

Form-factors for semi-leptonic B decays

Oliver Witzel
(RBC-UKQCD collaborations)



University of Colorado
Boulder

Beyond the Standard Model with precision flavour experiments
MIAPP München, Germany
April 30, 2019

introduction

Nonperturbative Lattice QCD calculation

- ▶ Lattice QCD allows for first principle calculations
- ▶ Also in the nonperturbative regime
- ▶ Systematical procedures to improve uncertainties
- ▶ Requires large scale computing facilities

Lattice QCD

- ▶ Wick-rotate to Euclidian time $t \rightarrow i\tau$
- ▶ Use path integral formalism

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] \mathcal{O}[\psi, \bar{\psi}, U] e^{-S_E[\psi, \bar{\psi}, U]}$$

- ▶ Discretize space-time
 - ⇒ Large but finite dimensional path integral
 - ⇒ Finite lattice spacing $a \rightarrow$ UV regulator
 - ⇒ Finite volume of length $L \rightarrow$ IR regulator



[ALCF]



[DiRAC]

Lattice QCD calculation

Simulate

- ▶ at finite lattice spacing a
- ▶ in a finite volume L^3
 - ⇒ discrete momenta $2\pi \vec{n}/L$
- ▶ lattice regularized
- ▶ bare input quark masses
 - $am_\ell, am_s, am_c, am_b$
- Mostly: $aM_\pi \neq aM_\pi^{\text{phys}}$

Desired result

- ▶ take $a \rightarrow 0$ limit
- ▶ take $L \rightarrow \infty$ limit
 - continuous momenta \vec{p}
- ▶ match to some continuum scheme
- ▶ physical quark masses
 - $m_l = m_{u/d}^{\text{phys}}, m_s = m_s^{\text{phys}}, m_c = m_c^{\text{phys}}, m_b = m_b^{\text{phys}}$

- ▶ Need to choose gauge and fermion action
- ▶ Need to control all limits keeping FV and discretization effects under control
 - u quarks want large volume (large L^3) such that $M_\pi \cdot L > 4$
 - b quarks want fine lattice (small a) i.e. $am_b \ll 1$

introduction
○○○●○○○

$B_S \rightarrow K\ell\nu$
○○○○○○○

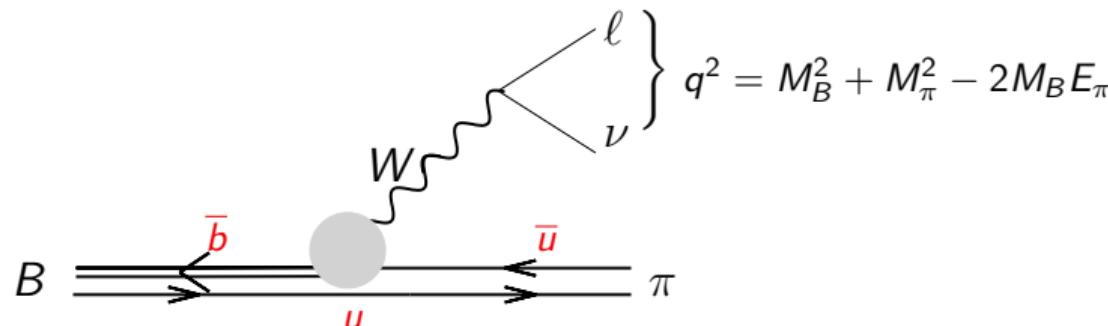
$B_S \rightarrow D_S\ell\nu$
○○○○○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

$|V_{ub}|$ from exclusive semileptonic $B \rightarrow \pi\ell\nu$ decay



- ▶ Conventionally parametrized by (B meson at rest)

$$\frac{d\Gamma(B \rightarrow \pi\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2} \right) M_B^2 (E_\pi^2 - M_\pi^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 |f_0(q^2)|^2 \right],$$

nonperturbative input

introduction
○○○○●○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

$B_s \rightarrow D_s\ell\nu$
○○○○○

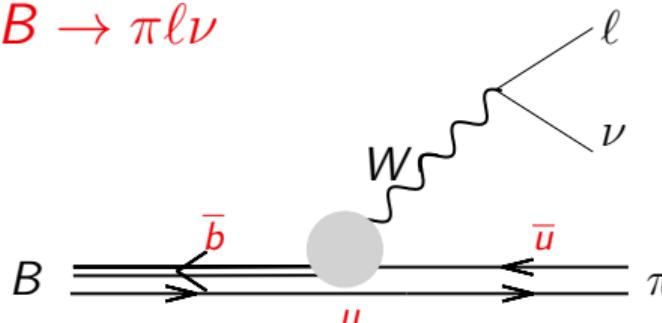
more FF
○○○○○○○○

FLAG
○○○○○○○

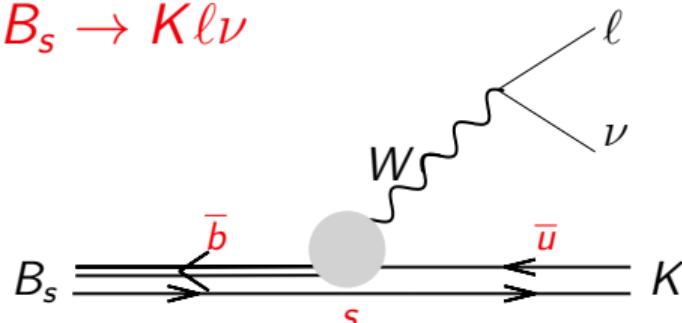
outlook
○○○

$|V_{ub}|$ from exclusive

$$B \rightarrow \pi \ell \nu$$



$$B_s \rightarrow K \ell \nu$$



- ▶ Only spectator quark differs
- ▶ Lattice QCD prefers s quark over u quark: statistically more precise, computationally cheaper
- ▶ B factories run at $\Upsilon(4s)$ threshold $\Rightarrow B$ mesons
- ▶ LHC collisions create many B and B_s mesons which decay \Rightarrow LHCb
 - LHCb prefers the ratio $(B_s \rightarrow D_s\ell\nu)/(B_s \rightarrow K\ell\nu) \Rightarrow |V_{cb}/V_{ub}|$

introduction
○○○○●○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

$B_s \rightarrow D_s\ell\nu$
○○○○○

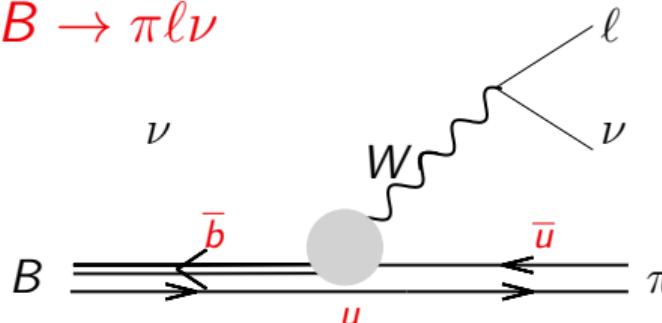
more FF
○○○○○○○

FLAG
○○○○○○○

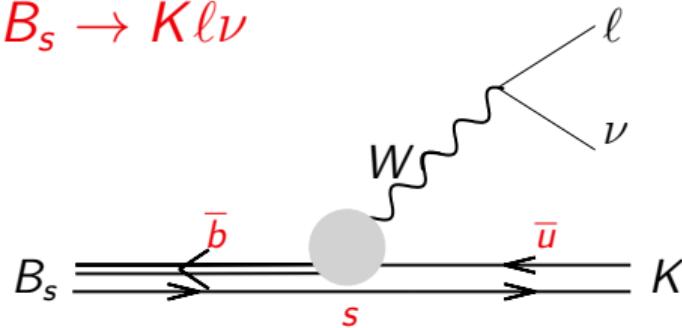
outlook
○○○

$|V_{ub}|$ from exclusive

$B \rightarrow \pi\ell\nu$

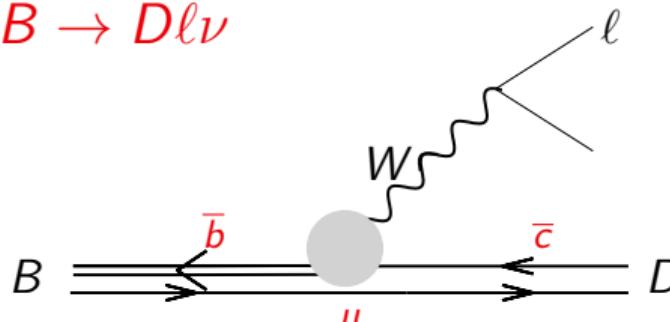


$B_s \rightarrow K\ell\nu$

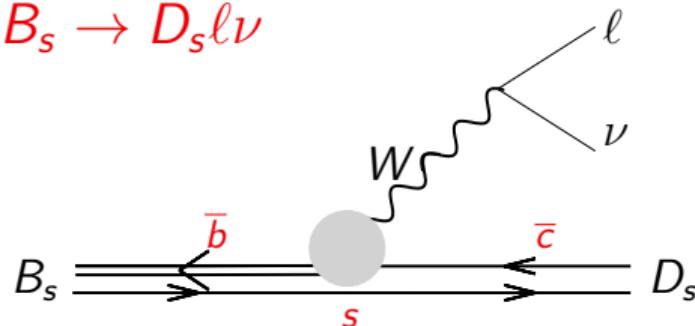


$|V_{cb}|$ from exclusive

$B \rightarrow D\ell\nu$



$B_s \rightarrow D_s\ell\nu$



introduction
○○○○○●○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

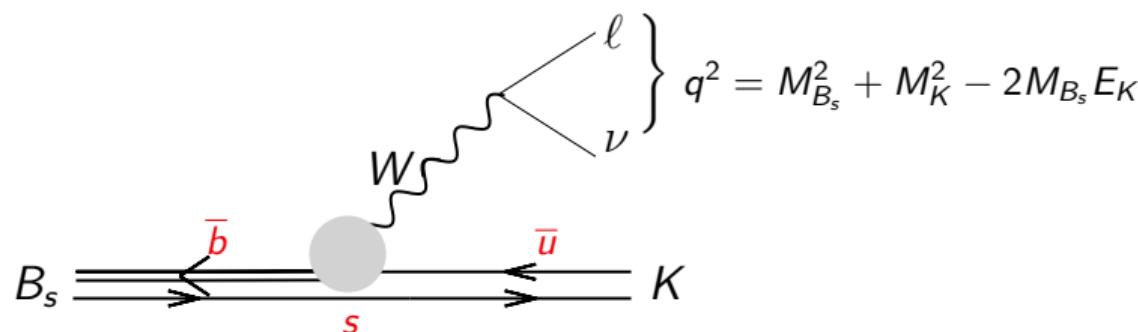
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

