Beautiful and charming physics on the lattice

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Lyon, September 7, 2016

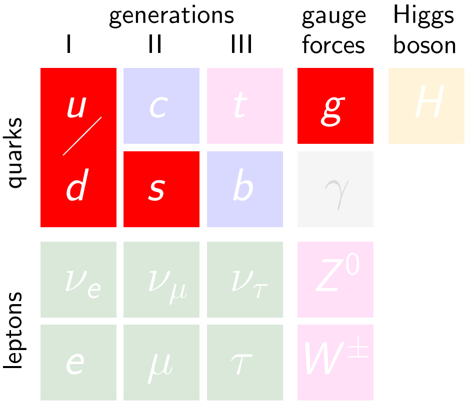
introduction

The Standard Model of Elementary Particle Physics

	generations I	generations II	generations III	gauge forces	Higgs boson
quarks	u	c	t	g	H
quarks	d	s	b	γ	
leptons	ν_e	ν_μ	ν_τ	Z^0	
leptons	e	μ	τ	$W^\pm$	

- Gravitation, dark matter, or dark energy not included
- Is the Higgs boson a fundamental scalar?

Typical lattice simulations (underlying most results to be shown later)



- 2+1 dynamical flavor QCD (fully relativistic action)

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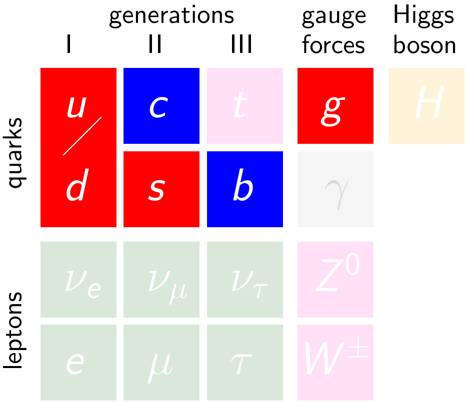

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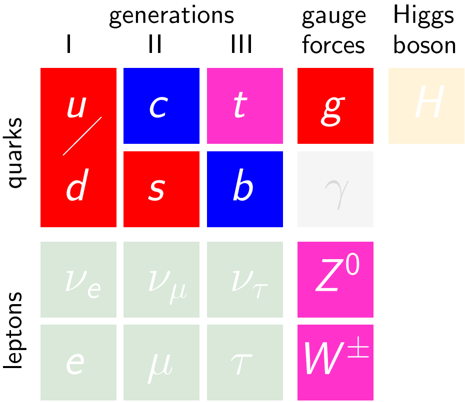

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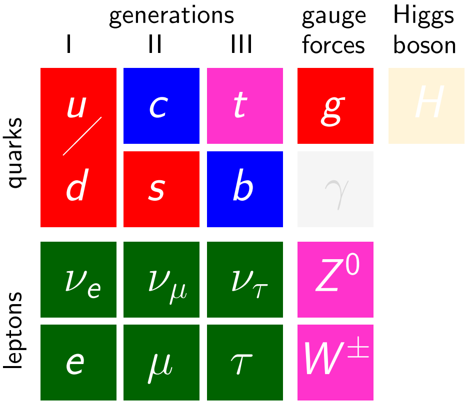

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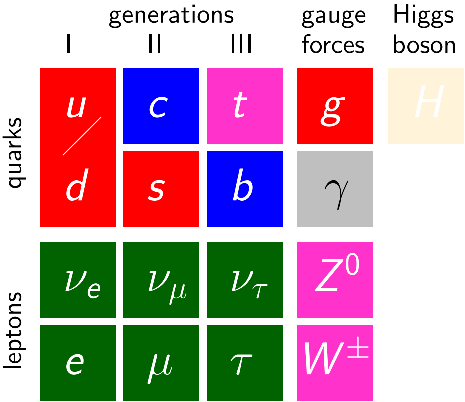

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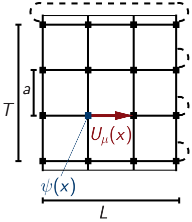
- Valence *c*- and *b*-quarks are simulated as valence quarks (dynamical charm is possible)
- Weak force carries and top enter only "in" point-like operators
- Leptons mainly in post-analysis steps
- QED and iso-spin breaking likely included in future calculations to achieve precision $\lesssim 1\%$

Lattice simulations



- Discretize Euclidean space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- Study physics in a finite box of volume $(aL)^3$
  - $\rightarrow$ The pion should not be squeezed $\Rightarrow M_\pi L \gtrsim 4$
- $1/a$ is the cutoff
  - $\rightarrow$ Traditionally quark-mass $am \ll 1$
  - $\rightarrow$ Improved actions quark-mass $am < 1$

- Today's lattices have an inverse lattice spacing of $a^{-1} \approx 1.7 \ldots 3 \ldots 4$ GeV
  - $\rightarrow$ Light quark masses: $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV
  - $\rightarrow$ Mass of the charm quark: $m_c = 1.29$ GeV
  - $\rightarrow$ Mass of the bottom quark: $m_b = 4.18$ GeV

