Heavy Quarks in the RBC/UKQCD lattice phenomenology programme

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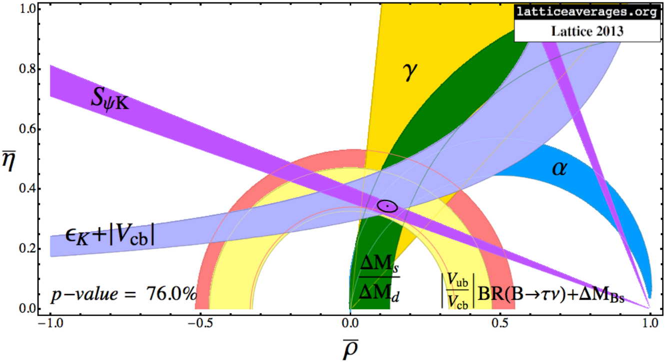
Overview

- Static Heavy Quarks
- Relativistic Heavy Quarks
- Domain-Wall Heavy Quarks

Motivation: Constraining the Standard Model

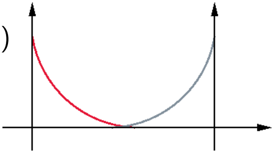
- Determination of CKM matrix elements V_{ub} and V_{cb}
- B-physics provides constraints on the apex of the CKM unitarity triangle
  - B^0–$\overline{B^0}$ mixing
  - V_{ub} and V_{cb} e.g. from $B \to \pi\ell\nu$ and $\overline{B} \to D^*\ell\nu$ form factors
- Experimental results and nonperturbative inputs are needed



http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it, http://www.latticeaverages.org

2+1 flavor domain-wall gauge field configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992], [Shamir 1993]
- Iwasaki gauge action [Iwasaki 1983]
- New ensembles with physical pions and Möbius DWF [Brower et al.2004][Brower et al. 2012]



L	a(fm)	m_l	m_h	M_π	
24	$\approx$ 0.11	0.005,0.010	0.040	$\gtrsim$ 330 MeV	[Allton et al. 2008]
32	$\approx$ 0.08	0.004,0.006,0.008	0.030	$\gtrsim$ 290 MeV	[Y. Aoki et al. 2011]
48	$\approx$ 0.11	0.00078	0.0362	139 MeV	[Blum et al. 2014]
64	$\approx$ 0.08	0.000678	0.02661	139 MeV	[Blum et al. 2014]

- Finer ensemble with $a^{-1} \approx 2.8$ GeV and $M_\pi \approx 200$ MeV in progress

Domain-wall fermions

- Good chiral properties
- Residual chrial symmetry breaking parameterized and controlled by m_{res}
- Strictly unitary: same action for the valence and the sea sector
- Continuum-like χPT expressions

- Algorithmic progress:
  - Möbius domain-wall fermion allow to reduce L_s by fixed m_{res} [Brower et al.2004][Brower et al. 2012]
  - HDCG significantly reduces costs in the valence sector [Boyle 2014]

- Why use DWF for heavy quarks?
  - Include charm sea-quarks
  - Simplified renormalization and improved discretization errors

Challenges for D- and B-physics on the lattice

- The mass of the b-quark introduces another scale
  - Light quark masses: $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_s = 95$ MeV
  - Mass of the c-quark: $m_c = 1.28$ GeV
  - Mass of the b-quark: $m_b = 4.18$ GeV
- Today's lattices have an inverse lattice spacing of $a^{-1} \approx 1.7 \ldots 3$ GeV
  - $am_c \lesssim 1$ but $am_b > 1$
  - Direct simulations of c-quarks become feasible
  - b-quarks require extrapolation or an effective action (e.g. HQET (static), NRQCD, Fermilab or RHQ action)

RBC-UKQCD's Heavy Quark Projects

	HQET (static)	relativistic (RHQ)	domain-wall
action	Eichten-Hill with link smearing	anisotropic Wilson-clover	Möbius DWF
heavy quarks	infinitely heavy, correction $1/m_H$	NP tuned using M_{B_s}, i.e. physical m_b	simulate near m_c, extrapolate to m_b
matching	perturbative NP in progress	mostly nonperturbative	nonperturbative
physics	f_B, f_{B_s}, $B^0-\overline{B^0}$	f_B, f_{B_s}, $g_{B^*B\pi}$, $B^0-\overline{B^0}$ $B\to\pi\ell\nu$, $B_s\to K\ell\nu$ rare B-decays	f_D, f_{D_s}, $D^0-\overline{D^0}$ f_B, f_{B_s}, $B^0-\overline{B^0}$ decays
references	PRD82(2010)01405 arXiv:1406.6192	PRD86 (2012) 116003 PoS(Lattice2013)408 arXiv:1404.4670 arXiv:1501.05373	arXiv:1412.6206 arXiv:1501.00660 PoS(Lattice2014)380

Static Heavy Quarks

HQET (static) [Y. Aoki et al. 2014]