$|V_{ub}|$ from exclusive semileptonic $B_s \rightarrow K\ell\nu$ decay



- ▶ Conventionally parametrized by (B_s meson at rest)

$$\frac{d\Gamma(B_s \rightarrow K\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}$$

experiment

CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2} \right) M_{B_s}^2 (E_K^2 - M_K^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_K^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

introduction
oooooooo●○

$B_S \rightarrow K \ell \nu$
oooooooo

$B_S \rightarrow D_S \ell \nu$
oooooo

more FF
oooooooo

FLAG
oooooooo

outlook
○○○

Nonperturbative input

- ▶ Parametrizes interactions due to the (nonperturbative) strong force
- ▶ Use operator product expansion (OPE) to identify short distance contributions
- ▶ Calculate the flavor changing currents as point-like operators using lattice QCD

RBC-UKQCD's set-up

- ▶ RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
 - Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
[PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][JHEP 1712 (2017) 008]

introduction
oooooooo● $B_s \rightarrow K\ell\nu$
oooooooo $B_s \rightarrow D_s\ell\nu$
oooooomore FF
ooooooooFLAG
oooooooooutlook
○○○

RBC-UKQCD's set-up

	L	$a^{-1}(\text{GeV})$	am_l	am_s	$M_\pi(\text{MeV})$	# config.	#sources	
C1	24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
C2	24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
<i>C0</i>	48	1.730	0.00078	0.0362	139	40	81 / 1*	[PRD 93 (2016) 074505]
M0	64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
F1	48	2.774	0.002144	0.02144	234	98	24	[JHEP 1712 (2017) 008]

* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

► Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]

► a : ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm

RBC-UKQCD's set-up

- ▶ RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
 - Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
[PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][JHEP 1712 (2017) 008]
- ▶ Unitary and partially quenched domain-wall up/down quarks
[Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- ▶ Domain-wall strange quarks at/near the physical value
- ▶ Additional challenge $m_c = 1.28\text{GeV} \sim 270 \times m_d$
 $m_b = 4.18\text{GeV} \sim 1000 \times m_d$

introduction
oooooooo●

$B_s \rightarrow K \ell \nu$
oooooooo

$B_s \rightarrow D_s \ell \nu$
oooooo

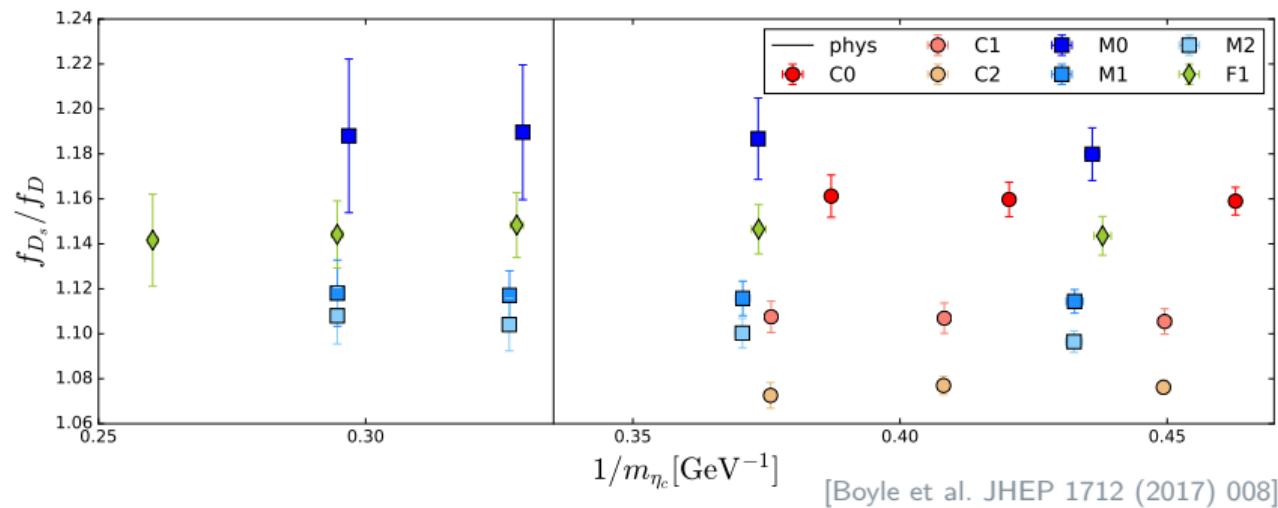
more FF
oooooooo

FLAG
ooooooo

outlook
○○○

RBC-UKQCD's set-up

- ▶ Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
 - Simulate 3 or 2 charm-like masses then extrapolate/interpolate



RBC-UKQCD's set-up

- ▶ Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
 - Simulate 3 or 2 charm-like masses then extrapolate/interpolate
- ▶ Effective relativistic heavy quark (RHQ) action for bottom quarks
 - [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
 - Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
 - Allows to tune the three parameters ($m_0 a$, c_P , ζ) nonperturbatively [PRD 86 (2012) 116003]
 - Smooth continuum limit; heavy quark treated to all orders in $(m_b a)^n$
 - Mostly nonperturbative renormalization [Hashimoto et al. PRD61 (1999) 014502]
[El-Khadra et al. PRD64 (2001) 014502]

$$Z_V^{bl} = \varrho \sqrt{Z_V^{ll} Z_V^{bb}}$$

B_s → Kℓν

in collaboration with

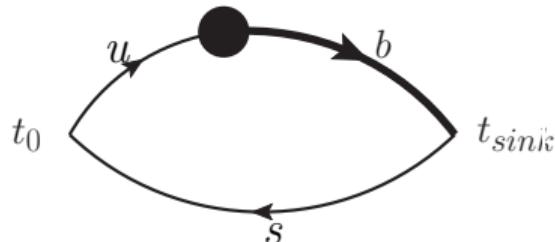
Jonathan M. Flynn, Ryan. C. Hill, Taku Izubuchi, Andreas Jüttner
Christoph Lehner, J. Tobias Tsang, Amarji Soni

introduction
○○○○○○○ $B_s \rightarrow K\ell\nu$
○●○○○○○ $B_s \rightarrow D_s\ell\nu$
○○○○○more FF
○○○○○○○○FLAG
○○○○○○○outlook
○○○

$B_s \rightarrow K\ell\nu$ form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K | V^\mu | B_s \rangle = f_+(q^2) \left(p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu$$



- ▶ Calculate 3-point function by
 - Inserting a quark source for a strange quark propagator at t_0
 - Allow it to propagate to t_{sink} , turn it into a sequential source for a b quark
 - Use a “light” quark propagating from t_0 and contract both at t with $t_0 \leq t \leq t_{sink}$

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○●○○○○

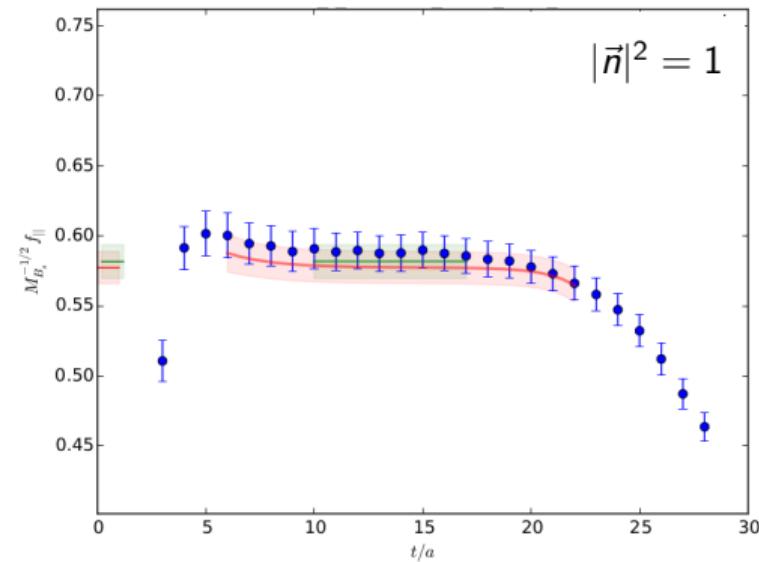
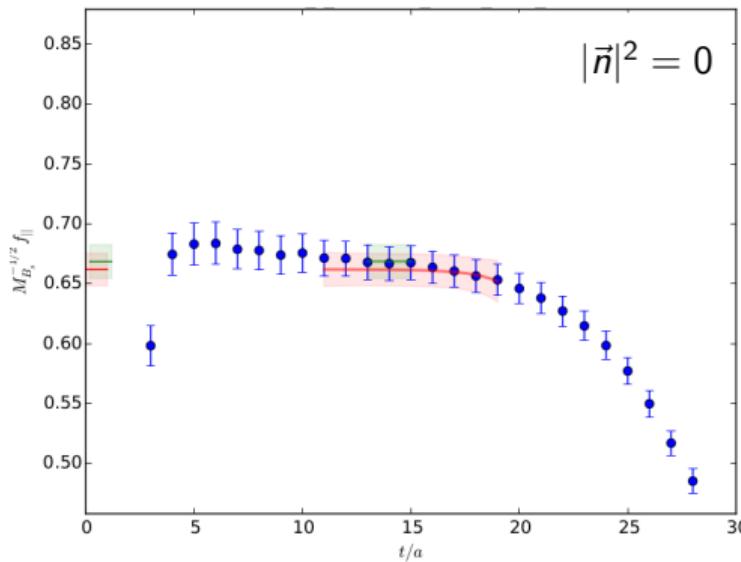
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○○○