Simulating charm and bottom (schematic)

$a^{-1} > 1.5$ GeV

charm: RHQ; extrapolations of fully relativistic actions (?)

bottom: HQET, NRQCD, RHQ

$a^{-1} > 2.2$ GeV

charm: fully relativistic action

bottom: (guided) extrapolation of fully relativistic action

$a^{-1} > 4.6$ GeV

bottom: fully relativistic action

HQET: static limit, relatively noisy

NRQCD: non-relativistic QCD, no continuum limit

RHQ or Fermilab: relativistic heavy quark action, complicated discretization errors

fully relativistic: (heavy) HISQ, (heavy) MDWF, . . .

Analyzing lattice simulations

- Compute expectation values of gauge invariant observables by

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U \, e^{-S(U)} \, \mathcal{O}(U), \quad \mathcal{Z} = \int \mathcal{D}U \, e^{-S(U)}$$

- Only statistical estimation possible: $\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(U_i)$

- Generate a sufficiently long sequence of configurations with probability distribution

$$P \propto \exp\{-S(U)\}$$

- Typically done by a Markov chain using the HMC algorithm

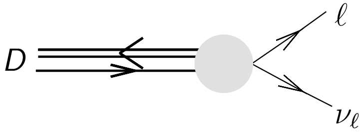
- Not focusing at averages of lattice calculations
  - → Overall lattice results are fairly consistent
  - → Some groups tend to report more aggressive, some more conservative errors

- For overview and averages see Flavor Lattice Averaging Group [FLAG 2016][website], see also [Rosner, Stone, Van de Water, arXiv:1509.02220]
  - → Please do cite the original publications

    — or no new calculations may happen

Beautiful and charming calculations

- Leptonic decays



Beautiful and charming calculations

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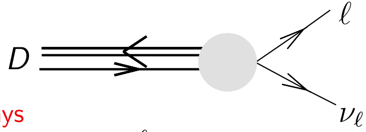
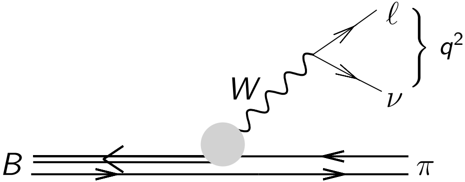
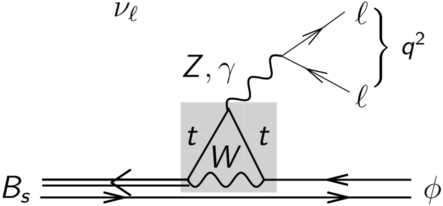


- Semi-leptonic decays





Beautiful and charming calculations

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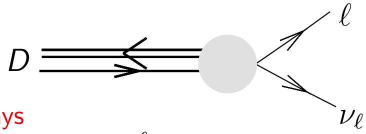

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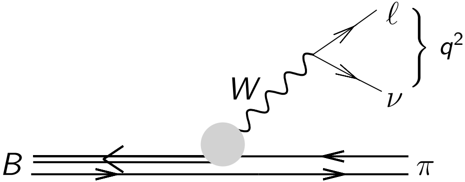
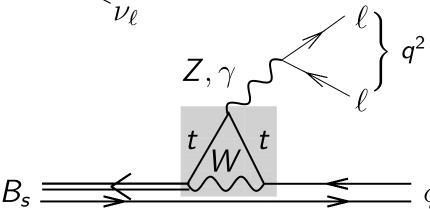
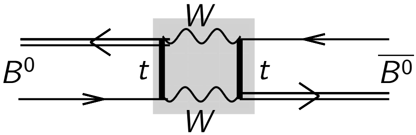
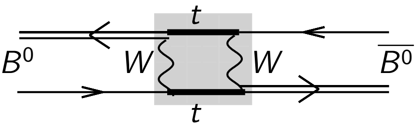




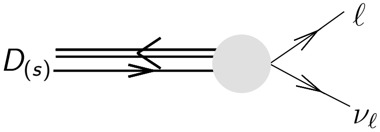
- (Short distance) neutral meson mixing





decay constants and mixing

Decay constants



- Allow to extract CKM matrix elements when combined with experimental measurements

$$\mathcal{B}(D_{(s)} \to \ell\nu_\ell) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_\ell^2 M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M_{D_{(s)}}^2}\right)^2$$

