Analyzing AMA data on $48^3 \times 96$ lattices

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The ensemble

- $48^3 \times 96$, MDWF, physical pions, $a^{-1} = 1.730$ GeV, spatial box 5.47 fm
- 1560 thermalized trajectories (configurations 640 - 2200)
  $\rightarrow$ Use 40 configurations [640:40:2200]
- Inverting physical light quarks is expensive
- Deflation/multi-grid methods make it affordable
- Favored to have many sources per configurations
  but smaller set of configurations

$\leadsto$ Not yet the of size Lüscher's master field simulation [Talk Lattice 2017]
$\leadsto$ Not going to restrict to sub-volumes

All-mode Averaging (AMA)

Idea: Reduce costs for inversions by exploiting translational invariance
[Blum, Izubuchi, Shintani PRD88 (2013) 094503][Shintani et al. PRD91 (2015) 114511]

→ Compute many lower precision propagators ("sloppy solves")

→ Compute a few high precision propagators ("exact solves")

→ Correct the result

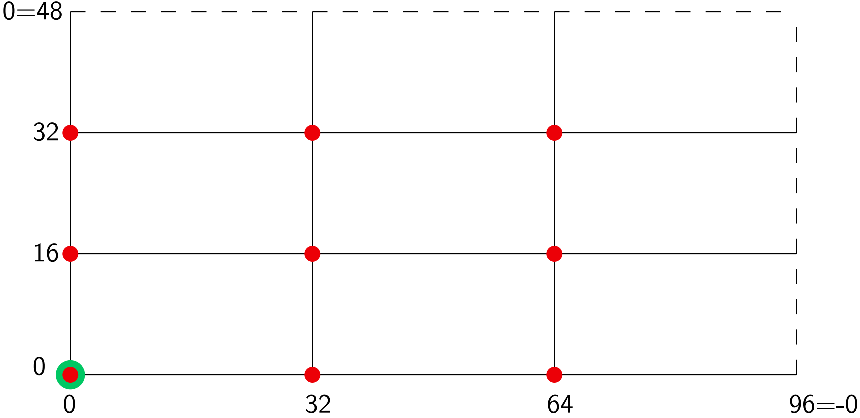
$$\mathcal{O}_G^{(\mathrm{appx}),g} = \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(appx),g}$$

$$\mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \mathcal{O}_G^{(\mathrm{appx})}$$

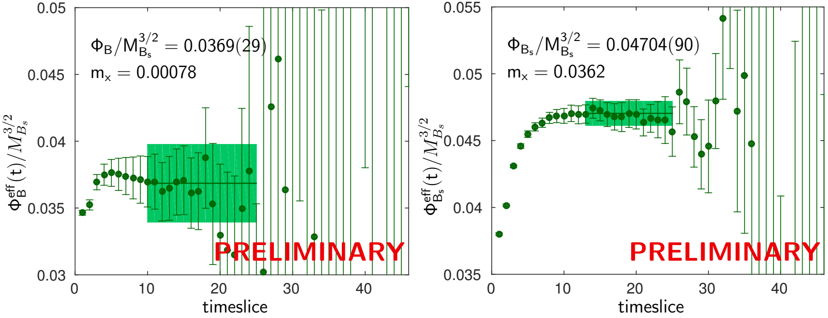
Set-up

- 81 sloppy solves and 1 exact solve at (0,0,0,0)
- evenly distributed on a hypercube; point-source u/d,s; Gaussian source b
- x, y, z = $\{0, 16, 32\}$
- t = $\{0, 32, 64\}$



First Results (Lattice 2015)

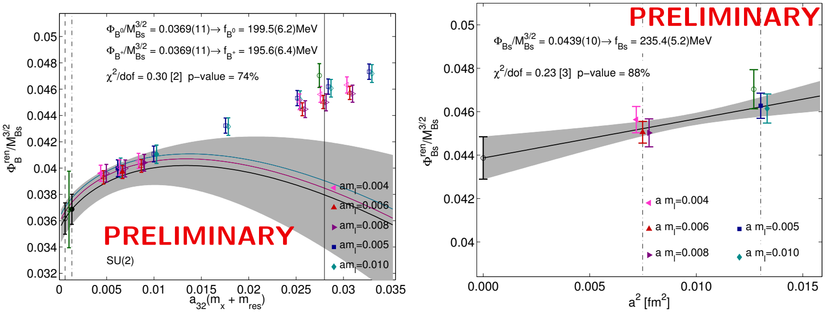
- Decay amplitudes: $Z_{\Phi_{B_q}} \Phi_{B_q} a^{-3/2} / \sqrt{M_{B_q}} = f_{B_{(s)}}$



