Semi-leptonic B_s decays

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(RBC-UKQCD collaborations)

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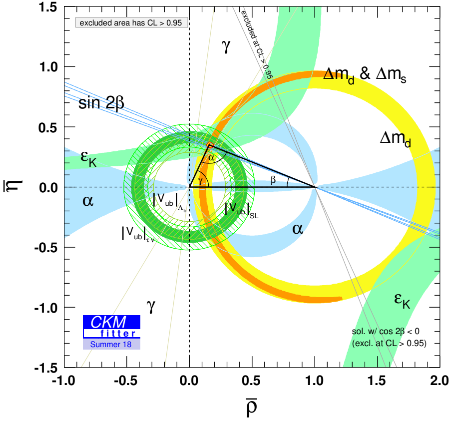
introduction

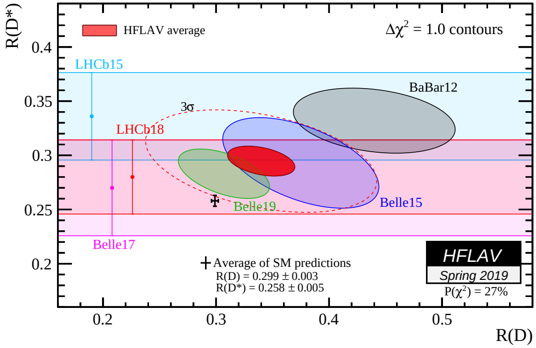
Motivation

- Determine CKM matrix elements, fundamental parameters of the Standard Model
- Predict processes to test Standard Model or discover new physics

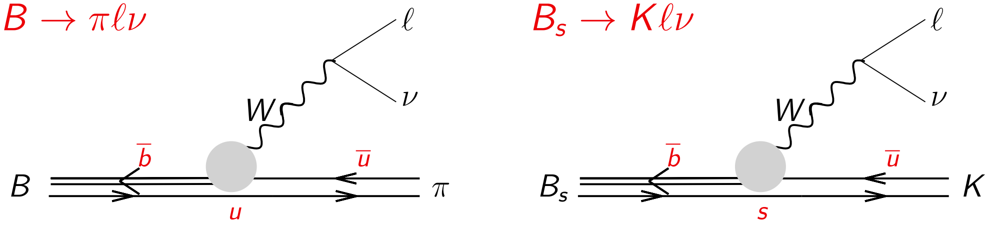
[http://ckmfitter.in2p3.fr]



[HFLAV]



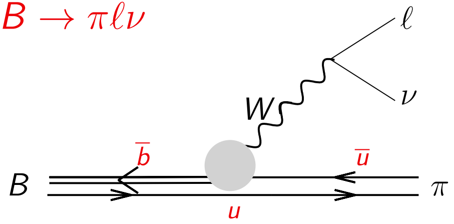
$|V_{ub}|$ from exclusive



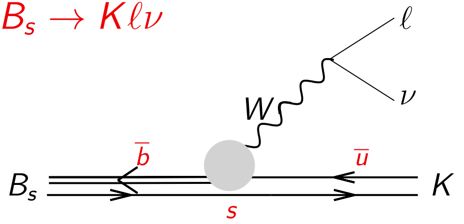
- $B \to \pi\ell\nu$ and $B \to D\ell\nu$ presented by Ryan C. Hill
- Only spectator quark differs
- Lattice QCD prefers s quark over u quark: statistically more precise, computationally cheaper
- B factories run at $\Upsilon(4s)$ threshold $\Rightarrow$ B mesons
- LHC collisions create many B and B_s mesons which decay $\Rightarrow$ LHCb
  $\rightarrow$ LHCb prefers the ratio $(B_s \to D_s\ell\nu)/(B_s \to K\ell\nu) \Rightarrow |V_{cb}/V_{ub}|$