- Simple lattice 2-point function of the axial vector current

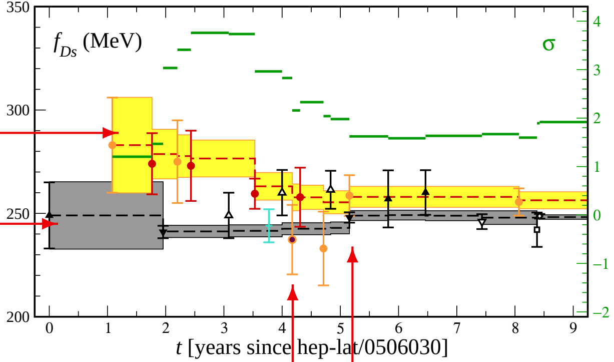
$$\langle 0|\mathcal{A}_{cq}^\mu|D_q(p)\rangle = i f_{D_q} p_{D_q}^\mu \quad \text{with} \quad \mathcal{A}_{cq}^\mu = \bar{c}\gamma_\mu\gamma_5 q \quad \text{and} \quad q = d, s$$

History of the f_{Ds} puzzle

[Kronfeld arXiv:1312.2930]

experiments

2+1 flavor lattice QCD



change of experimental normalization

change of scale setting

- 2σ difference between lattice QCD and experiment + unitarity
  [Rosner, Stone, Van de Water, arXiv:1509.02220]

New Physics in rare B-decays?

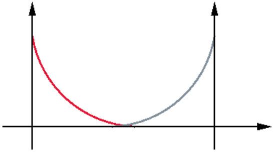
$B \to \tau\nu$ [UTfit PLB 687 (2010) 61]

- f_B is needed for the Standard-Model prediction of $BR(B \to \tau\nu)$
- Potentially sensitive to charged-Higgs exchange due to large τ mass

$B_s \to \mu_+\mu_-$ [Buras et al. EPJ C72 (2012) 2172], [Buras et al. JHEP07 (2013) 077]

- f_{B_s} is needed for Standard-Model prediction of $BR(B_s \to \mu_+\mu_-)$
- Strong sensitivity to NP because FCNC processes are suppressed by the Glashow-Iliopoulos-Maiani (GIM)-mechanism in the SM
- Measured by CMS and LHCb: combined analysis of 7 and 8 TeV runs yields $> 6\sigma$ significance — in agreement with SM [CMS and LHCb arXiv:1411.4413]

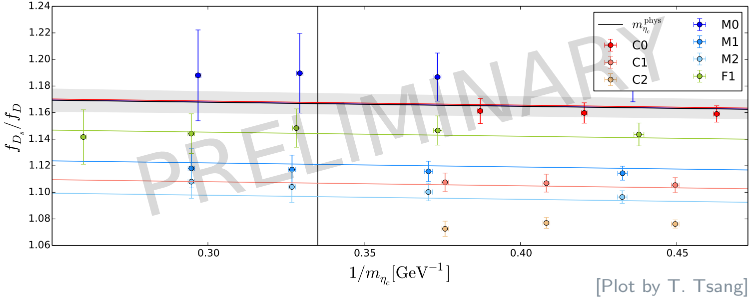
Heavy Möbius domain-wall fermions



- Domain-wall fermions
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
  → 5 dimensional formulation
  → Perfect chirality for $L_s \to \infty$
  → Residual chiral symmetry breaking: m_{res}
- Möbius domain-wall fermions [Brower, Neff Orginos, arXiv:1206.5214]
  → Smaller m_{res} for same L_s
- Möbius DWF optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  → Discretization errors well under control for $am_c \leq 0.4$
    → On coarse ($a^{-1} = 1.785$ GeV) ensembles we simulate just below m_c^{phys}
    → Simulate 3–4 charm-like masses and then extrapolate/interpolate
- Smeared Möbius domain-wall fermions [Hashimoto et al.]
  → Allows to reach larger am_c values

D-meson decay constants [Boyle et al., PoS LATTICE2015 336]

- Renormalization factors not finalized ⇒ ratios
  → Nonperturbative renormalization [Boyle, Del Debbio, Khamseh, arXiv:1608.07982]
- Linear extrapolation/interpolation in m_{η_c}
- Two ensembles with physical pions
- Preliminary value: $f_{D_s}/f_D = 1.167(8)$ (statistical error only)
  FLAG 2016: $f_{D_s}/f_D = 1.187(12)^{2+1}$ and $1.1716(32)^{2+1+1}$



[Plot by T. Tsang]

Renormalization

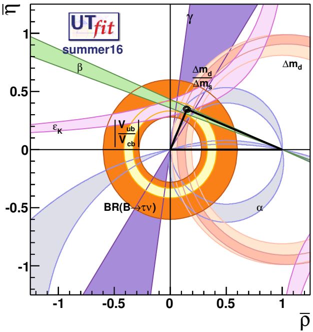
- Lattice measurements result in bare quantities
- Renormalization needed to match to physical quantities
- Lattice perturbation theory is challenging
  → Typically only 1-loop calculations ⇒ large uncertainties

- Nonperturbative renormalization established for light-light quantites (massless scheme)
  → Heavy-light: mostly nonperturbative renormalization
  $Z_V^{hl} = \varrho^{hl}\sqrt{Z_V^{hh} \cdot Z_V^{ll}}$ [El-Khadra et al. PRD 64 (2001) 014502]