- Statistical errors too large
- Difficult to carry our correlated fits
  $\rightarrow 1/N = 1/40 \rightsquigarrow$ rather large fluctuations of the variance-covariance matrix
- All analysis carried out using single-elimination jackknife
- Fixed fit interval [13:25] for bottom-strange correlators

Comparison with data at unphysically heavy pion masses

[PRD 91 (2015) 054502]



- 48^3 data points not included in the fit
- Besides other issues, error bars too large

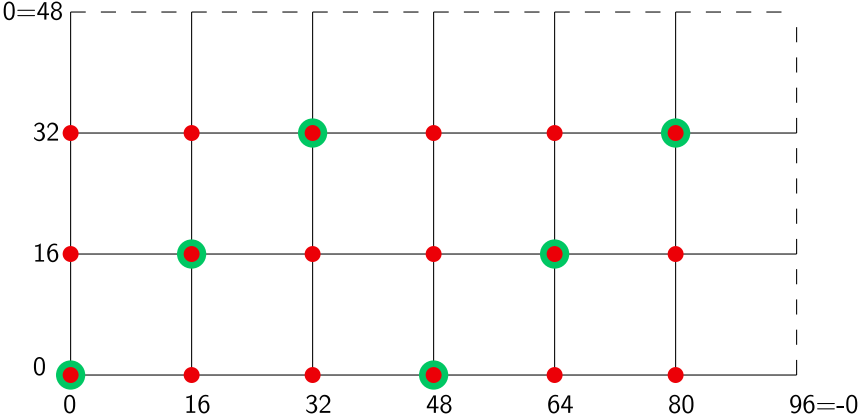
Options for improvement

- Fill-in: calculate on every 20^{th} configuration
→ Issue with autocorrelation between configurations? decorrelated?
→ Requires to compute eigenvectors on more configurations (higher costs)

- More sources per configuration
→ Reuse expensive eigenvectors
→ How independent are the sources?
→ Will that improve the fitting difficulties?

More sources per configuration

- 162 sloppy and 6 exact solves per configuration
- x, y, z = {0, 16, 32}
- t = {0, 16, 32, 48, 64, 80}
- ONLY strange quark propagators generated yet



Looking at data

- When computing 2-point or 3-point correlation functions, we are always performing a spatial sum
  - → Lüscher's sub-volumes may help
  - → Our volume would likely be too small
- How independent are the different time planes?
- At which data should we look?

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- At which data should we look?
  - → Depends on your problem of interest
  - → Keep it simply: look at 2-point correlators

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- At which data should we look?
  - → $B \to \pi \ell \nu$: bottom-light and light-light 2-point correlators
  - → M_B, f_B: bottom-light 2-point correlators
  - ⇝ Worst signal to noise ratio in bottom-light correlators
  - ⇝ Slowest exponential decay in light-light correlators
  - ↯ Insufficient light quark propagators available ↯

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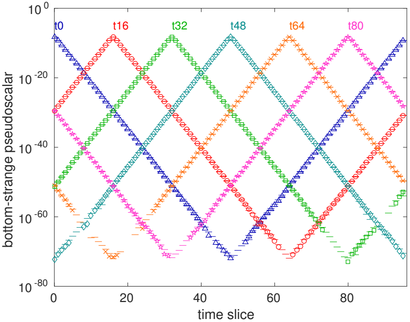
  - ⇝ bottom-strange 2-point correlators readily available

Bottom-strange 2-point function (ps-ps)



