009	0.07%	0.0010	0.08%	0.0001	0.01%
finite size	0.0021	0.18%	0.0021	0.18%	0.0021	0.18%	0.0018	0.18%
total systematic	$^{+0.0114}_{-0.0039}$	$^{+0.98}_{-0.34}\%$	$^{+0.0125}_{-0.0137}$	$^{+1.06}_{-1.16}\%$	$^{+0.0102}_{-0.0146}$	$^{+0.86}_{-1.24}\%$	$^{+0.0041}_{-0.0070}$	$^{+0.41}_{-0.70}\%$
total sys+stat	$^{+0.0120}_{-0.0052}$	$^{+1.03}_{-0.45}\%$	$^{+0.0134}_{-0.0145}$	$^{+1.13}_{-1.22}\%$	$^{+0.0116}_{-0.0156}$	$^{+0.97}_{-1.32}\%$	$^{+0.0060}_{-0.0082}$	$^{+0.60}_{-0.82}\%$

$\Rightarrow$ **Systematically Improvable** with finer lattices at (near) physical m_π.

Comparison to literature - ratio of decay constants

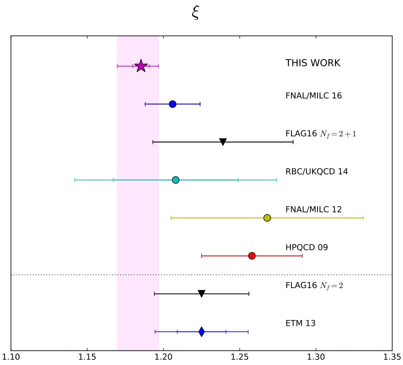


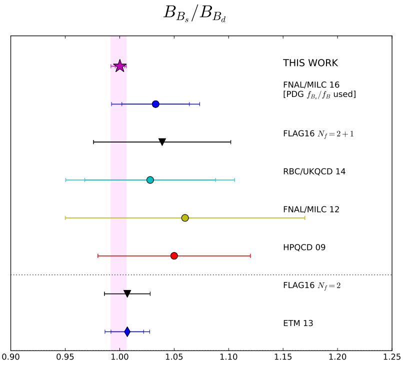


- Self consistent with RBC/UKQCD17: JHEP **12** (2017) 008
- Complimentary to (most) literature - no effective action for b.
- One of few results with physical pion masses.

$$|V_{cd}/V_{cs}| = 0.2148(56)_{\rm exp}\,\left({}^{+22}_{-10}\right)_{\rm lat}$$

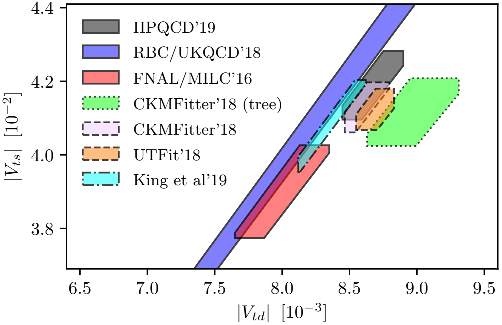
Comparison to literature - ratio of mixing parameters





- Complimentary - no effective action needed for b
- Complimentary - **no operator mixing**!
- **First time with physical pion masses**
- New results: HPQCD'19, King et al. '19

Comparison to literature - V_{td}/V_{ts}



Plot taken from HPQCD'19

$$|V_{td}/V_{ts}| = 0.2018(\ 4)_{\mathrm{e}} \left({}^{+20}_{-27}\right)_{\mathrm{t}}$$

- Slight "discrepancy" between tree-only and loop determinations
- Error still dominated by theory
- Requires more work, but groups are active
- Our next target: V_{td} and V_{ts}

Next steps: Decay constants and bag parameters

1. Different choice of (domain wall) action between light/strange and heavy quarks leads to a mixed action
   Mixed action renormalisation constants cancel for appropriate ratios (f_{B_s}/f_B, B_{B_s}/B_B), but are needed for individual decay constants and bag parameters

$\Rightarrow$ Need to carry out the fully non-perturbative mixed action renormalisation as outlined in JHEP **12** (2017) 008.