- Massive momentum-subtraction scheme [Boyle, Del Debbio, Khamseh, arXiv:1608.07982]
  → Based on preserving renormalized Ward identities
  → Reormalization condition defined at $\overline{m}$ (reduces to SMOM for $\overline{m} \to 0$)
  → Properties verfied in 1-loop perturbation theory with dim. regularization
  → Extended to the heavy-light quantities ($m_R \to 0$, $M_R \to \overline{m}$)

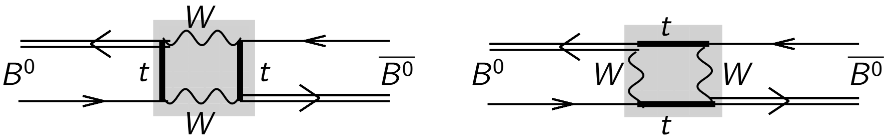
Neutral meson mixing

- D^0–$\overline{D^0}$ mixing has long and short contributions
- B^0–$\overline{B^0}$ mixing constraints the apex of the CKM unitarity triangle
  $\rightarrow$ Short distance dominated
- Experimental results and nonperturbative inputs are needed
  $\rightarrow$ Lattice computes short distance, 4-quark operators



[UTfit website] [CKM fitter website] [latticeaverages.org]

Example: B^0–$\overline{B^0}$ mixing



- Dominant contribution in SM: box diagram with top quarks

$$\left.\begin{array}{r}|V_{td}^* V_{tb}| \text{ for} B_d\text{–mixing}\\ |V_{ts}^* V_{tb}| \text{ for} B_s\text{–mixing}\end{array}\right\} \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$
- CKM matrix elements are extracted by

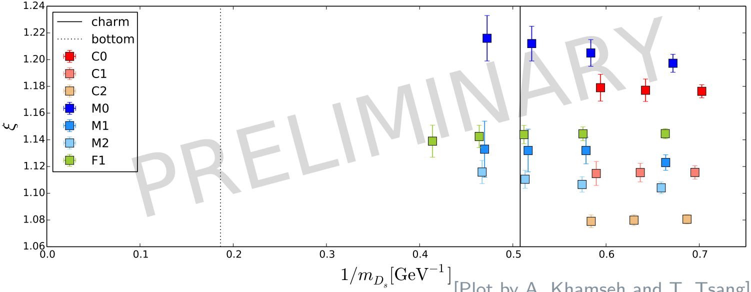
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

- Experimental uncertainty $< 1\%$; best lattice determination 1.6%

  [Fermilab/MILC PRD93 (2016) 113016]

Neutral meson mixing: ξ [Boyle et al., PoS LATTICE2015 336]

- Computation of the short distance operators only
- Linear extrapolation/interpolation in m_{D_s}
- Two ensembles with physical pions
- Combined analysis / continuum extrapolation in progress

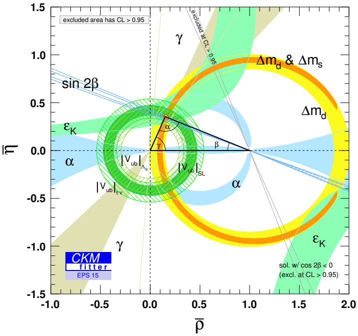


[Plot by A. Khamseh and T. Tsang]

semi-leptonic decays

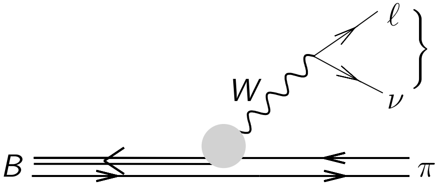
Semi-leptonic decays: $|V_{ub}|$

- $|V_{ub}|$ is another constrain of the apex of the CKM unitarity triangle
- Longstanding $2-3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$) measurements
- Alternative, exclusive ($\Lambda_b \to p\ell\nu$) determination
  [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- $B \to \tau\nu$ has larger error



[http://ckmfitter.in2p3.fr]

Example: V_{ub} from exclusive semileptonic decay $B \to \pi \ell \nu$



$$q^2 = M_B^2 + M_\pi^2 - 2 M_B E_\pi$$

- Conventionally parametrized by

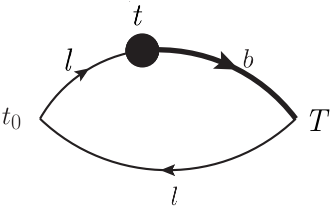
$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2}{192 \pi^3 M_B^3} \left[\left(M_B^2 + M_\pi^2 - q^2 \right)^2 - 4 M_B^2 M_\pi^2 \right]^{3/2} \times \left| f_+(q^2) \right|^2 \times \left| V_{ub} \right|^2$$

experiment known nonperturbative input CKM

Relativistic Heavy Quark action for the *b*-quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
- Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

$B \to \pi \ell \nu$ form factors [PRD 91 (2015) 074510]



- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

- Re-use point-source light quark propagators and generate Gaussian smeared-source sequential heavy quark propagators