$B_s \rightarrow K\ell\nu$ form factors: F1 ensemble



- ▶ Comparison of fit to the ground state only with fit including one excited state term for K and B_s

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○●○○○

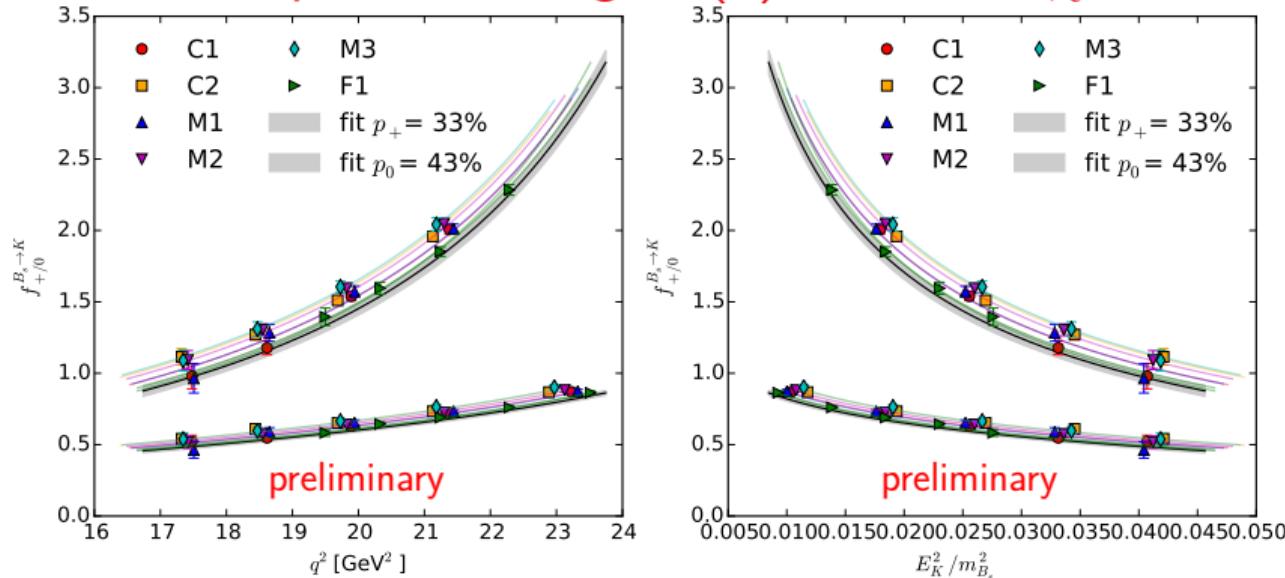
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Chiral-continuum extrapolation using SU(2) hard-kaon χ PT



- ▶ Updating calculation [PRD 91 (2015) 074510] with improved values for a^{-1} and RHQ parameters
- ▶ $f_{pole}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c^{(1)} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c^{(2)} \frac{M_K^2}{\Lambda^2} + c^{(3)} \frac{E_K}{\Lambda} + c^{(4)} \frac{E_K^2}{\Lambda^2} + c^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right]$
- ▶ δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \rightarrow 0$

introduction
○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○●○○

$B_s \rightarrow D_s \ell \nu$
○○○○○

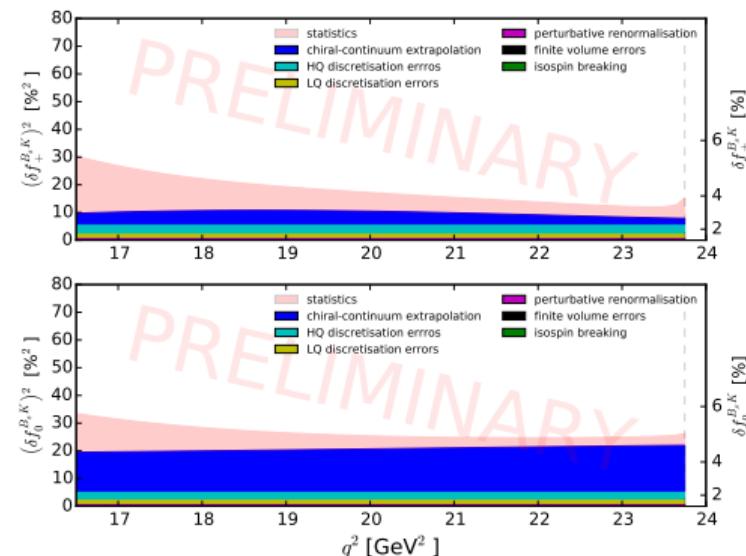
more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Estimate systematic errors due to

- ▶ Chiral-continuum extrapolation
 - Use alternative fit functions
 - Impose different cuts on the data
- ▶ Discretization errors of light and heavy quarks
 - Estimate via power-counting
- ▶ Uncertainty of the renormalization factors
 - Estimate effect of higher loop corrections
- ▶ Finite volume, iso-spin breaking, ...
- ▶ Uncertainty due to RHQ parameters and lattice spacing (a^{-1})
 - Carry out additional simulations to test effects on form factors
- ▶ Uncertainty of strange quark mass
 - Repeat simulation with different valence quark mass



(plot still incomplete)

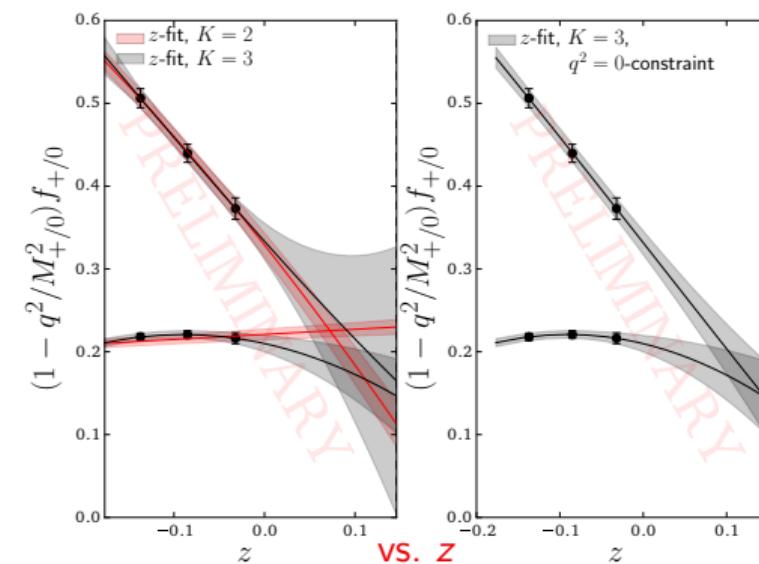
⇒ full error budget

introduction
○○○○○○○ $B_s \rightarrow K\ell\nu$
○○○○○●○ $B_s \rightarrow D_s \ell\nu$
○○○○○more FF
○○○○○○○FLAG
○○○○○○○outlook
○○○

Kinematical extrapolation (z-expansion)

- Map q^2 to z with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with}$$



$$\begin{aligned} t_{\pm} &= (M_B \pm M_\pi)^2 \\ t_0 &\equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2 \end{aligned}$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

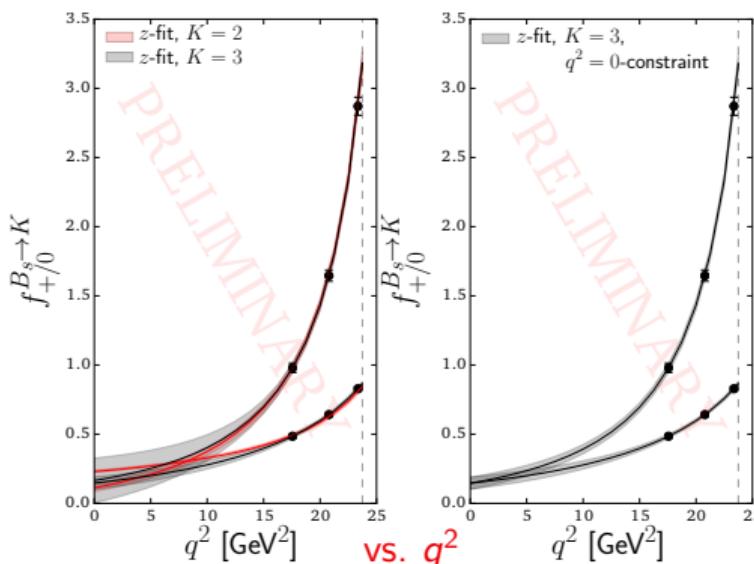
[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Express f_+ as convergent power series
- f_0 is analytic, except for B^* pole
- BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV
- Exploit kinematic constraint $f_+ = f_0 \Big|_{q^2=0}$
- Include HQ power counting to constrain size of f_+ coefficients
- Only some systematics included!

Kinematical extrapolation (z-expansion)

- Map q^2 to z with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with}$$



$$t_{\pm} = (M_B \pm M_\pi)^2 \\ t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Allows to compare shape of form factors

→ Obtained by other lattice calculations

[Bouchard et al. PRD 90 (2014) 054506]
[Bazavov et al. arXiv:1901.02561]

→ Predicted by QCD sum rules and alike

- Combination with experiment leads to the overall normalization: $|V_{ub}|$

- Only some systematics included!