$|V_{ub}|$ from exclusive

$B \to \pi\ell\nu$

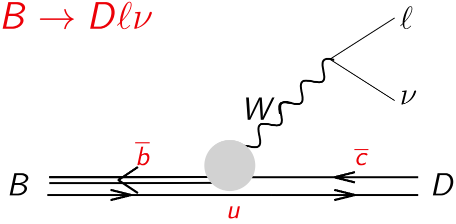


$B_s \to K\ell\nu$

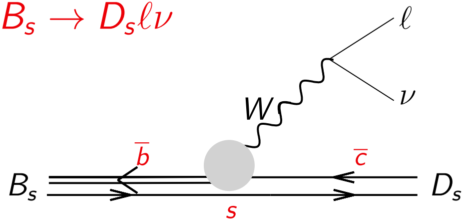


$|V_{cb}|$ from exclusive

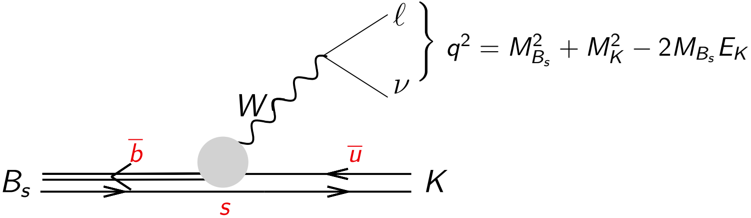
$B \to D\ell\nu$



$B_s \to D_s\ell\nu$



$|V_{ub}|$ from exclusive semi-leptonic $B_s \to K\ell\nu$ decay



- Conventionally parametrized by (B_s meson at rest)

$$\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}\frac{(q^2 - m_\ell^2)^2\sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2(E_K^2 - M_K^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_{B_s}^2 - M_K^2)^2|f_0(q^2)|^2\right]$$

nonperturbative input

Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

RBC-UKQCD's set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions

    [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][JHEP 1712 (2017) 008]

RBC-UKQCD's set-up

	L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources	
C1	24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
C2	24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
C0	*48*	*1.730*	*0.00078*	*0.0362*	*139*	*40*	*81/1**	[PRD 93 (2016) 074505]
M0	64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
F1	48	2.774	0.002144	0.02144	234	98	24	[JHEP 1712 (2017) 008]

* All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. PRD 88 (2012) 094503]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm

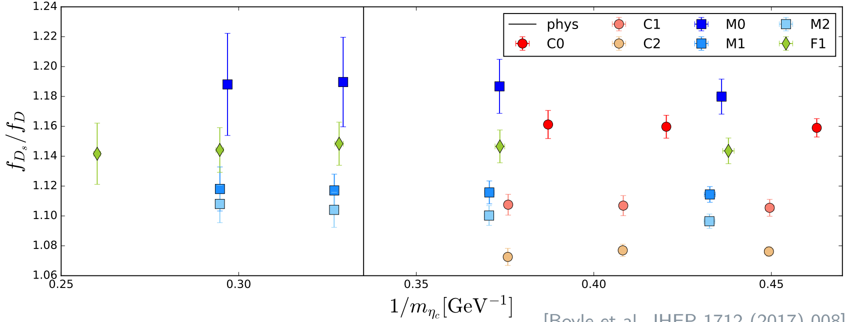
RBC-UKQCD's set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
  [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][JHEP 1712 (2017) 008]
- Unitary and partially quenched domain-wall up/down quarks
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Domain-wall strange quarks at/near the physical value
- Additional challenge $m_c = 1.28\text{GeV} \sim 270 \times m_d$
  $m_b = 4.18\text{GeV} \sim 1000 \times m_d$

RBC-UKQCD's set-up

- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]

  $\rightarrow$ Simulate 3 or 2 charm-like masses then extrapolate/interpolate



[Boyle et al. JHEP 1712 (2017) 008]

RBC-UKQCD's set-up

- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  → Simulate 3 or 2 charm-like masses then extrapolate/interpolate

- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  → Allows to tune the three parameters (m_0a, c_P, ζ) nonperturbatively [PRD 86 (2012) 116003]
  → Smooth continuum limit; heavy quark treated to all orders in $(m_ba)^n$
  → Mostly nonperturbative renormalization [Hashimoto et al. PRD61 (1999) 014502]
  [El-Khadra et al. PRD64 (2001) 014502]