- Improve vector current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)

Relating form factors f_+ and f_0 to $f_\parallel$ and $f_\perp$

- On the lattice we prefer using the B-meson rest frame and compute

$$f_\parallel(E_\pi) = \langle\pi|V^0|B\rangle/\sqrt{2M_B} \quad \text{and} \quad f_\perp(E_\pi)p_\pi^i = \langle\pi|V^i|B\rangle/\sqrt{2M_B}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2-M_\pi^2}\left[(M_B - E_\pi)f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2)f_\perp(E_\pi)\right]$$

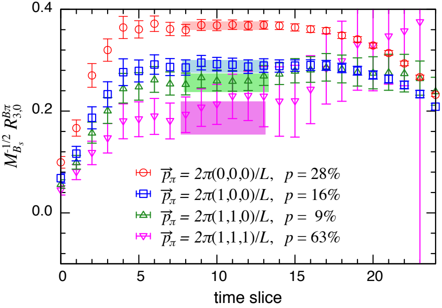
$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_\parallel(E_\pi) + (M_B - E_\pi)f_\perp(E_\pi)\right]$$

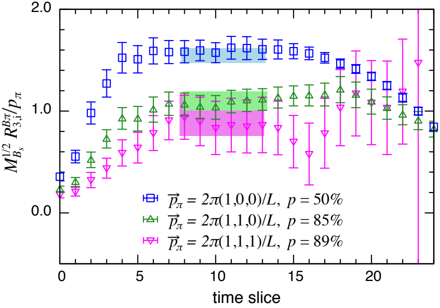
Lattice results for form factors $f_\parallel$ and $f_\perp$ [PRD 91 (2015) 074510]

$$f_\parallel = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_\perp = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

$$R_\mu^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{C_2^\pi(t)C_2^B(T-t)}\sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T-t)}}}$$





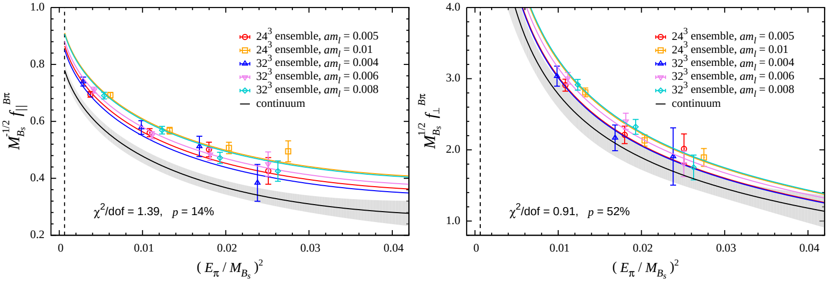
Chiral-continuum extrapolation using SU(2) hard-pion χPT

$$f_{\parallel}(M_\pi, E_\pi, a^2) = c_{\parallel}^{(1)} \left[1 + \left(\frac{\delta f_{\parallel}}{(4\pi f)^2} + c_{\parallel}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\parallel}^{(3)} \frac{E_\pi}{\Lambda} + c_{\parallel}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\parallel}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

$$f_{\perp}(M_\pi, E_\pi, a^2) = \frac{1}{E_\pi + \Delta} c_{\perp}^{(1)} \left[1 + \left(\frac{\delta f_{\perp}}{(4\pi f)^2} + c_{\perp}^{(2)} \frac{M_\pi^2}{\Lambda^2} + c_{\perp}^{(3)} \frac{E_\pi}{\Lambda} + c_{\perp}^{(4)} \frac{E_\pi^2}{\Lambda^2} + c_{\perp}^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right)\right]$$

with δf non-analytic logs of the pion mass and hard-pion limit is taken by $\frac{M_\pi}{E_\pi} \to 0$

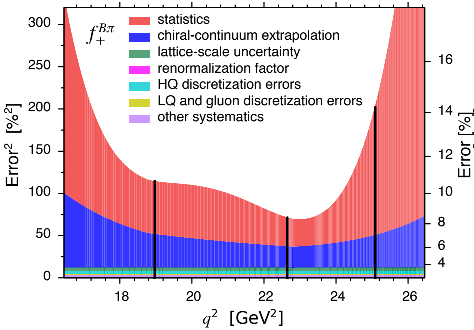
- Again we perform the analysis in terms of dimensionless ratios over M_{B_s}

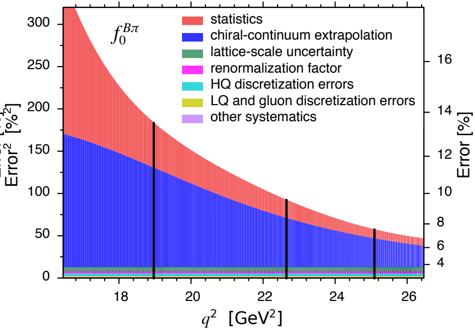


[PRD 91 (2015) 074510]

Obtaining form factors f_+ and f_0 [PRD 91 (2015) 074510]