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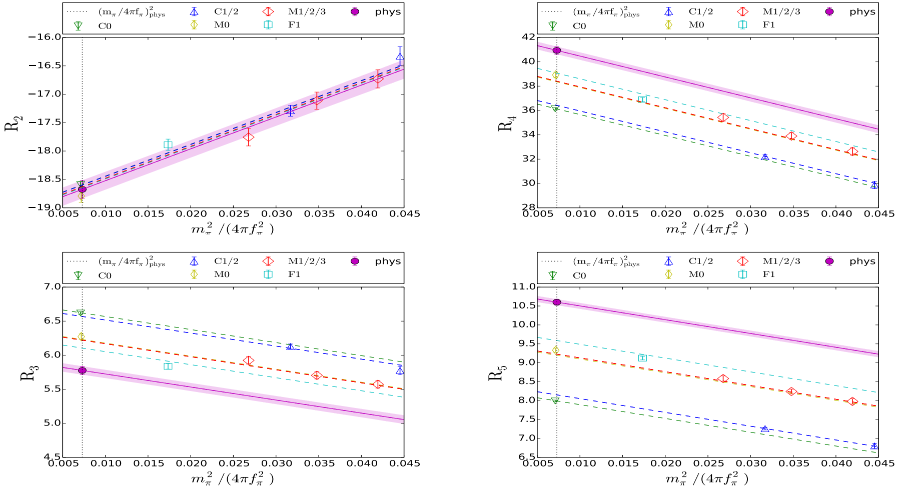
3. Supplement data set with JLQCD ensemble
   (in collaboration with S. Hashimoto and T. Kaneko)

$\Rightarrow$ further reach in m_H due to finer lattice spacing

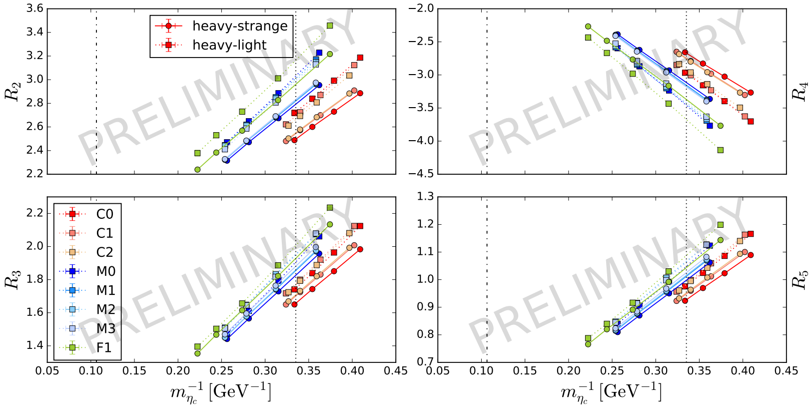
preliminary $K^0 - \bar{K}^0$ results [1812.04981,in preparation]

$$R_i \equiv \langle \bar{P}^0 | \mathcal{O}_i | P^0 \rangle / \langle \bar{P}^0 | \mathcal{O}_1 | P^0 \rangle$$



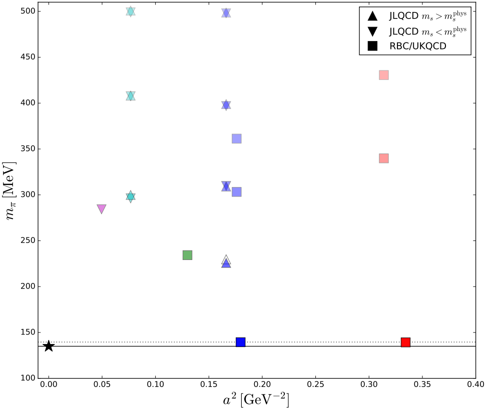
PRELIMINARY RESULTS in $\overline{MS}$ at 3 GeV

$B^0_{(s)} - \bar{B}^0_{(s)}$ (and $D^0 - \bar{D}^0$) **PRELIMINARY and BARE**



- "quite linear" in m_H^{-1}
- similar slopes for h-l and h-s $\Rightarrow SU(3)$ breaking rat's?
- renormalisation to be done (mixed action + op mixing)
- analogous analysis to $K - \bar{K}$ paper + m_H dependence