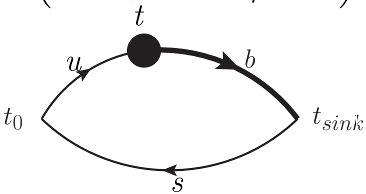
$$Z_V^{bl} = \varrho\sqrt{Z_V^{ll} Z_V^{bb}}$$

$$B_s \to K\ell\nu$$

$B_s \to K\ell\nu$ form factors

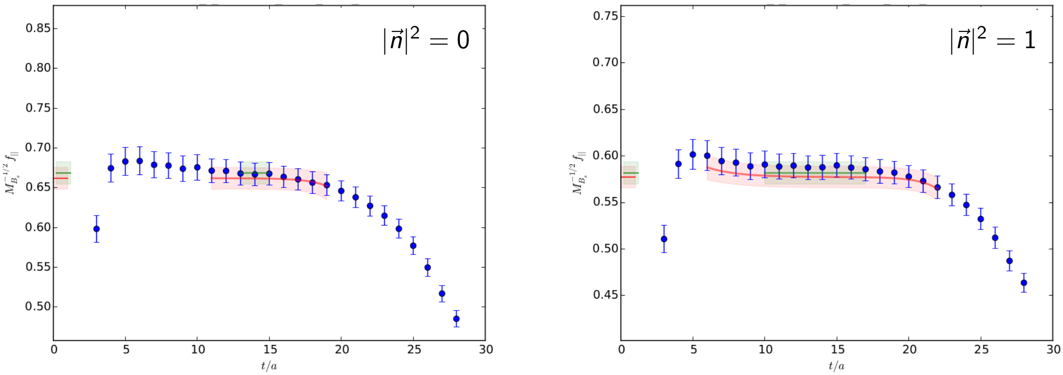
- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$



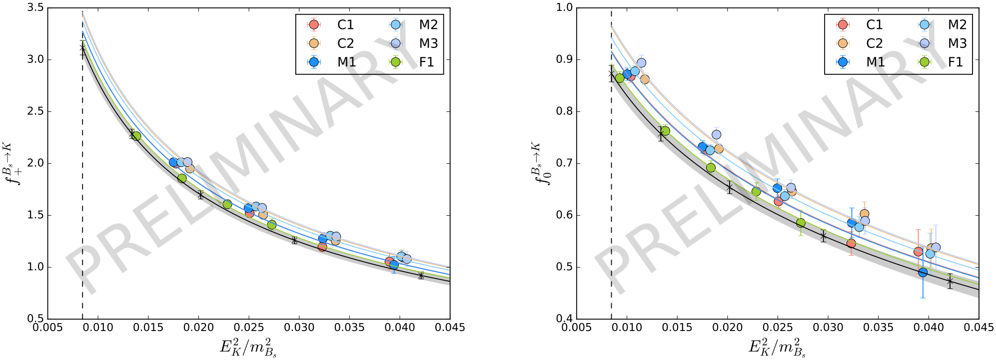
- Calculate 3-point function by
  - → Inserting a quark source for a strange quark propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use a light quark propagating from t_0 and contract both at t with $t_0 \le t \le t_{sink}$

$B_s \to K\ell\nu$ form factors: F1 ensemble



- Comparison of fit to the ground state only with fit including one excited state term for K and B_s

Chiral-continuum extrapolation using SU(2) hard-kaon χPT



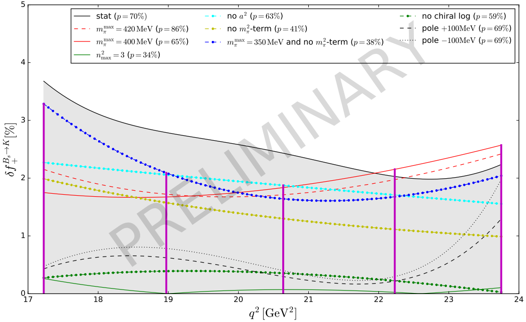
- Updating calculation [PRD 91 (2015) 074510] with improved values for a^{-1} and RHQ parameters
- $f_{pole}(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c^{(1)} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c^{(2)}\frac{M_\pi^2}{\Lambda^2} + c^{(3)}\frac{E_K}{\Lambda} + c^{(4)}\frac{E_K^2}{\Lambda^2} + c^{(5)}\frac{a^2}{\Lambda^2 a_{32}^4}\right]$
- δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$