- Extract $f_\parallel$ and $f_\perp$ for three different q^2 values (synthetic data points)
- Estimate all systematic errors and them add in quadrature
- Convert results to f_+ and f_0





z-expansion [PRD 91 (2015) 074510]

- Use the model-independent z-expansion fit to extrapolate lattice results to the full kinematic range [Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+}-\sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+}+\sqrt{1-t_0/t_+}}$$

with $t_\pm = (M_B \pm M_\pi)^2$ and $t_0 \equiv t_{\rm opt} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$

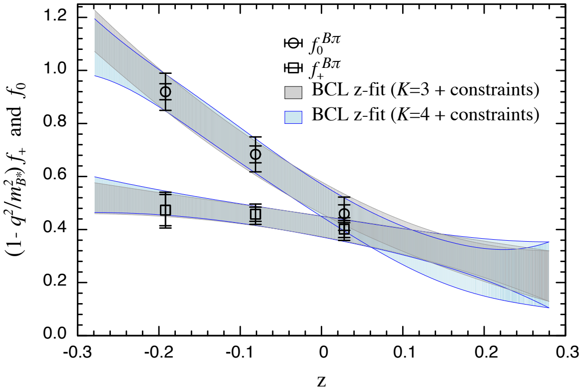
- Minimizes the magnitude of z in the semi-leptonic region: $|z| \leq 0.279$
- $f_0(q^2)$ is analytic in the semi-leptonic region except at the B^* pole
- $f_+(q^2)$ can be expressed as convergent power series

$$f_+(q^2) = \frac{1}{1-q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - (-1)^{k-K} \frac{k}{K} z^k\right]$$

and use for $f_0(q^2)$ the functional form $f_0(q^2) = \sum_{k=0}^{K-1} b_0^{(k)} z^k$

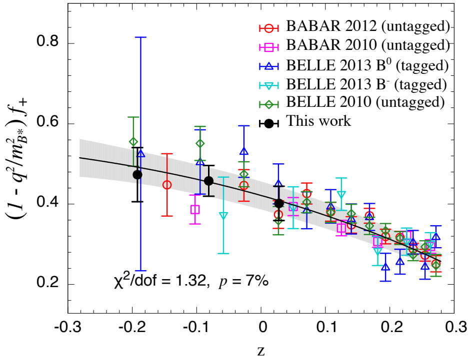
- Exploit the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$ and use HQ power counting to constrain the size of the f_+ coefficients

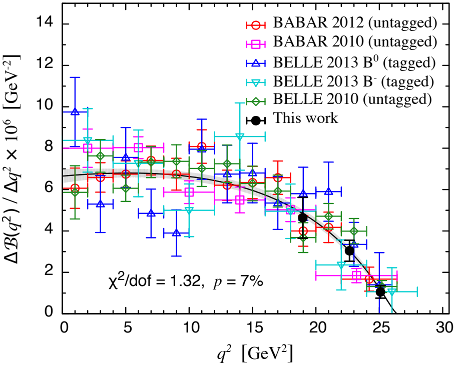
z-expansion fit [PRD 91 (2015) 074510]



Combine with experimental data to determine $|V_{ub}|$

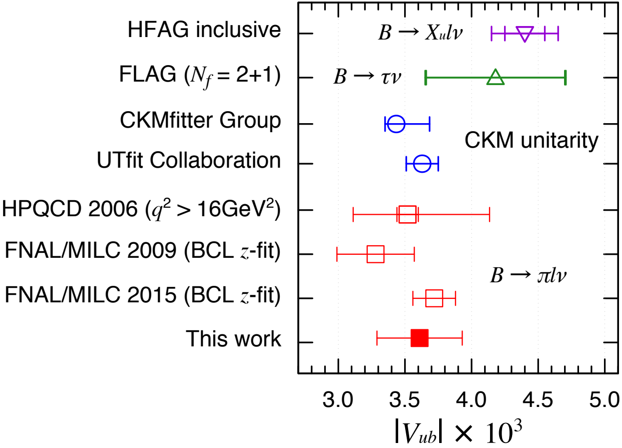
[PRD 91 (2015) 074510]





- Result: $|V_{ub}| = 3.61(32) \cdot 10^{-3}$

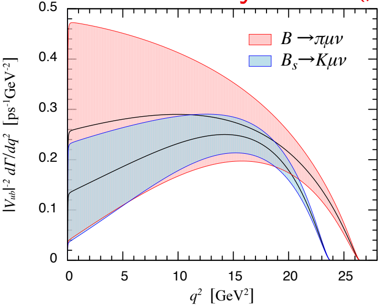
Comparison with other determinations

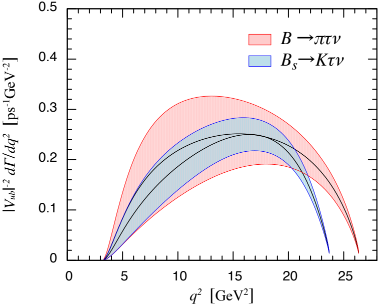


- In good agreement with existing and new FNAL/MILC result
- Result agrees with value obtained CKM unitarity
- Exhibits 2σ tension to inclusive results