- Steep, monotonic exponential decay
- T=96, anti-periodic BC

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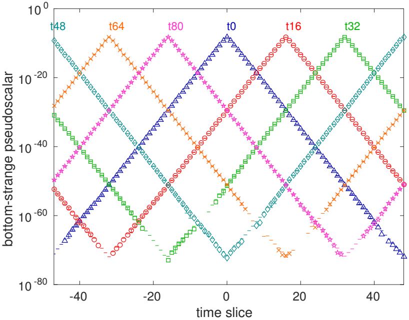
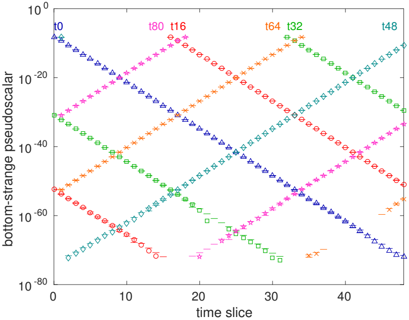


- Steep, monotonic exponential decay
- T=96, anti-periodic BC
- Centered for $t = 0$
  $\rightarrow$ Forward and backward propagation
- Signal has decayed by ~ 10 orders at first "crossing" with another time source

Bottom-strange 2-point function (ps-ps)



- Steep, monotonic exponential decay
- T=96, anti-periodic BC
- Folded at T/2
- Signal region $t_{\text{src}} + [13:25]$

How correlated are the six different time planes?

- Compute AMA values for six time planes i.e. $6 \times (1 \oplus 27)$ sources: $N = 40$; $r, s = \{0, 16, 32, 48, 64, 80\}$, $t = 0, 1, 2, \ldots, 48$

- Mean value

$$\bar{y}_r(t) = \frac{1}{N} \sum_{n=1}^{N} y_r(t, n)$$

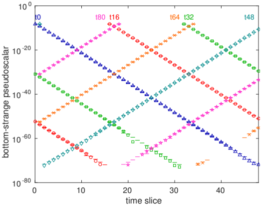
- Variance-covariance matrix

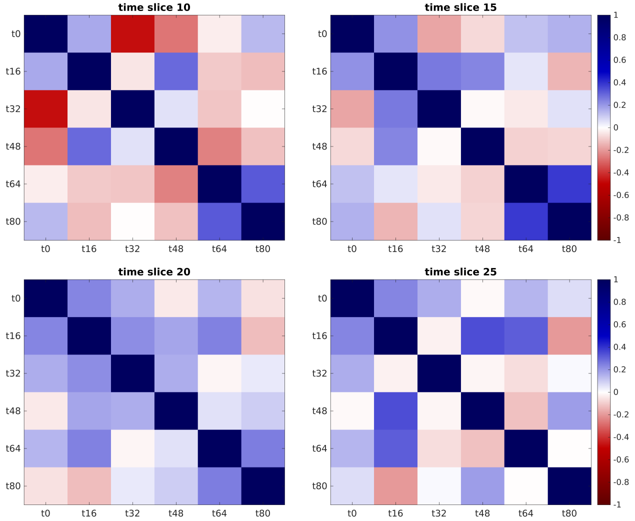
$$V_{rs}(t) = \frac{1}{N(N-1)} \sum_{n=1}^{N} \left(\bar{y}_r(t) - y_r(t, n)\right) \left(\bar{y}_s(t) - y_s(t, n)\right)$$

- Correlation matrix (normalized values from -1, . . . , 1)