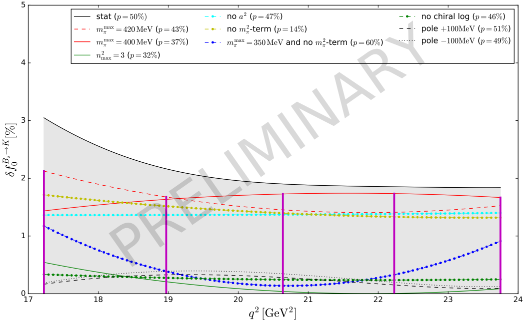
Estimate systematic errors due to

- Chiral-continuum extrapolation
  - → Use alternative fit functions, vary pole mass, etc.
  - → Impose different cuts on the data
- Discretization errors of light and heavy quarks
  - → Estimate via power-counting
- Uncertainty of the renormalization factors
  - → Estimate effect of higher loop corrections
- Finite volume, iso-spin breaking, ...
- Uncertainty due to RHQ parameters and lattice spacing (a^{-1})
  - → Carry out additional simulations to test effects on form factors
- Uncertainty of strange quark mass
  - → Repeat simulation with different valence quark mass

$\Longrightarrow$ full error budget

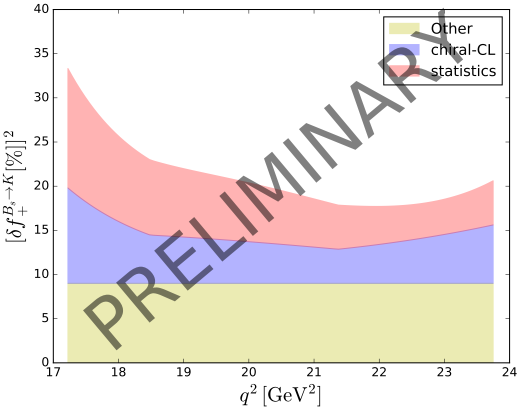
PRELIMINARY error budget $B_s \to K\ell\nu$

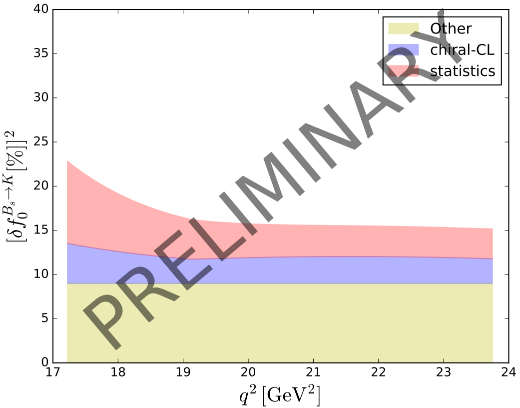




- $\delta f = \left| f^{\text{variation}} - f^{\text{central}} \right| / f^{\text{central}}$

PRELIMINARY error budget $B_s \to K\ell\nu$





- "Other": 3% placeholder to cover higher order corrections, lattice spacing, finite volume, ...

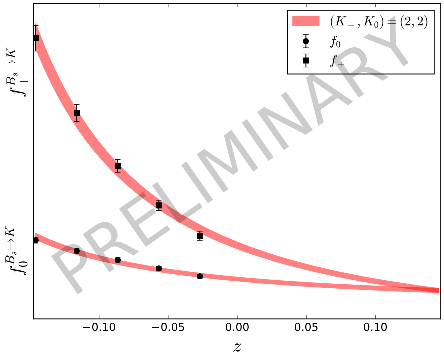
Kinematical extrapolation (z-expansion)

- Map q^2 to z with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with} \quad \begin{array}{l} t_\pm = (M_B \pm M_\pi)^2 \\ t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2 \end{array}$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]



- Express f_+ as convergent power series
- f_0 is analytic, except for B^* pole
- BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV
- Exploit kinematic constraint $f_+ = f_0\big|_{q^2=0}$

→ Include HQ power counting to constrain size of f_+ coefficients

- Systematic errors subject to changes!