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○●

$B_s \rightarrow D_s\ell\nu$
○○○○○

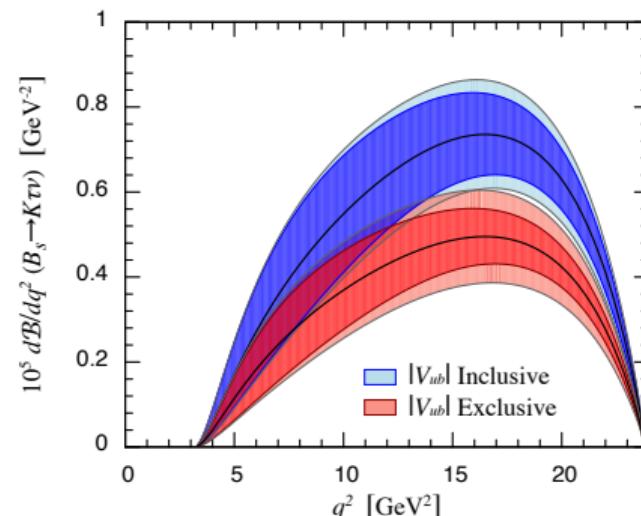
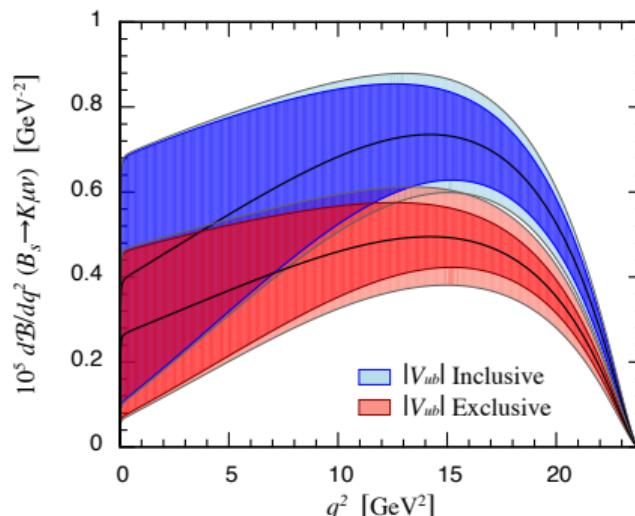
more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

- ▶ Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ



introduction
○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○○●

$B_s \rightarrow D_s \ell \nu$
○○○○○

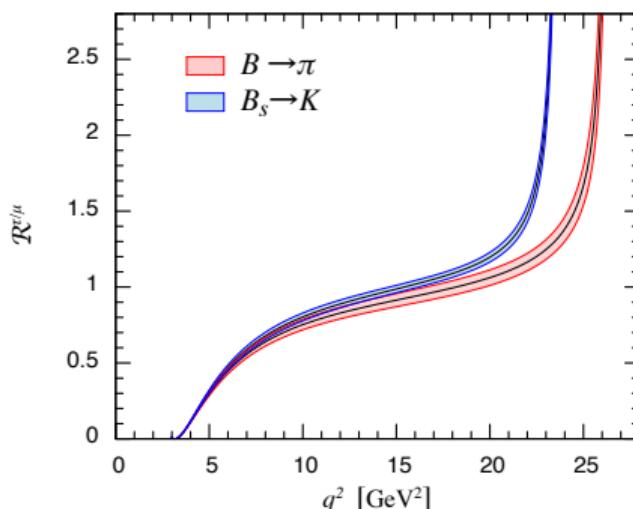
more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

- ▶ Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ
- ▶ Predict ratio of branching fractions \leadsto LFUV



$$R_\pi^{\tau/\mu} = 0.69(19)$$

$$R_K^{\tau/\mu} = 0.77(12)$$

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○●

$B_s \rightarrow D_s \ell\nu$
○○○○○

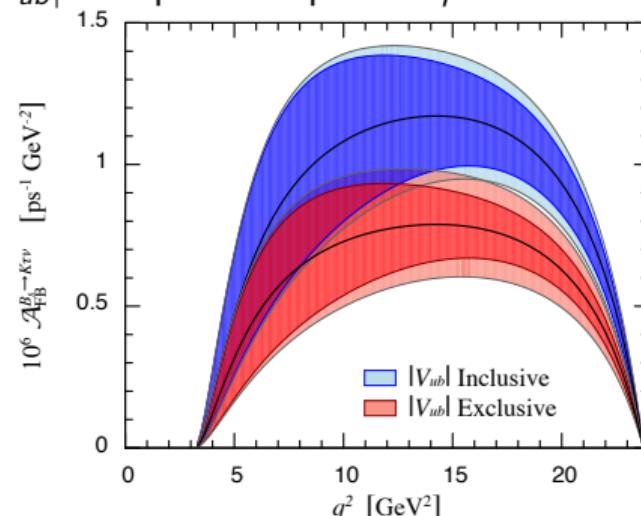
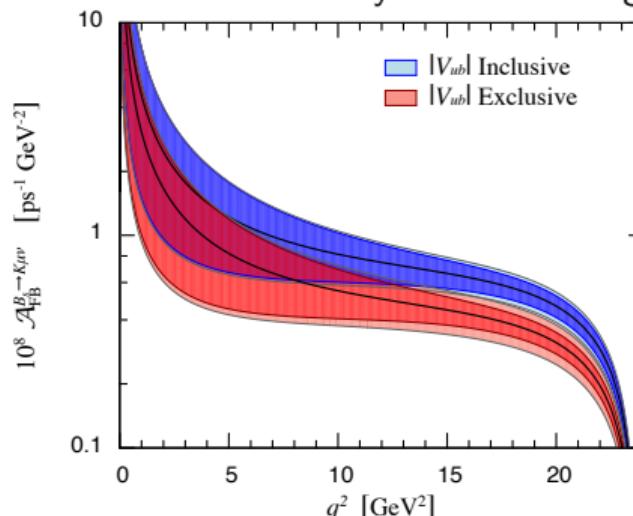
more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Phenomenological interpretation (2015) [PRD 91 (2015) 074510]

- ▶ Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = μ or τ
- ▶ Predict ratio of branching fractions \sim LFUV
- ▶ Predict forward-backward asymmetries using $|V_{ub}|$ as input for lepton = μ or τ



$B_s \rightarrow D_s \ell \nu$

in collaboration with

Jonathan M. Flynn, Ryan. C. Hill, Taku Izubuchi, Andreas Jüttner
Christoph Lehner, J. Tobias Tsang, Amarji Soni

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

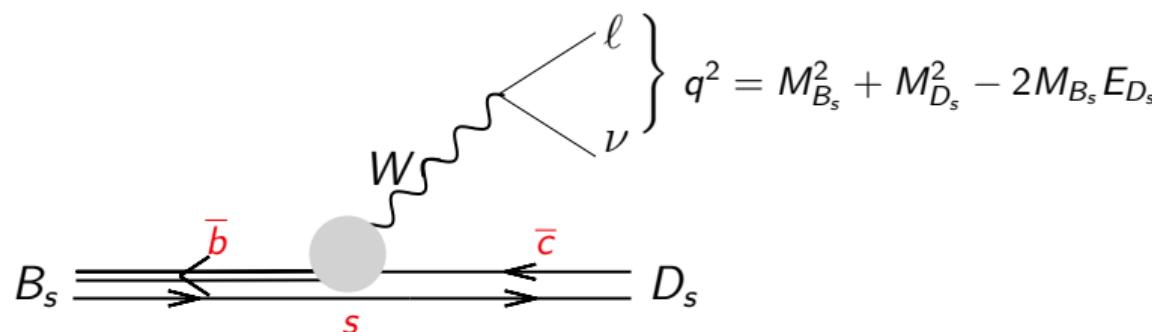
$B_s \rightarrow D_s\ell\nu$
○●○○○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

$|V_{cb}|$ from exclusive semileptonic $B_s \rightarrow D_s\ell\nu$ decay



- ▶ Conventionally parametrized by (B_s meson at rest)

$$\frac{d\Gamma(B_s \rightarrow D_s\ell\nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_{D_s}^2 - M_{D_s}^2}}{q^4 M_{B_s}^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_{D_s}^2 - M_{D_s}^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_{D_s}^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

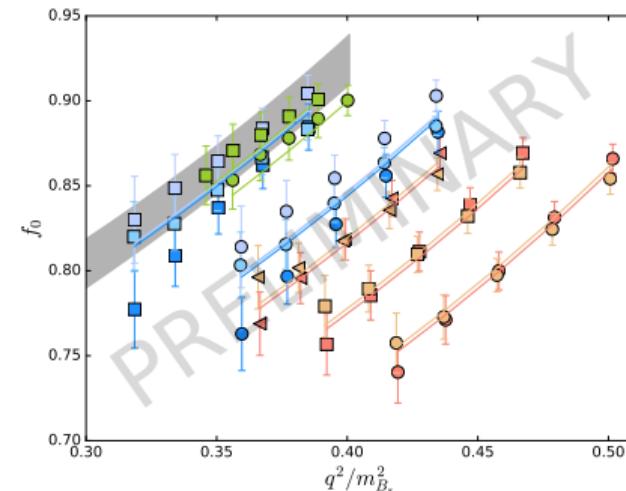
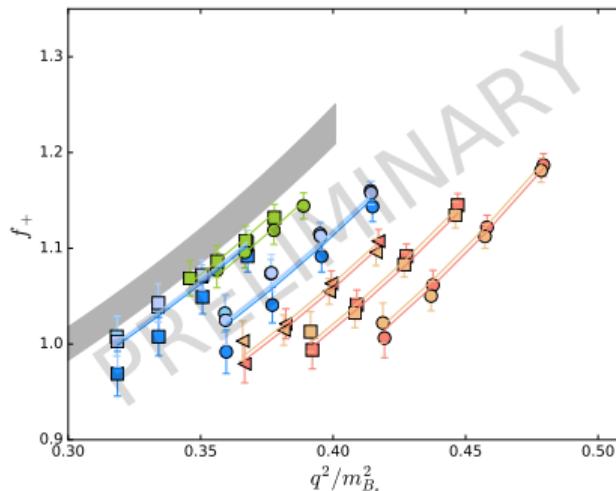
$B_s \rightarrow D_s\ell\nu$
○○●○○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

Global fit $B_s \rightarrow D_s\ell\nu$



►
$$f(q^2, a, M_\pi, M_{D_s}) = \frac{\alpha_0 + \alpha_1 M_{D_s} + \alpha_2 a^2 + \alpha_3 M_\pi^2}{1 + \alpha_4 q^2/M_{B_s}^2}$$