Differential decay rates $(\mu = e, \mu)$

[PRD 91 (2015) 074510]



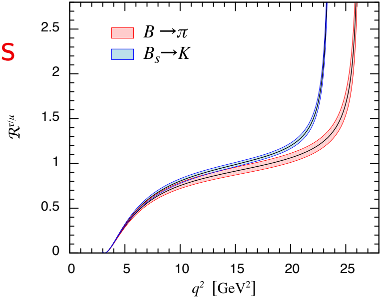


Ratio of differential decay rates

$$\mathcal{R}_\pi^{\tau/\mu} \equiv \frac{d\Gamma(B \to \pi\tau\nu_\tau)/d_q^2}{d\Gamma(B \to \pi\mu\nu_\mu)/d_q^2} = 0.60(19)$$

Similar computation for $B_s \to K\ell\nu$:

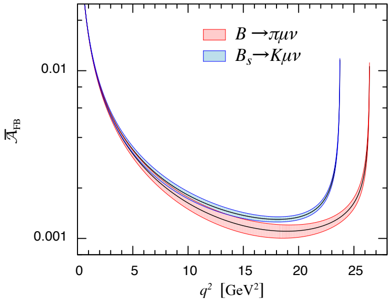
$\mathcal{R}_K^{\tau/\mu} = 0.77(12)$

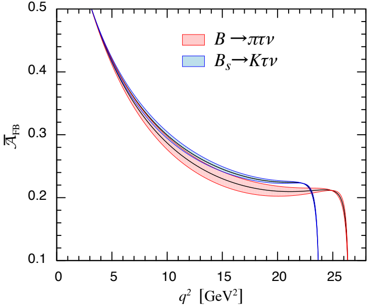


Normalized forward-backward asymmetry [PRD 91 (2015) 074510]

$$\bar{\mathcal{A}}_{\rm FB}^{B_{(s)}\to P\ell\nu} \equiv \frac{\int_{m_\ell^2}^{q^2_{\rm max}} dq^2 \mathcal{A}_{\rm FB}^{B_{(s)}\to P\ell\nu}(q^2)}{\int_{m_\ell^2}^{q^2_{\rm max}} dq^2\, d\Gamma(B_{(s)} \to P\ell\nu)/dq^2}$$

$$\mathcal{A}_{FB}^{B_{(s)}\to P\ell\nu}(q^2) = \frac{G_F^2|V_{ub}|^2}{32\pi^3 M_{B_{(s)}}}\left(1-\frac{m_\ell^2}{q^2}\right)^2 |\vec{p}_P|^2 \times \frac{m_\ell^2}{q^2}(M^2_{B_{(s)}} - M_P^2) f_+(q^2) f_0(q^2)$$





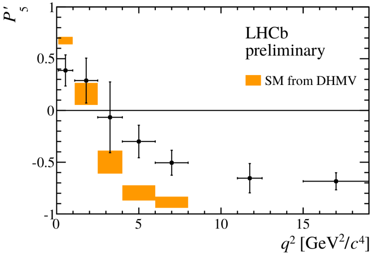
$$\bar{\mathcal{A}}_{\rm FB}^{B\to\pi\mu\nu} = 0.0044(13)$$
$$\bar{\mathcal{A}}_{\rm FB}^{B_s\to K\mu\nu} = 0.0039(11)$$

$$\bar{\mathcal{A}}_{\rm FB}^{B\to\pi\tau\nu} = 0.252(12)$$
$$\bar{\mathcal{A}}_{\rm FB}^{B_s\to K\tau\nu} = 0.2650(79)$$

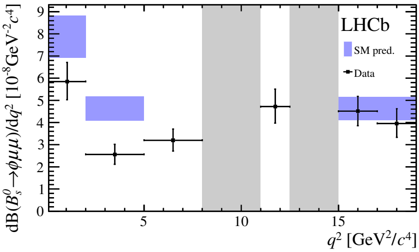
outlook & conclusion

Rare B decays (FCNC)

- GIM suppressed in the Standard Model $\Rightarrow$ sensitive to new physics
- Angular observable P_5' in $B \to K^* \mu^+ \mu^-$ received a lot of attention