$$C_{rs}(t) = \frac{V_{rs}(t)}{\sqrt{V_{rr}(t)}\sqrt{V_{ss}(t)}}$$

Correlations between time planes using $6 \times (1 \oplus 27)$

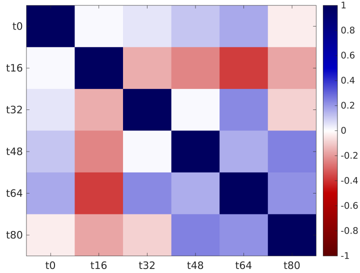




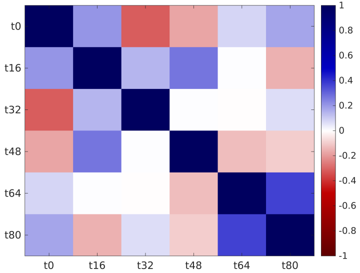
Correlations between the time planes

- $N = 40,\ t = 14$

6×1 exact

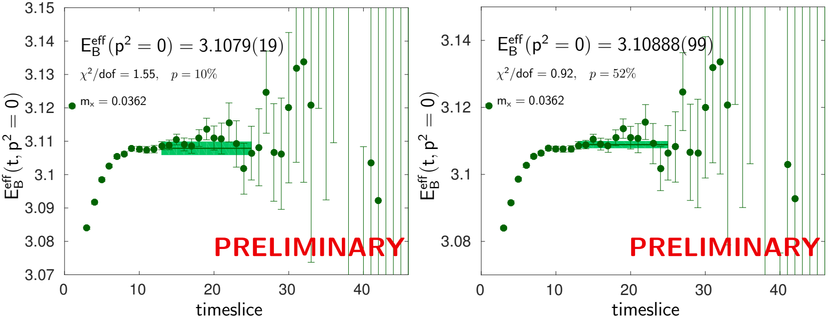


$6 \times (1 \oplus 27)$



Comparison for M_{B_s}

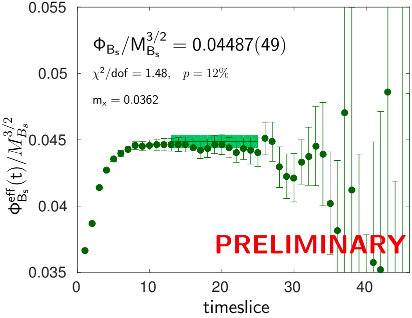
- Average time planes $40 \times (6 \oplus 162)$
- Treat time planes as independent $(40 \times 6) \times (1 \oplus 27)$

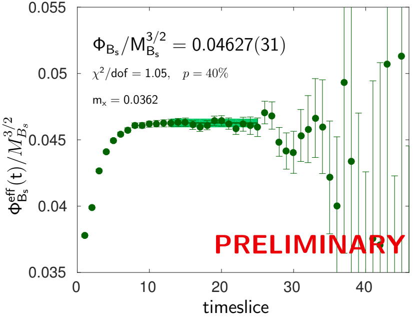


- Central values agree (2015: $E_B^{eff}(t, p^2 = 0) = 3.1037(26), p = 8\%$)
- Treating time planes independently leads to $1/2$ of the statistical error and fit quality improves

Comparison for Φ_{B_s}

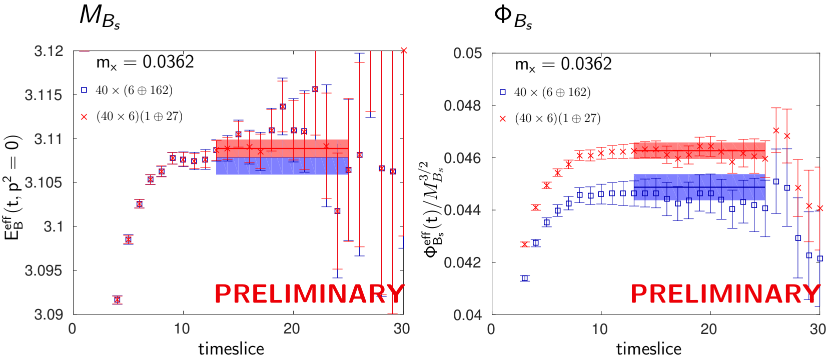
- Average time planes $40 \times (6 \oplus 162)$
- Treat time planes as independent $(40 \times 6) \times (1 \oplus 27)$





- 2015: $\Phi_s/M_{B_s}^{3/2} = 0.04704(90)$
- Central values differ by $\sim$2 sigmas
- Statistical errors are similar; fit quality improved

$[40 \times (6 \oplus 162)]$ vs. $[(40 \times 6) \times (1 \oplus 27)]$



- Data points have similar size errors
- Correlated fit improved

- Why shift in central values???
- Data points have smaller errors
- Correlated fit looks better

Remarks

- Doubling the number of sources reduces statistical uncertainties!

- Different time planes appear to be relatively independent
- Treating time sources as independent, improves correlated fits

- Need to understand shift in decay amplitudes

- Next check correlations for strange-strange 2-point functions
- Try "replica analysis" based on `UWerr` (Γ function method)
- Bootstrap analysis may also be superior given the small sample size

- Will it carry over to pions? ($B \to \pi \ell \nu$)

$\Rightarrow$ Generate more physical light quark propagators!

Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)
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