- Extrapolation to the continuum limit with physical quark masses
- Error budget still work in progress

introduction
○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○○○○

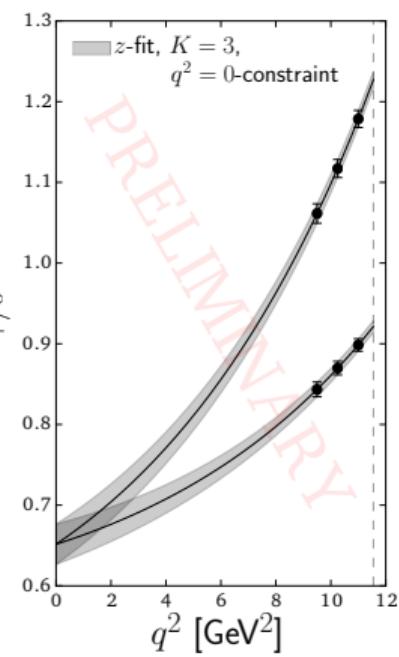
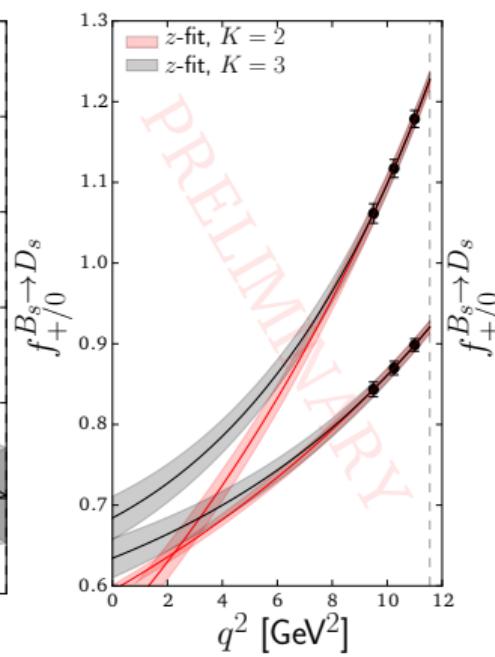
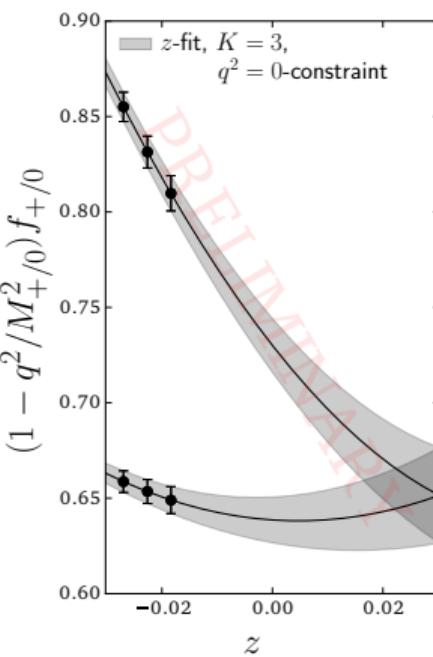
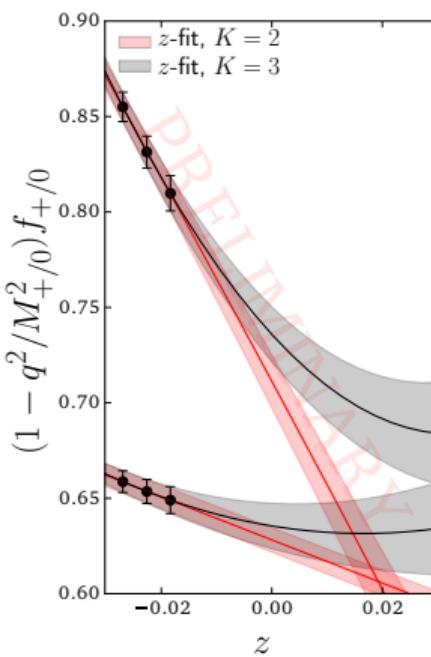
$B_s \rightarrow D_s \ell \nu$
○○○●○

more FF
○○○○○○○

FLAG
○○○○○○○

outlook
○○○

z-expansion



- BCL with poles $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV
- Comparison with CLN in progress

introduction
oooooooo

$B_s \rightarrow K\ell\nu$
ooooooo

$B_s \rightarrow D_s\ell\nu$
oooo●

more FF
oooooooo

FLAG
ooooooo

outlook
○○○

Status $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$

- ▶ $B_s \rightarrow K\ell\nu$ chiral-continuum extrapolation
- ▶ $B_s \rightarrow D_s\ell\nu$ global fit (M_π , M_{D_s} , a^2 , q^2)
- ▶ Extract synthetic data points

- ▶ Full systematic error budget
 - RHQ parameter tuning
 - Continuum extrapolation:
cut to data set, different fit functions, ...
 - Charm extrapolation
 - FV, higher order disc. effects, isospin,
 s -quark mass tuning, ...

- ▶ z-expansion over full q^2 range
 - BGL vs. BCL
 - Test CLN for $B_s \rightarrow D_s\ell\nu$
 - Number of synthetic data points
 - Different truncations
 - Incl. vs. excludig $f_+ = f_0 \Big|_{q^2=0}$

- ▶ Phenomenology: $R(K)$, $R(D_s)$, ...

more form factors

introduction
○○○○○○○

$B_S \rightarrow K\ell\nu$
○○○○○○○

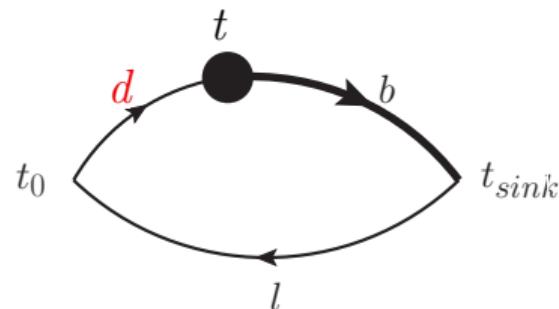
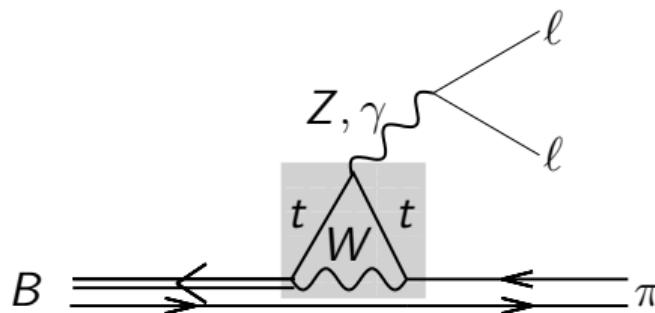
$B_S \rightarrow D_S \ell\nu$
○○○○○

more FF
○●○○○○○

FLAG
○○○○○○○

outlook
○○○

$B \rightarrow \pi \ell^+ \ell^-$ form factor (FCNC)



- If the daughter quark is a d -quark, we have a FCNC decay at loop-level
 - Need to implement additional operators
- Dominant contributions at short distance: f_0 , f_+ , and f_T

$$\langle \pi(k) | i \bar{d} \sigma^{\mu\nu} b(p) | B \rangle = 2 \frac{p^\mu k^\nu - p^\nu k^\mu}{M_B + M_\pi} f_T(q^2)$$

introduction
○○○○○○○

$B_S \rightarrow K \ell \nu$
○○○○○○○

$B_S \rightarrow D_S \ell \nu$
○○○○○

more FF
○○●○○○○○

FLAG
○○○○○○○

outlook
○○○

$B \rightarrow D^* \ell \nu$ form factors

- Vector final state with narrow width approximation

$$\begin{aligned}\langle D^*(k, \lambda) | \bar{c} \gamma^\mu b | B(p) \rangle &= f_V \frac{2i\epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_B + M_D^*} \\ \langle D^*(k, \lambda) | \bar{c} \gamma^\mu \gamma_5 b | B(p) \rangle &= f_{A_0}(q^2) \frac{2M_{D^*} \varepsilon^* \cdot q}{q^2} q^\mu \\ &\quad + f_{A_1}(q^2) (M_B + M_{D^*}) \left[\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\ &\quad - f_{A_2}(q^2) \frac{\varepsilon^* \cdot q}{M_B + M_{D^*}} \left[k^\mu + p^\mu - \frac{M_B^2 - M_{D^*}^2}{q^2} q^\mu \right]\end{aligned}$$

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

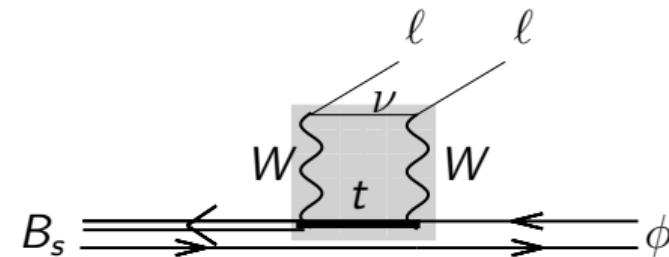
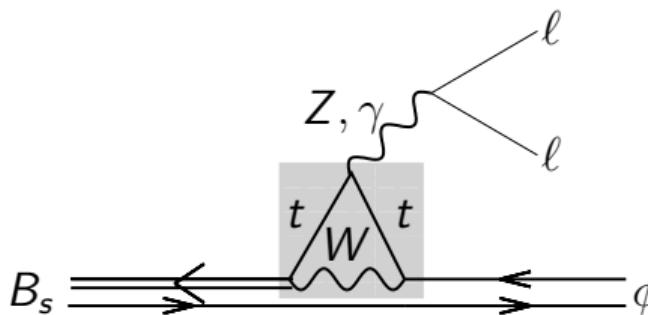
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○●○○○

FLAG
○○○○○○○

outlook
○○○

$B_s \rightarrow \phi\ell^+\ell^-$ form factors (FCNC)



- Vector final state treated in narrow width approximation
- Effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{b \rightarrow s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i^{10} C_i O_i^{(\prime)}$$

- Leading contributions at short distance

$$O_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}$$

$$O_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \ell$$

$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s} \gamma^\mu P_{L(R)} b \bar{\ell} \gamma_\mu \gamma^5 \ell$$