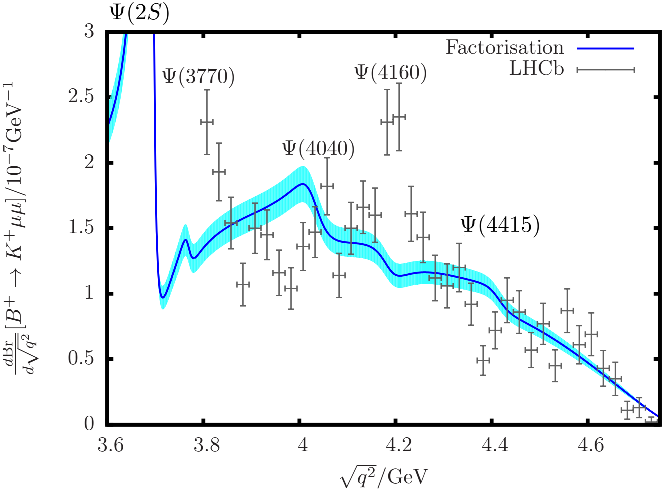
[LHCb-CONF-2015-002]



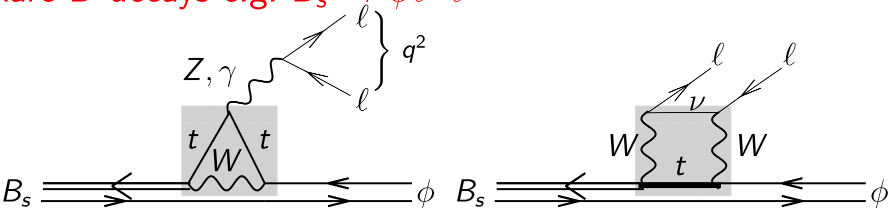
[LHCb JHEP 1509 (2015) 179]

- Lattice QCD: [Horgan et al. PRD 89 (2013) 094501]

▶ Charm resonances under control? [Lyon and Zwicky, arXiv:1406.0566]



Rare B decays e.g. $B_s \to \phi\ell^+\ell^-$



- Pseudoscalar or vector final state (narrow width approximation)
- Effective Hamiltonian

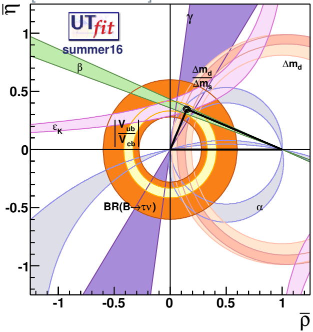
$$\mathcal{H}_{\text{eff}}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{20} C_i O_i$$

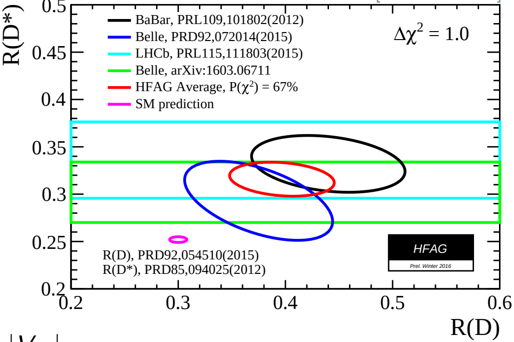
- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$
$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^{\mu} P_{L(R)} b \bar{\ell}\gamma_\mu \ell$$
$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^{\mu} P_{L(R)} b \bar{\ell}\gamma_\mu \gamma^5 \ell$$

⇒ 7 form factors, further details [E. Lizarazo, Lattice 2016]

$B \to D\ell\nu$ and $B \to D^*\ell\nu$





- Determine CKM matrix element $|V_{cb}|$
- Ratio of branching fractions

$$\mathcal{R}^{\tau/\mu}_{D^{(*)}} \equiv \frac{d\Gamma(B \to D^{(*)}\tau\nu_\tau)/d_q^2}{d\Gamma(B \to D^{(*)}\mu\nu_\mu)/d_q^2}$$

Update: $B \to D\ell\nu$

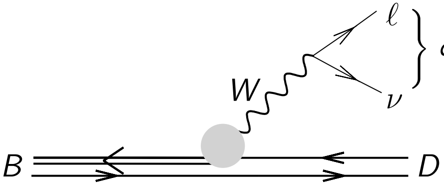
- Two lattice form factor calculations with $q^2 \leq q^2_{max}$
  [Fermilab/MILC PRD92 (2015) 035606] [HPQCD PRD92 (2015) 054510]
- New more precise Belle measurement [Belle PRD93 (2016) 032006]

- Result of new analysis [Bigi and Gambino, arXiv:1606.08030]

$$|V_{cb}|^{incl} = 42.00(65) \cdot 10^{-3} \text{ vs. } |V_{cb}|^{excl} = 40.49(97) \cdot 10^{-3} \sim 1.5\sigma$$

$$R_D^{exp} = 0.397(49) \text{ vs. } R_D^{SM} = 0.299(3) \sim 2.0\sigma$$

Computational setup



$$q^2 = M_B^2 + M_D^2 - 2M_B E_D$$

- RHQ *b*-quarks
- MDWF *c*-quarks

  ⇒ computation rather similar to $B \to \pi\ell\nu$ or $B_s \to K\ell\nu$

- Challenge: resolve q^2 dependence because $m_c \gg m_s$

- Further details: [OW Lattice 2016]

Conclusion

To find new physics in the flavor secctor,

we need to understand the Standard Model contributions.