Increased set of ensembles



JLQCD (triangles)

Fine lattices: $a^{-1} = 2.4 - 4.5\,\mathrm{GeV}$

UKQCD (squares)

+RBC Physical Pion masses

Both: $N_f = 2 + 1$ DWF

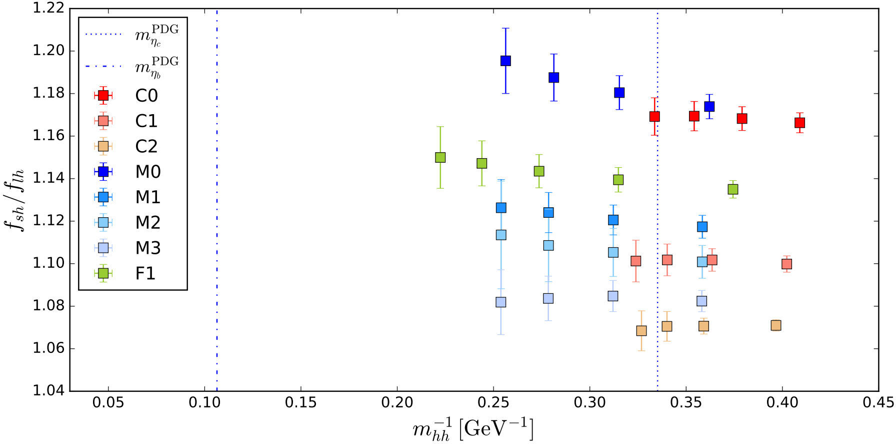
3+3 Lattice Spacings

⇒ Fine lattices: Further heavy quark reach on JLQCD ensembles

⇒ Chiral extrapolation stabilised by m_π^{phys} ensembles

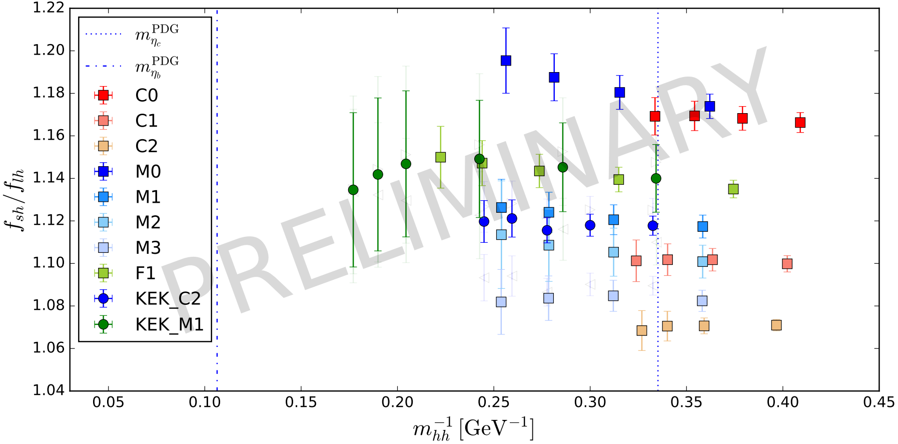
⇒ **Combined physics analysis with S. Hashimoto and T. Kaneko**

JLQCD + RBC/UKQCD data: ratio of decay constants I



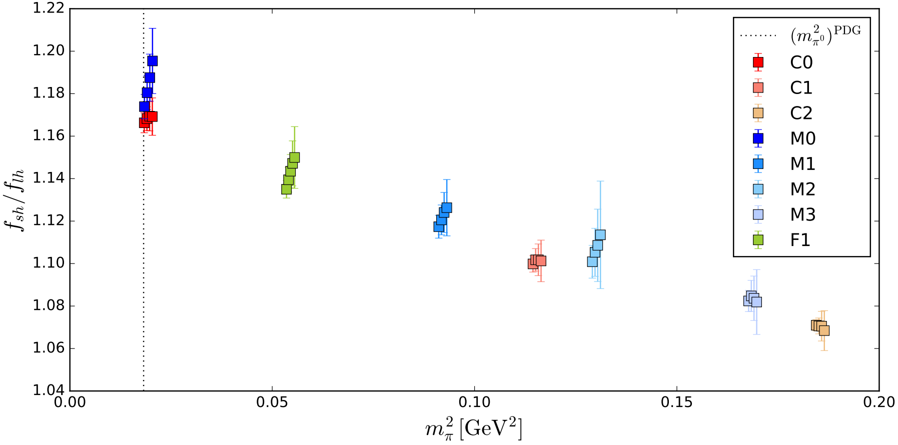
RBC-UKQCD data set from arXiv:1812.08791