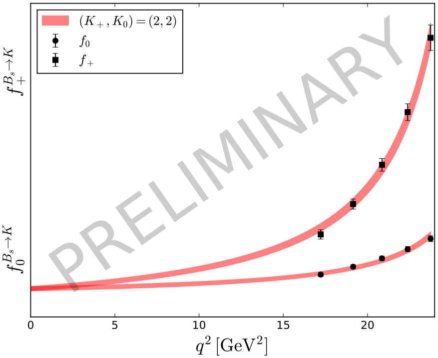
Kinematical extrapolation (z-expansion)

- Map q^2 to z with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+}-\sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+}+\sqrt{1-t_0/t_+}} \quad \text{with} \quad \begin{aligned} t_\pm &= (M_B \pm M_\pi)^2 \\ t_0 \equiv t_{\text{opt}} &= (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2 \end{aligned}$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

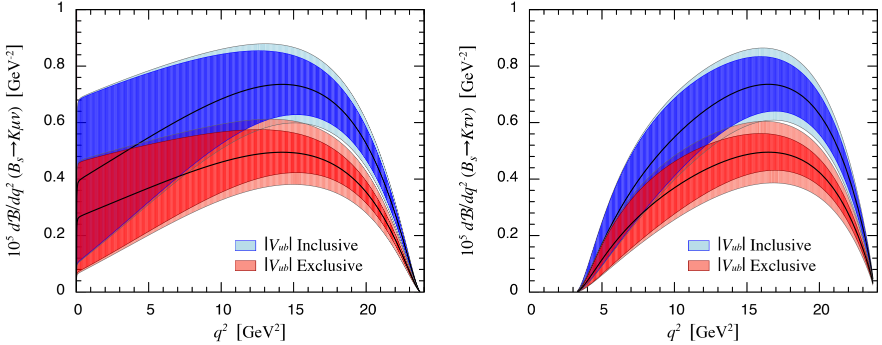
[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]



- Allows to compare shape of form factors
  - → Obtained by other lattice calculations
    [Bouchard et al. PRD 90 (2014) 054506]
    [Bazavov et al. arXiv:1901.02561]
  - → Predicted by QCD sum rules and alike
- Combination with experiment leads to the overall normalization: $|V_{ub}|$
- Systematic errors subject to changes!

Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

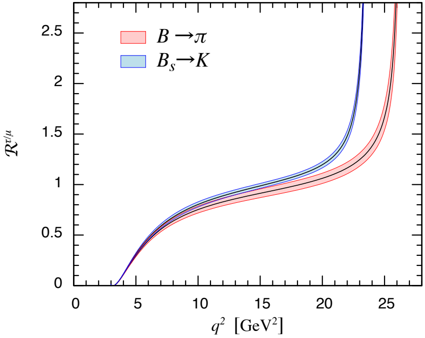
- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton $= \mu$ or τ



Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton $= \mu$ or τ

- Predict ratio of branching fractions $\rightsquigarrow$ LFUV

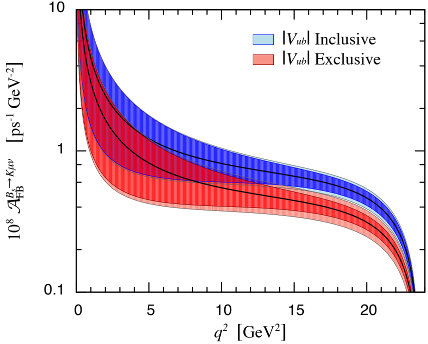


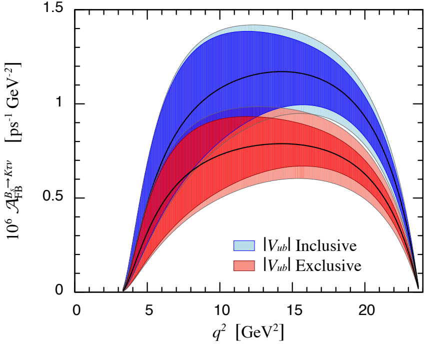
$$R_\pi^{\tau/\mu} = 0.69(19)$$

$$R_K^{\tau/\mu} = 0.77(12)$$

Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

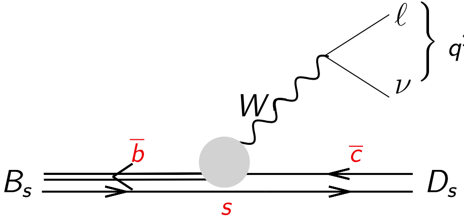
- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton $= \mu$ or τ
- Predict ratio of branching fractions $\rightsquigarrow$ LFUV
- Predict forward-backward asymmetries using $|V_{ub}|$ as input for lepton $= \mu$ or τ





$$B_s \to D_s \ell \nu$$

$|V_{cb}|$ from exclusive semi-leptonic $B_s \to D_s\ell\nu$ decay



$$q^2 = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}E_{D_s}$$

- Conventionally parametrized by (B_s meson at rest)