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

$B_s \rightarrow D_s \ell\nu$
○○○○○

more FF
○○○○●○○○

FLAG
○○○○○○○

outlook
○○○

Seven form factors

$$\langle \phi(k, \lambda) | \bar{s} \gamma^\mu b | B_s(p) \rangle = f_V(q^2) \frac{2i \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu^* k_\rho p_\sigma}{M_{B_s} + M_\phi}$$

$$\begin{aligned} \langle \phi(k, \lambda) | \bar{s} \gamma^\mu \gamma_5 b | B_s(p) \rangle &= f_{A_0}(q^2) \frac{2M_\phi \varepsilon^* \cdot q}{q^2} q^\mu \\ &\quad + f_{A_1}(q^2) (M_{B_s} + M_\phi) \left[\varepsilon^{*\mu} - \frac{\varepsilon^* \cdot q}{q^2} q^\mu \right] \\ &\quad - f_{A_2}(q^2) \frac{\varepsilon^* \cdot q}{M_{B_s} + M_\phi} \left[k^\mu + p^\mu - \frac{M_{B_s}^2 - M_\phi^2}{q^2} q^\mu \right] \end{aligned}$$

$$q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} b | B_s(p) \rangle = 2f_{T_1}(q^2) \epsilon^{\mu\rho\tau\sigma} \varepsilon_\rho^* k_\tau p_\sigma ,$$

$$\begin{aligned} q_\nu \langle \phi(k, \lambda) | \bar{s} \sigma^{\nu\mu} \gamma^5 b | B_s(p) \rangle &= i f_{T_2}(q^2) [\varepsilon^{*\mu} (M_{B_s}^2 - M_\phi^2) - (\varepsilon^* \cdot q)(p + k)^\mu] \\ &\quad + i f_{T_3}(q^2) (\varepsilon^* \cdot q) \left[q^\mu - \frac{q^2}{M_{B_s}^2 - M_\phi^2} (p + k)^\mu \right] \end{aligned}$$

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

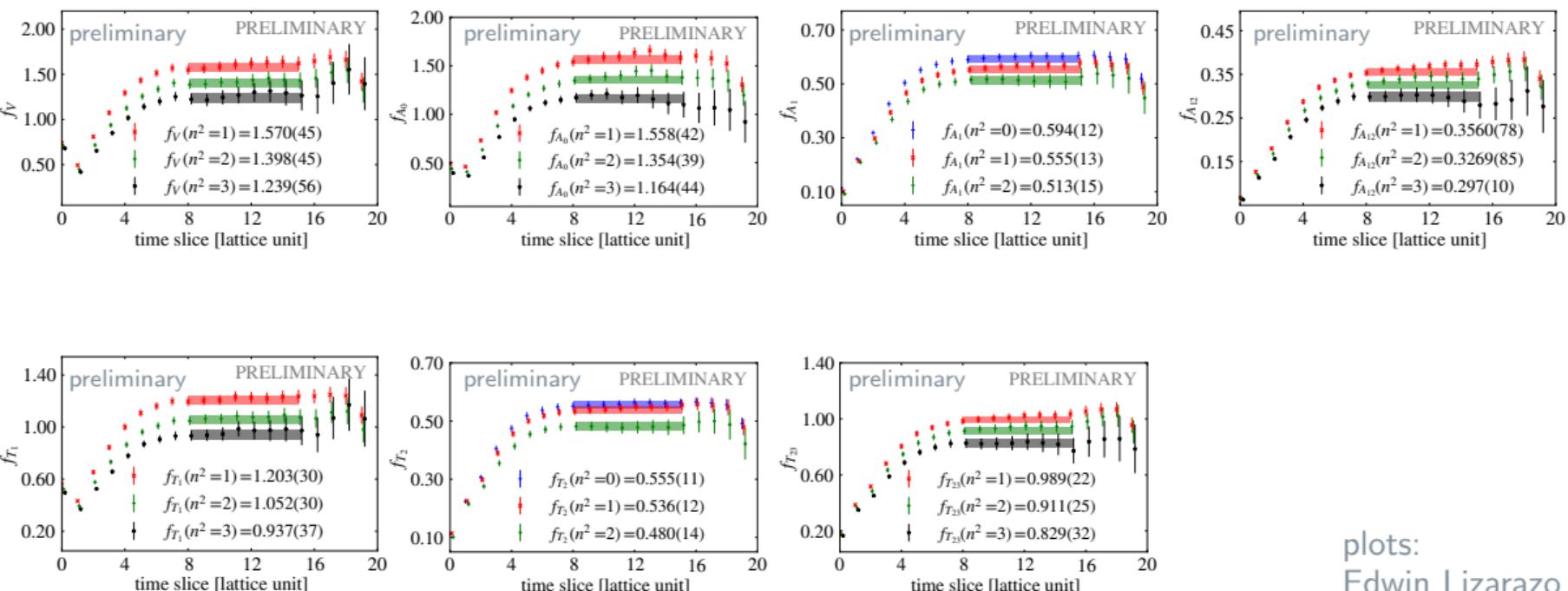
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○●○○

FLAG
○○○○○○○

outlook
○○○

$B_s \rightarrow \phi\ell\ell$: seven form factors ($a^{-1} = 1.784$ GeV, $am_l^{\text{sea}} = 0.005$, $am_s = 0.03224$)



plots:
Edwin Lizarazo

introduction
○○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○○

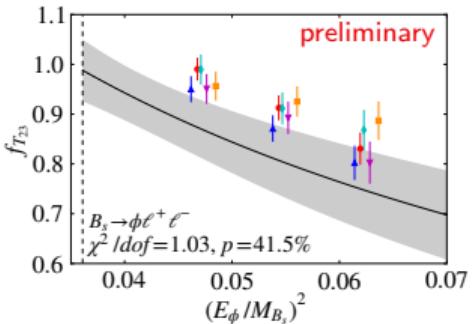
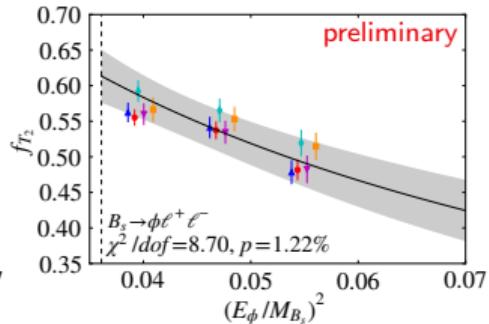
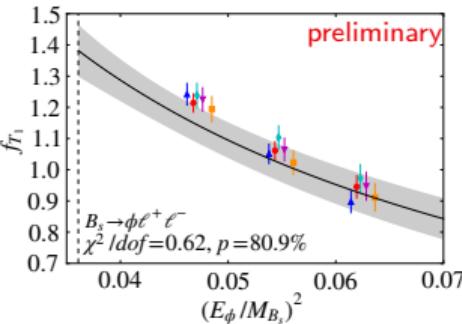
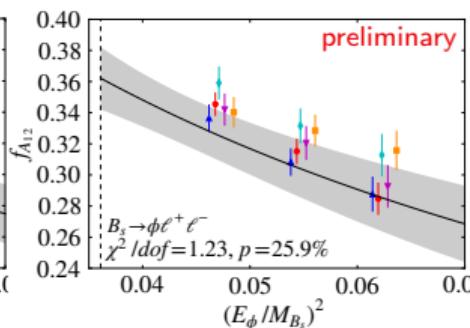
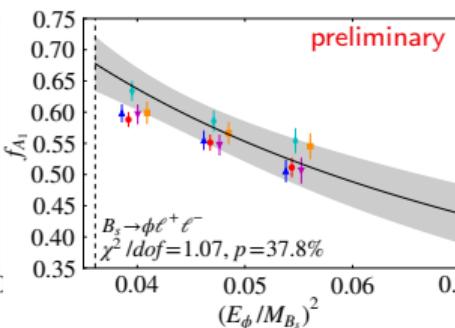
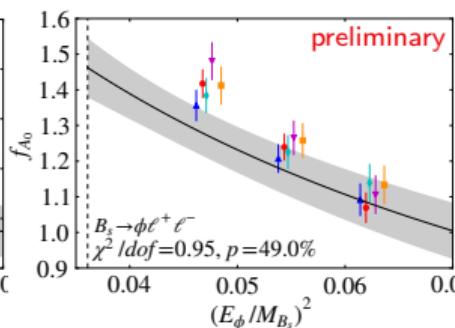
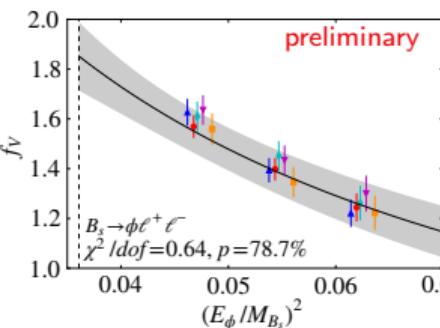
$B_s \rightarrow D_S\ell\nu$
○○○○○○○○

more FF
○○○○○○●○

FLAG
○○○○○○○○

outlook
○○○○○○○○

$B_s \rightarrow \phi\ell^+\ell^-$: seven form factors vs. q^2



$am_l = 0.008$ $am_l = 0.010$
 $am_l = 0.006$ $am_l = 0.005$
 $am_l = 0.004$ $am_l = 0.003$