JLQCD + RBC/UKQCD data: ratio of decay constants I

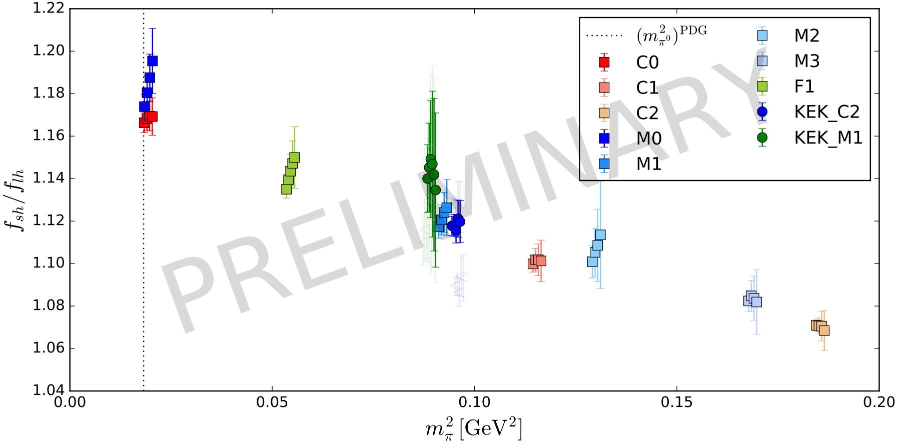


Increased reach in the heavy-mass. The fine KEK ensemble is yet to come!

JLQCD + RBC/UKQCD data: ratio of decay constants II



JLQCD + RBC/UKQCD data: ratio of decay constants II



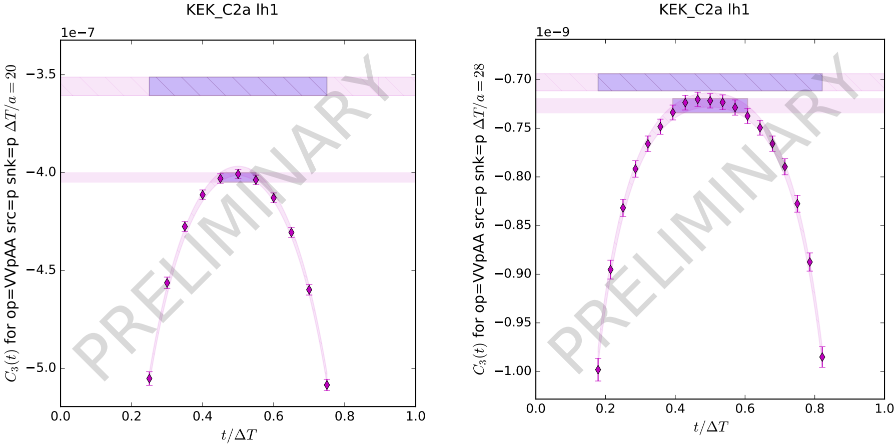
JLQCD + RBC/UKQCD data: bag parameter fit strategy

Write out the excited state contribution to the three point functions

$$
\begin{aligned}
C_3^{\mathcal{O}}(t;\Delta T) \approx &+ \frac{M^0_{\mathrm{snk}} M^0_{\mathrm{src}}}{4E_0E_0} \langle gr| \mathcal{O} |gr\rangle \, e^{-E_0\Delta T} \\
&+ \frac{M^0_{\mathrm{snk}} M^1_{\mathrm{src}}}{4E_0E_1} \langle gr| \mathcal{O} |ex\rangle \, e^{-(E_0+E_1)\Delta T/2} e^{-(E_1-E_0)(t-\Delta T/2)} \\
&+ \frac{M^1_{\mathrm{snk}} M^0_{\mathrm{src}}}{4E_0E_1} \langle ex| \mathcal{O} |gr\rangle \, e^{-(E_0+E_1)\Delta T/2} e^{-(E_0-E_1)(t-\Delta T/2)}
\end{aligned}
$$