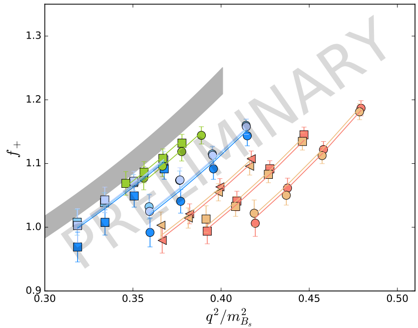
$$\frac{d\Gamma(B_s \to D_s\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2}{24\pi^3}\frac{(q^2 - m_\ell^2)^2\sqrt{E_{D_s}^2 - M_{D_s}^2}}{q^4 M_{B_s}^2}$$

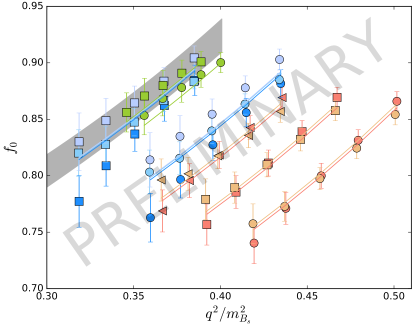
experiment — CKM — known

$$\times\left[\left(1 + \frac{m_\ell^2}{2q^2}\right)M_{B_s}^2(E_{D_s}^2 - M_{D_s}^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_{B_s}^2 - M_{D_s}^2)^2|f_0(q^2)|^2\right]$$

nonperturbative input

Global fit $B_s \to D_s\ell\nu$

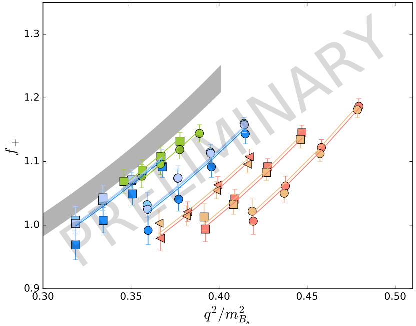


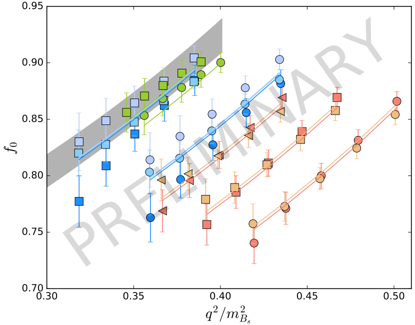


- $$f(q^2, a, M_\pi, M_{D_s}) = \left[\alpha_1 + \alpha_2 M_\pi^2 + \sum_{j=1}^{n_{D_s}} \alpha_{3,j} \left[\Delta M_{D_s}^{-1}\right]^j + \alpha_4 a^2\right] P_{a,b}\left(\frac{q^2}{M_{B_s}^2}\right)$$

with $$\Delta M_{D_s}^{-1} \equiv \left(\frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\rm phys}}\right), \quad P_{a.b}(x) = \frac{1 + \sum_{i=1}^{N_a} a_i x^i}{1 + \sum_{i=1}^{N_b} b_i x^i}$$