plots:
Edwin Lizarazo

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

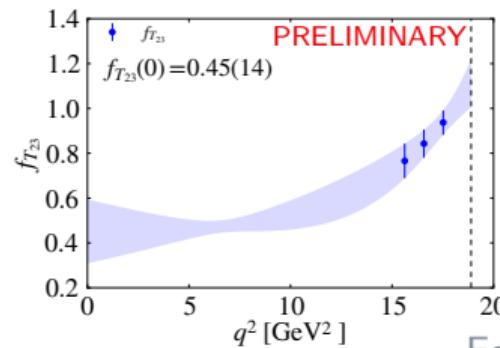
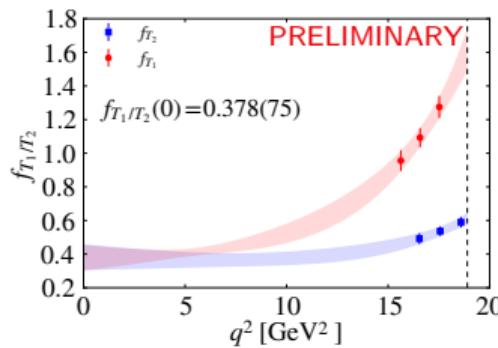
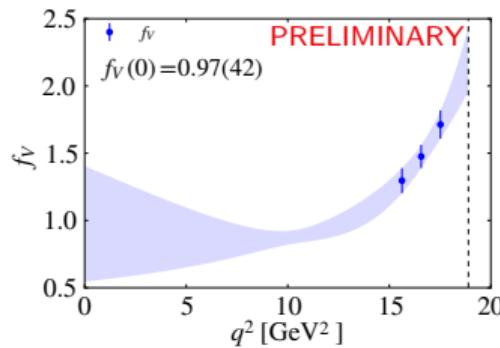
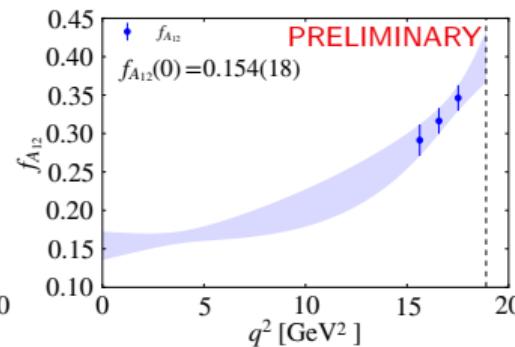
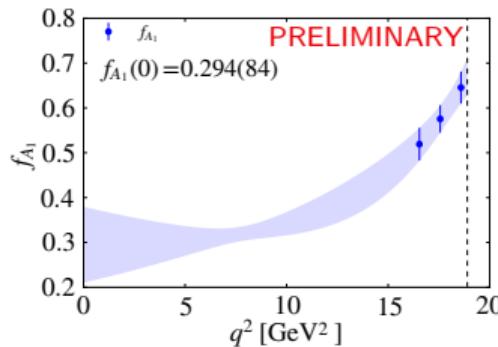
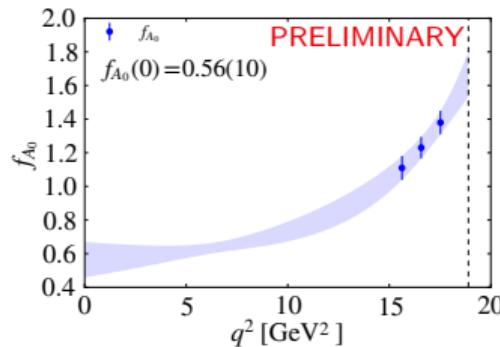
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○●

FLAG
○○○○○○

outlook
○○○

$B_s \rightarrow \phi\ell^+\ell^-$: first attempt to use z-parametrization



plots:
Edwin Lizarazo

Flavor Lattice Averaging Group

[FLAG 2019]

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

$B_s \rightarrow D_s\ell\nu$
○○○○○

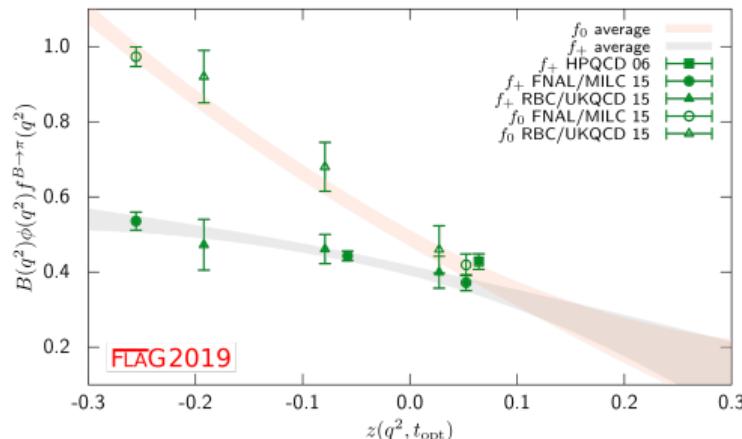
more FF
○○○○○○○

FLAG
○●○○○○○

outlook
○○○

$B \rightarrow \pi\ell\nu$

Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	$\Delta\zeta^{B\pi}$
FNAL/MILC 15	[575]	2+1	A	★	○	★	○	✓	BCL
RBC/UKQCD 15	[576]	2+1	A	○	○	○	○	✓	BCL
HPQCD 06	[573]	2+1	A	○	○	○	○	✓	n/a 1.77(34) 2.07(41)(39)



- ▶ Summary table indicating quality of key features
- ▶ Combined analysis accounting for correlations (if needed)
- ▶ FNAL/MILC also $B \rightarrow \pi\ell^+\ell^-$ (FCNC)
- ▶ Please do cite calculations feeding into FLAG averages

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

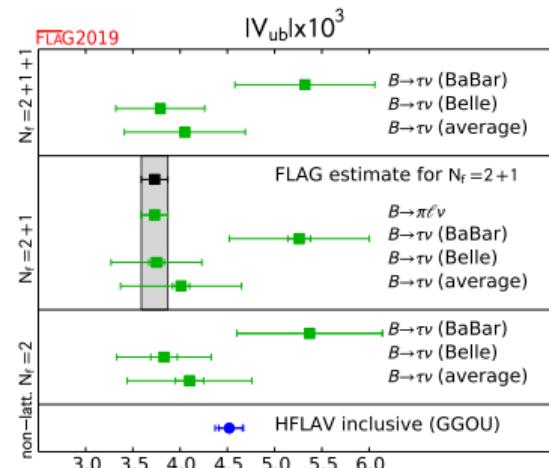
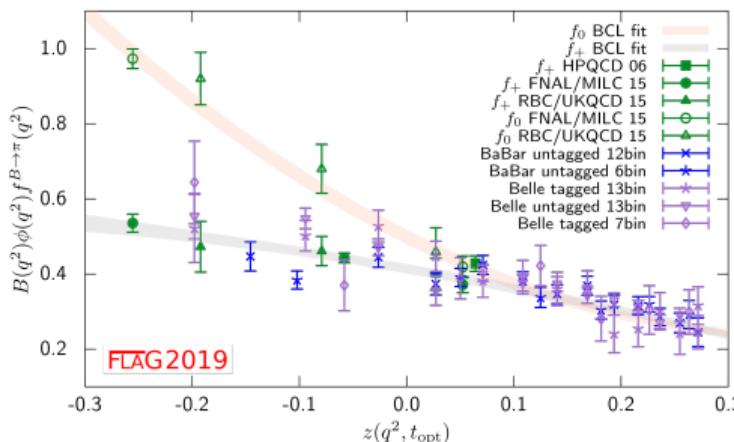
$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○○

FLAG
○●○○○○○

outlook
○○○

$B \rightarrow \pi\ell\nu$



- ▶ Combination of lattice average with experimental results
- ▶ Extraction of $|V_{ub}|$
- ▶ FNAL/MILC also $B \rightarrow \pi\ell^+\ell^-$ (FCNC)
- ▶ Please do cite calculations feeding into FLAG averages

introduction
○○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○○

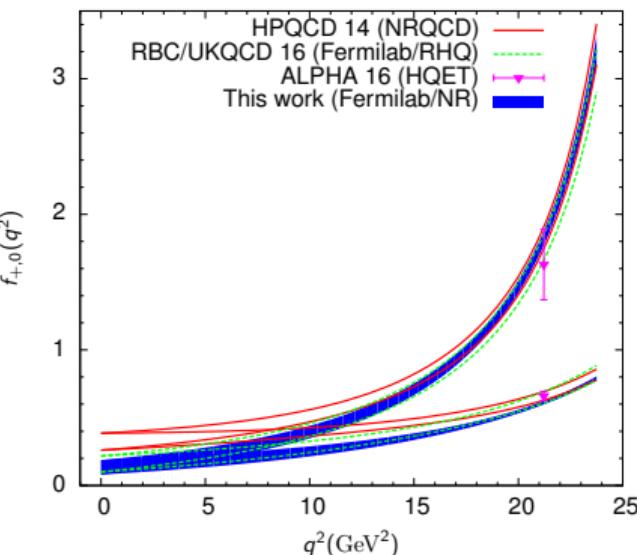
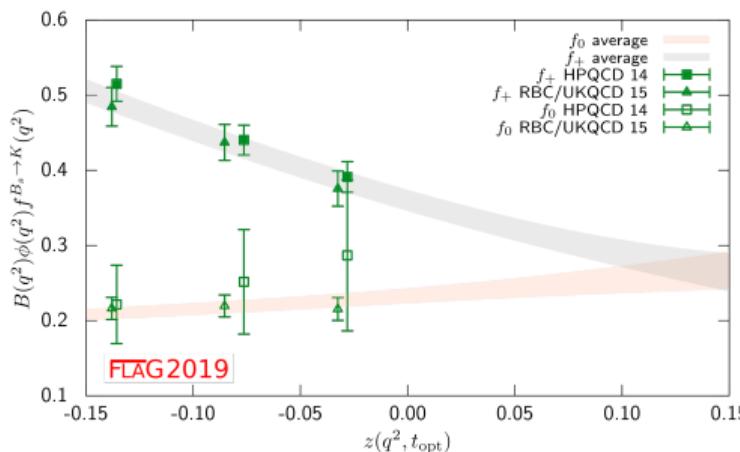
$B_s \rightarrow D_s\ell\nu$
○○○○○○○

more FF
○○○○○○○○

FLAG
○○●○○○○

outlook
○○○

$B_s \rightarrow K\ell\nu$



► New FNAL/MILC [arXiv:1901.02561]