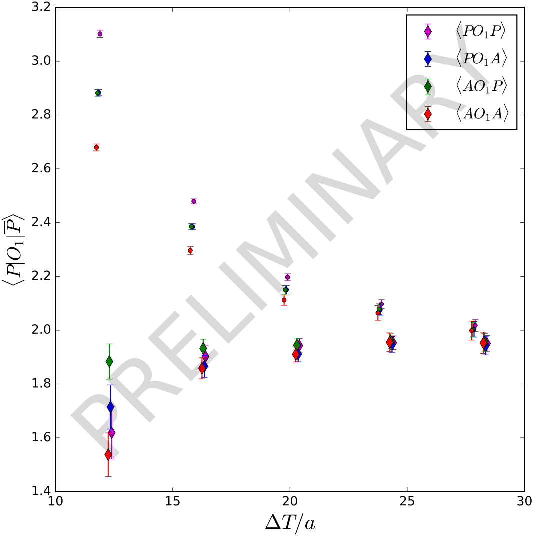
- Use the excited state information from the two-point fits.
- Compare ground state only vs. ground + excited as a function of ΔT

JLQCD + RBC/UKQCD data: bag parameter fitting



- Ground state fit: to a constant (range indicated in plot)
- Excited state fit: to functional form above for all data points shown.

JLQCD + RBC/UKQCD data: bag parameter fitting



- Combined fit to 2-point and 3-point functions possible
- Small symbols: ground state only
- Large symbols: excited state fit
- Each data point is a separate fit

- Stable for $\Delta T/a \gtrsim 20$
- Stable for different channels

Conclusions and Outlook

$SU(3)$ breaking ratios

- **arXiv:1812.08791**
- f_{D_s}/f_D, f_{B_s}/f_B, B_{B_s}/B_B and ξ
- $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$
- 3 lattice spacings, 2 m_π^{phys}
- First result for ξ and B_{B_s}/B_B with m_π^{phys}
- m_h from below m_c to $\sim m_b/2$
  $\Rightarrow$ extrapolation to b for ratios
  $\Rightarrow$ fully relativistic
- Good continuum scaling and self-consistent
- Competitive precision

Ongoing

- Supplementing dataset with very fine JLQCD ensembles
- Combined fit with universality constraint
- Mixed action renormalisation underway
- First results look promising

$\Rightarrow$ Determine $f_{B_{(s)}}$, $f_{D_{(s)}}$

$\Rightarrow$ Full mixing operator basis for $B_{(s)}$ and D (short distance).

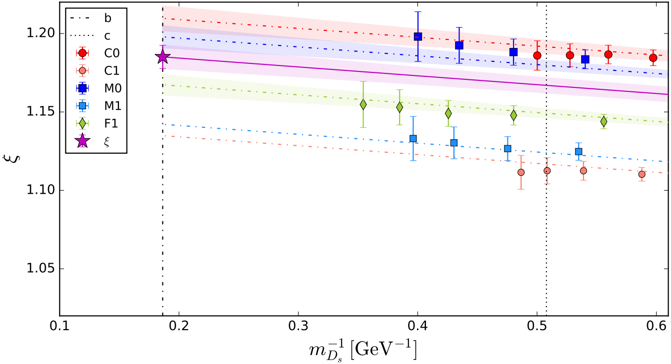
Outlook

- $a^{-1} = 2.8\,\mathrm{GeV}$, $m_\pi = m_\pi^{\mathrm{phys}}$

ADDITIONAL SLIDES

Global fit results for ξ

$$O(a, m_\pi, m_H) = O(0, m_\pi^{\mathrm{phys}}, m_H^{\mathrm{phys}}) + C_{CL}a^2 + C_\chi \Delta m_\pi^2 + C_H \Delta m_H^{-1}$$



Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

Global fit results for ξ

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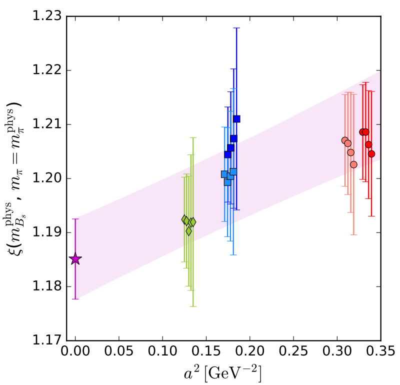
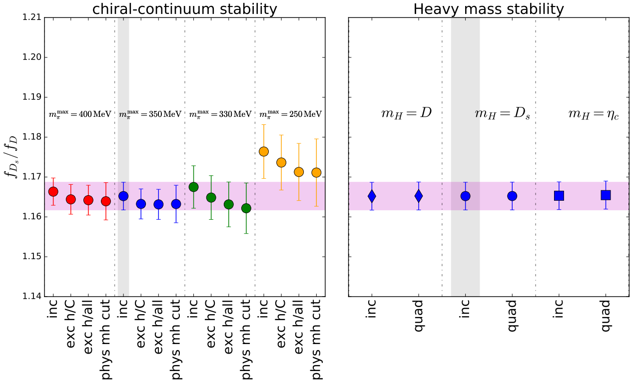


Ratio of decay constants for $m_\pi \leq 350\,\mathrm{MeV}$

Systematic Errors - variations of cuts to data for f_{D_s}/f_D

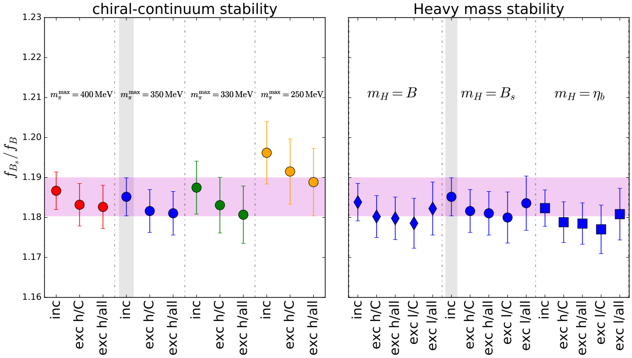
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{D_s}/f_D = 1.1652(35)_{\text{stat}} \left(\begin{matrix}+120 \\ -\ 52\end{matrix}\right)_{\text{sys}}$$

Systematic Errors - variations of cuts to data for f_{B_s}/f_B

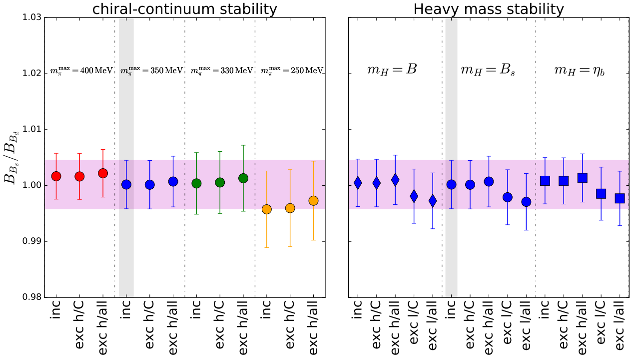
- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$f_{B_s}/f_B = 1.1852(48)_{\mathrm{stat}} \left({}^{+134}_{-145}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for B_{B_s}/B_B

- Global fits all correlated with satisfying p-values.
- sys error: includes chiral-CL (left), heavy mass (right), H.O. terms, $m_u \neq m_d$ and FV.



$$B_{B_s}/B_B = 1.0002(43)_{\mathrm{stat}} \left({}^{+60}_{-82}\right)_{\mathrm{sys}}$$

Systematic Errors - variations of cuts to data for ξ

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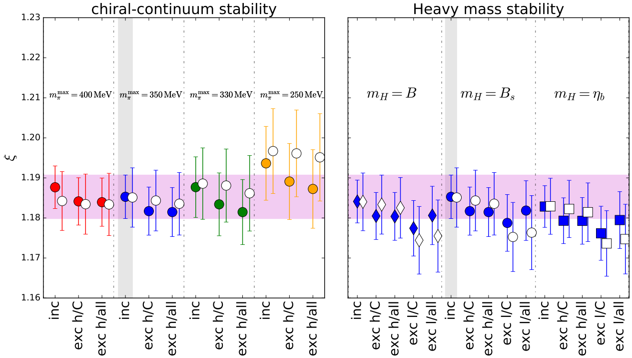


$$\xi = 1.1853(54)_{\rm stat} \left(^{+116}_{-156}\right)_{\rm sys}$$

Non-Perturbative Renormalisation of mixed action

SMOM ren. conds. relates amputated vertex functions to Z factors.