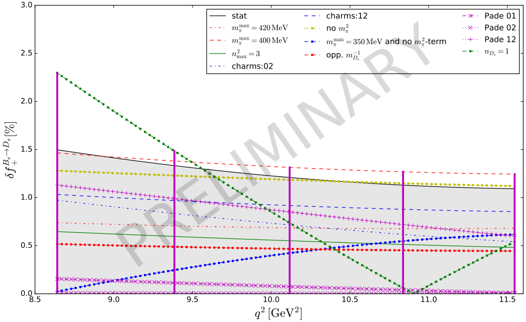
Global fit $B_s \to D_s\ell\nu$

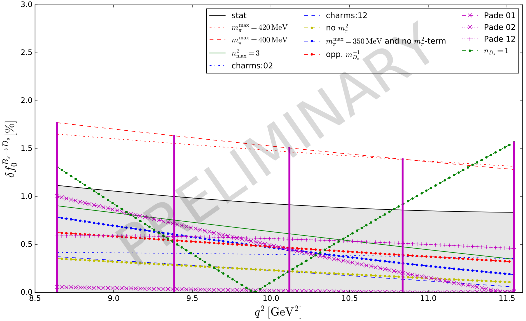




- $f(q^2, a, M_\pi, M_{D_s}) = \left[\alpha_1 + \alpha_2 M_\pi^2 + \sum_{j=1}^{n_{D_s}} \alpha_{3,j} \left[\Delta M_{D_s}^{-1}\right]^j + \alpha_4 a^2\right] P_{a,b}\left(\frac{q^2}{M_{B_s}^2}\right)$
- Extrapolation to the continuum limit with physical quark masses

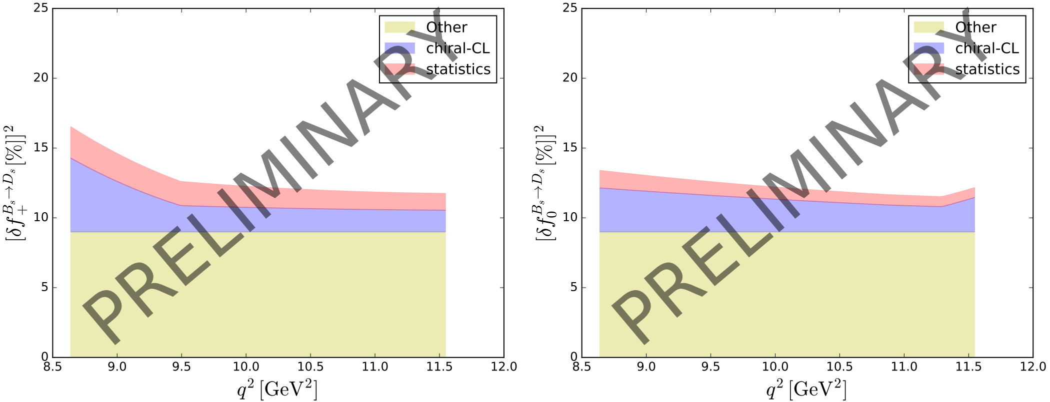
PRELIMINARY error budget $B_s \to D_s\ell\nu$





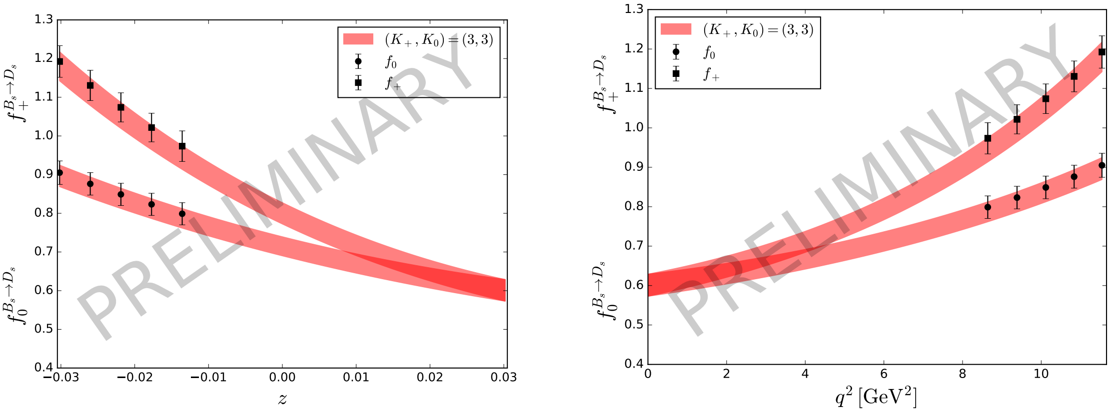
- $\delta f = \left| f^{\text{variation}} - f^{\text{central}} \right| / f^{\text{central}}$

PRELIMINARY error budget $B_s \to D_s\ell\nu$



- "Other": 3% placeholder to cover higher order corrections, lattice spacing, finite volume, ...

z-expansion



- BCL with poles $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV
  kinematical constraint $f_0^{B_s\to D_s}(0) = f_+^{B_s\to D_s}(0)$

Status $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$

- $B_s \to K\ell\nu$ chiral-continuum extrapolation
- $B_s \to D_s\ell\nu$ global fit (M_π, M_{D_s}, a^2, q^2)
- Extract synthetic data points

- Full systematic error budget
  - → RHQ parameter tuning
  - → Continuum extrapolation: cut to data set, different fit functions, ...
  - → Charm extrapolation
  - → FV, higher order disc. effects, isospin, s-quark mass tuning, ...