► Please do cite calculations feeding into FLAG averages

introduction
○○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○○

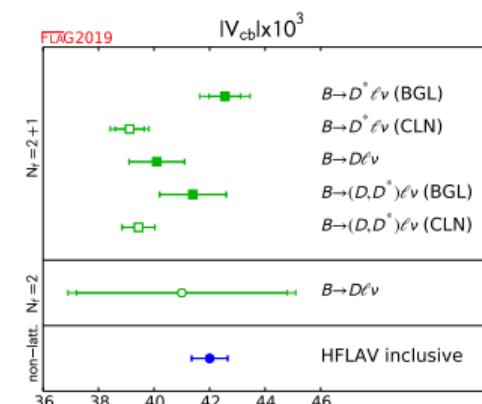
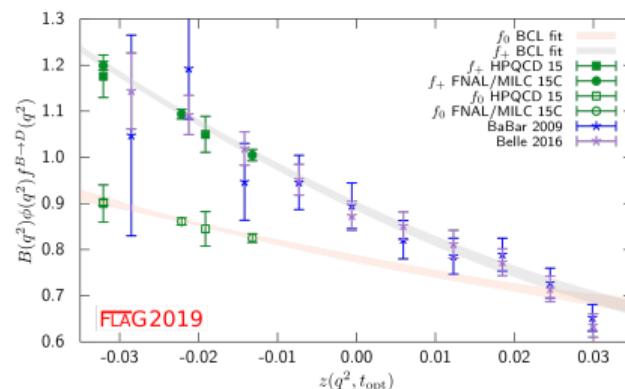
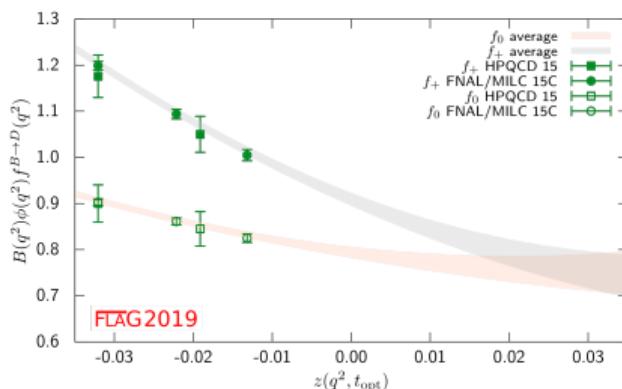
$B_s \rightarrow D_s\ell\nu$
○○○○○○○

more FF
○○○○○○○○○○

FLAG
○○○●○○○

outlook
○○○

$B \rightarrow D\ell\nu$



► Please do cite calculations feeding into FLAG averages

introduction
○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○○○○

$B_s \rightarrow D_s \ell \nu$
○○○○○

more FF
○○○○○○○

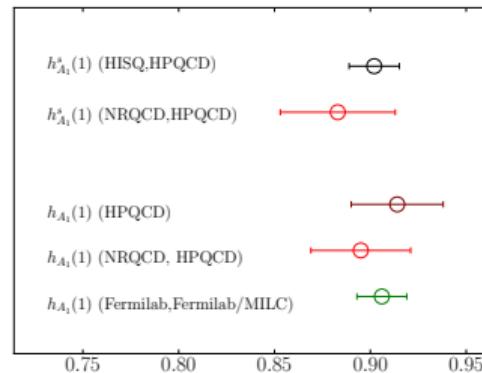
FLAG
○○○●●○○

outlook
○○○

$B \rightarrow D_{(s)}^{(*)} \ell \nu$

Collaboration	Ref.	N_f	publication status	w = 1 form factor / ratio					
				continuum extrapolation	chiral extrapolation	finite volume	renormalization	heavy-quark treatment	
HPQCD 15, HPQCD 17 [614, 616]	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.035(40)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.054(4)(8)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B \rightarrow D}(1)$ 1.033(95)
HPQCD 15, HPQCD 17 [614, 616]	[614, 616]	2+1	A	○	○	○	○	✓	$\mathcal{G}^{B_s \rightarrow D_s}(1)$ 1.068(40)
Atoui 13	[610]	2	A	★	○	★	—	✓	$\mathcal{G}^{B_s \rightarrow D_s}(1)$ 1.052(46)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$ 0.895(10)(24)
FNAL/MILC 14	[612]	2+1	A	★	○	★	○	✓	$\mathcal{F}^{B \rightarrow D^*}(1)$ 0.906(4)(12)
HPQCD 17B	[618]	2+1+1	A	○	★	★	○	✓	$\mathcal{F}^{B_s \rightarrow D_s^*}(1)$ 0.883(12)(28)
HPQCD 15, HPQCD 17 [614, 616]	[614, 616]	2+1	A	○	○	○	○	✓	$R(D)$ 0.300(8)
FNAL/MILC 15C	[613]	2+1	A	★	○	★	○	✓	$R(D)$ 0.299(11)

► New HPQCD $B_s \rightarrow D_s^* \ell \nu$ [arXiv:1904.02046]



► Please do cite calculations feeding into FLAG averages

[FLAG 2019]

introduction
○○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○○○○○

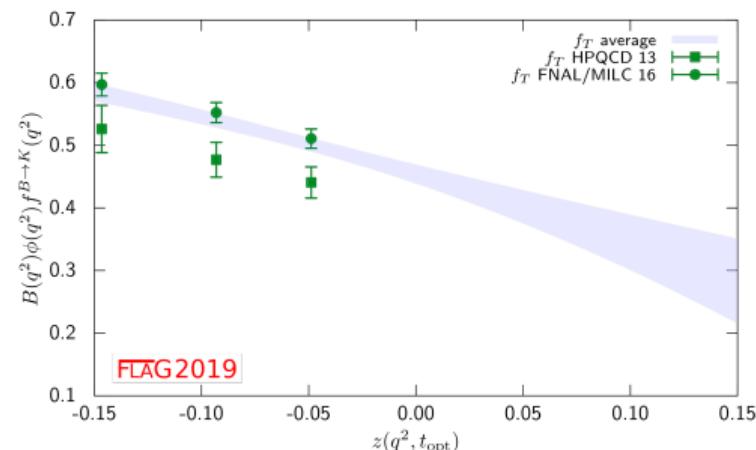
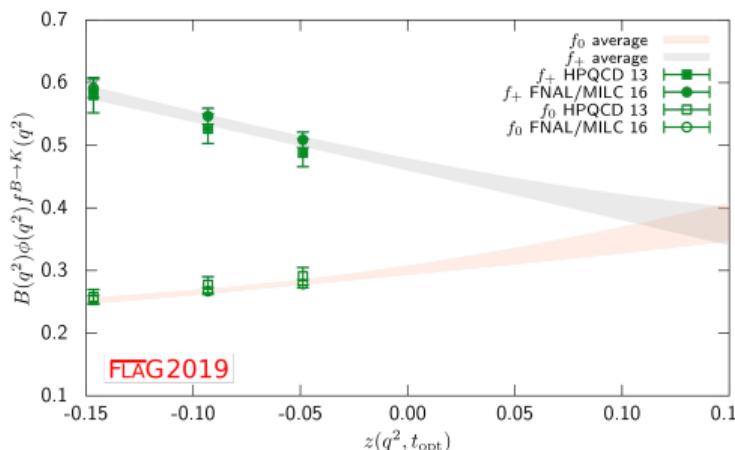
$B_s \rightarrow D_s \ell \nu$
○○○○○○○

more FF
○○○○○○○○○○

FLAG
○○○○○●○

outlook
○○○

$B \rightarrow K \ell^+ \ell^-$ (FCNC)



► Please do cite calculations feeding into FLAG averages

introduction
○○○○○○○

$B_s \rightarrow K \ell \nu$
○○○○○○○

$B_s \rightarrow D_s \ell \nu$
○○○○○

more FF
○○○○○○○

FLAG
○○○○○●

outlook
○○○

Other calculations (likely incomplete)

► FCNC decays with vector final state

→ Seven form factors $B \rightarrow K^* \ell^+ \ell^-$ and $B_s \rightarrow \phi \ell^+ \ell^-$

[Horgan, Liu, Meinel, and Wingate PRD89 (2014) 090501][PoS Lattice2014 (2015) 372]

→ Angular analysis [PRL112 (2014) 212003]

► Exclusive baryonic decays

→ $\Lambda_b \rightarrow \Lambda_c \ell \nu$ and $\Lambda_b \rightarrow p \ell \nu \Rightarrow |V_{cb}|/|V_{ub}|$ [Detmold, Lehner, Meinel, PRD92 (2015) 034503]

→ $\Lambda_b \rightarrow \Lambda_c \tau \nu$ [Datta et al. JHEP08(2017)131]

outlook

introduction
○○○○○○○

$B_S \rightarrow K \ell \nu$
○○○○○○○

$B_S \rightarrow D_S \ell \nu$
○○○○○

more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○●○

Outlook

- ▶ Many calculation for exclusive decays are on it's way
- ▶ Updates will be reported Lattice 2019 (June 16–22, 2019)
- ▶ New ideas to compute inclusive decays using lattice techniques

[Hashimoto PTEP 2017 (2017) 053B03] [Hansen, Meyer, Robaina arXiv:1704.08993]

introduction
○○○○○○○

$B_s \rightarrow K\ell\nu$
○○○○○○○

$B_s \rightarrow D_s\ell\nu$
○○○○○

more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○●○

Outlook: RBC-UKQCD

- ▶ Second (third) entirely independent analysis completed
- ▶ In the final stages to complete $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$ form factor calculation
 - As usual, carefully estimating all systematic uncertainties is tedious
- ▶ Our lattice calculation also includes
 - $B \rightarrow \pi\ell\nu$, $B \rightarrow \pi\ell^+\ell^-$
 - $B \rightarrow K^*\ell^+\ell^-$
 - $B \rightarrow D^{(*)}\ell\nu$
 - $B_s \rightarrow K^*\ell^+\ell^-$
 - $B_s \rightarrow D_s^*\ell\nu$
 - $B_s \rightarrow \phi\ell^+\ell^-$
 - ...
- ▶ Current status $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$:
[arXiv:1903.02100]
- ▶ Future
 - Add $48^3 \times 96$ ensemble with physical pions
- ▶ Parallel efforts: SU(3) breaking ratios
[arXiv:1812.08791]

introduction
○○○○○○○

$B_S \rightarrow K \ell \nu$
○○○○○○○

$B_S \rightarrow D_S \ell \nu$
○○○○○

more FF
○○○○○○○○

FLAG
○○○○○○○

outlook
○○●

Resources for RBC-UKQCD's calculation

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab), skylake cluster (BNL)

RBC qcdcl (RIKEN) and cuth (Columbia U)

UK: ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)