$$\begin{aligned} 1 &= \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_A^{\mathrm{ren}}\right) \gamma_5 \not{q}\right] |_{\mathrm{sym}} \\ &= \frac{Z_A}{Z_q} \lim_{\bar{m}\to 0} \frac{1}{12q^2} \mathrm{Tr}\left[\left(q \cdot \Lambda_A^{\mathrm{bare}}\right) \gamma_5 \not{q}\right] |_{\mathrm{sym}} \\ &\equiv \frac{Z_A}{Z_q} \mathcal{P}[\Lambda_A^{\mathrm{bare}}] \end{aligned}$$

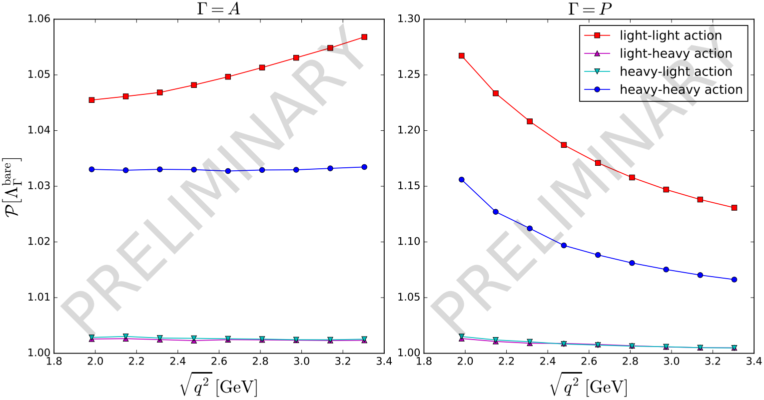
So for actions i,j

$$\frac{\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ii} \mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{jj}}{\left(\mathcal{P}[\Lambda_A^{\mathrm{bare}}]^{ij}\right)^2} = \frac{(Z_A^{ij})^2}{Z_A^{ii} Z_A^{jj}}$$

But for non-mixed actions we can determine Z_A^{ii} from conserved current.

Preliminary mixed action renormalisation

First study on single configuration

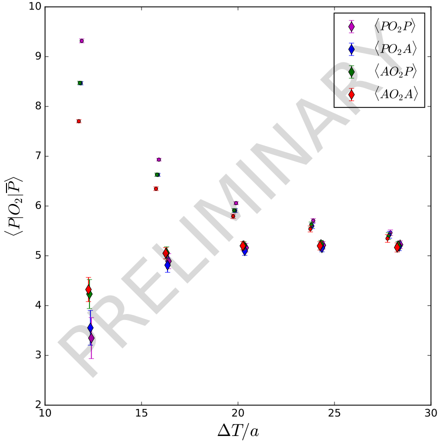


⇒ mixed NPR is feasible

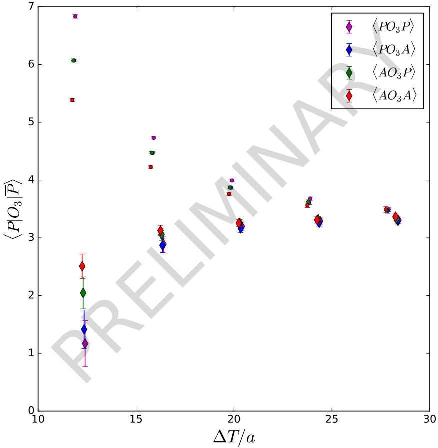
⇒ need to compute Z_A^{hh} from conserved current to obtain Z_A^{hl}

JLQCD + RBC/UKQCD data: additional operators

$VV - AA$



$SS + PP$



JLQCD + RBC/UKQCD data: additional operators