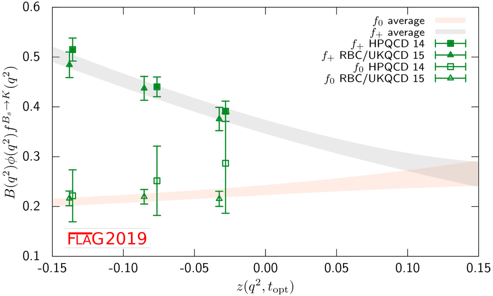
- z-expansion over full q^2 range
  - → BGL vs. BCL
  - → Test CLN for $B_s \to D_s\ell\nu$
  - → Number of synthetic data points
  - → Different truncations
  - → Incl. vs. excluding $f_+ = f_0\big|_{q^2=0}$

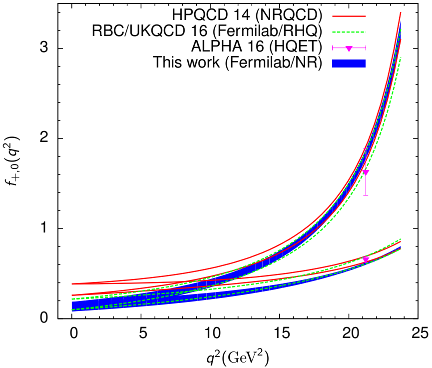
- Phenomenology: $R(K)$, $R(D_s)$, ...

Flavor Lattice Averaging Group

[FLAG 2019]

$B_s \to K\ell\nu$





- New FNAL/MILC [arXiv:1901.02561]

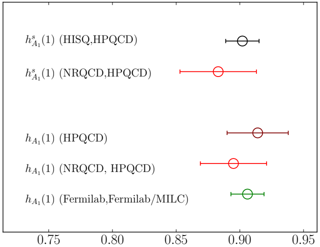
- Please do cite calculations feeding into FLAG averages

[FLAG 2019]

$B \to D^{(*)}_{(s)}\ell\nu$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$w=1$ form factor / ratio	
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B\to D}(1)$	1.035(40)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$\mathcal{G}^{B\to D}(1)$	1.054(4)(8)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B\to D}(1)$	1.033(95)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B_s\to D_s}(1)$	1.068(40)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B_s\to D_s}(1)$	1.052(46)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.895(10)(24)
FNAL/MILC 14	[612]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B\to D^*}(1)$	0.906(4)(12)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B_s\to D_s^*}(1)$	0.883(12)(28)
HPQCD 15, HPQCD 17	[614, 616]	2+1	A	○	○	○	○	✓	$R(D)$	0.300(8)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$R(D)$	0.299(11)

- New HPQCD $B_s \to D_s\ell\nu$ [arXiv:1906.00701]
- New HPQCD $B_s \to D_s^*\ell\nu$ [arXiv:1904.02046]



- Please do cite calculations feeding into FLAG averages

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outlook

Outlook

- Second (third) entirely independent analysis completed
- In the final stages to complete $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ form factor calculation
  - → As usual, carefully estimating all systematic uncertainties is tedious
- Our lattice calculation also includes
  - → $B \to \pi\ell\nu$, $B \to \pi\ell^+\ell^-$
  - → $B \to K^*\ell^+\ell^-$
  - → $B \to D^{(*)}\ell\nu$
  - → $B_s \to K^*\ell^+\ell^-$
  - → $B_s \to D_s^*\ell\nu$
  - → $B_s \to \phi\ell^+\ell^-$
  - → ...
- Current status $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$: [arXiv:1903.02100]
- Future
  - → Add $48^3 \times 96$ ensemble with physical pions
- Parallel efforts: SU(3) breaking ratios [arXiv:1812.08791]
  - → Talk by J. Tobias Tsang

Resources for RBC-UKQCD's calculation

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab), skylake cluster (BNL)
RBC qcdcl (RIKEN) and cuth (Columbia U)
UK: ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)

[USQCD]

[DiRAC]