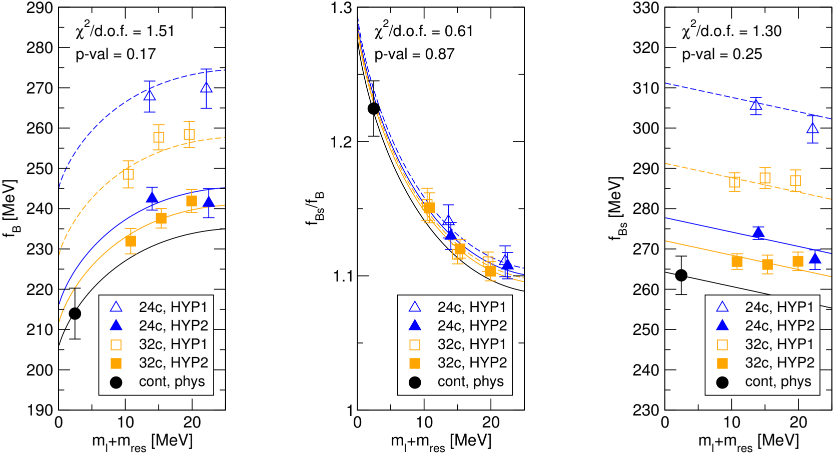
(Tomomi Ishikawa, Yasumichi Aoki)

- Effective action for b-quarks in the limit $m_b \to \infty$ [Eichten and Hill 1990]
  i.e. neglect corrections of $\mathcal{O}(1/m_b)$
- Static quark propagator is given by a product of gauge links
- Improve poor signal-to-noise ratio applying link smearing [Della Morte et al. 2005]
  HYP1: (0.75,0.6,0.3) [Hasenfratz and Knechtli 2001]
  HYP2: (1.0,1.0,0.5) [Della Morte et al. 2005]
- Two-step matching: lattice HQET → continuum HQET → continuum QCD
- Matching factors perturbatively computed at 1-loop
- Gaussian smeared sources/sinks

- Physics program
  - Decay constants: f_B, f_{B_s}, f_{B_s}/f_B
  - B^0–$\overline{B^0}$ mixing: ξ, $f_B\sqrt{\hat{\mathcal{B}}_B}$, $f_{B_s}\sqrt{\hat{\mathcal{B}}_{B_s}}$

Chiral- and continuum extrapolation for f_B, f_{B_s}, and f_{B_s}/f_B

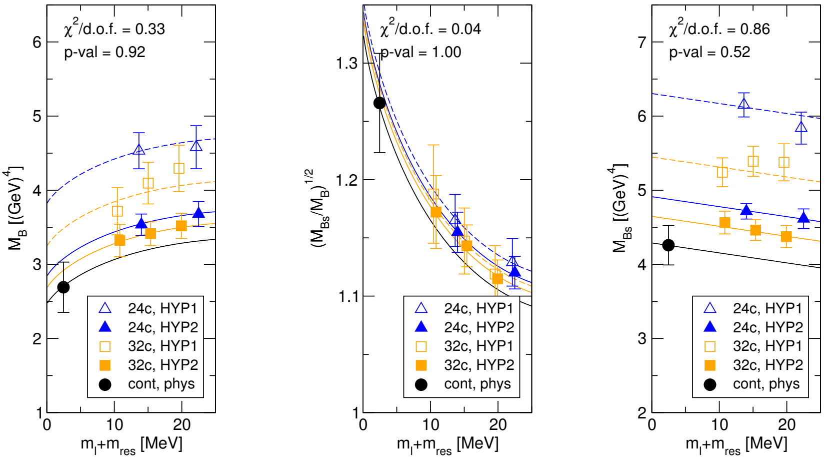
[Y. Aoki et al. 2014]



- NLO SU(2) HMχPT ▸ $g_b^{SU(2)} = 0.449$ ▸ $f^{SU(2)} = 110$ MeV ▸ $\Lambda_\chi = 1$ GeV
- Statistical errors only

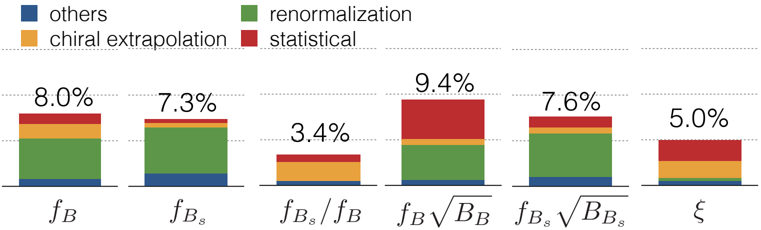
Chiral- and continuum extrapolation for $\mathcal{M}_B$, $\mathcal{M}_{B_s}$, and ξ

[Y. Aoki et al. 2014]



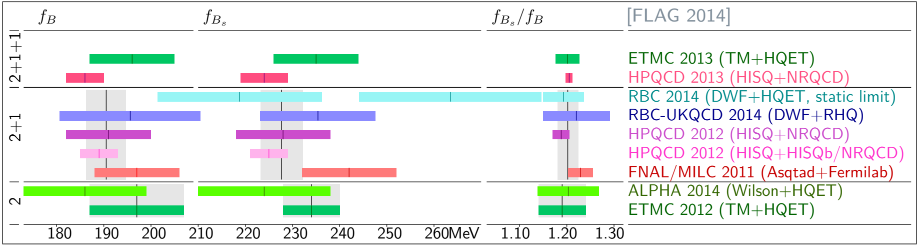
- NLO SU(2) HMχPT ▸ $g_b^{SU(2)} = 0.449$ ▸ $f^{SU(2)} = 110$ MeV ▸ $\Lambda_\chi = 1$ GeV
- Statistical errors only

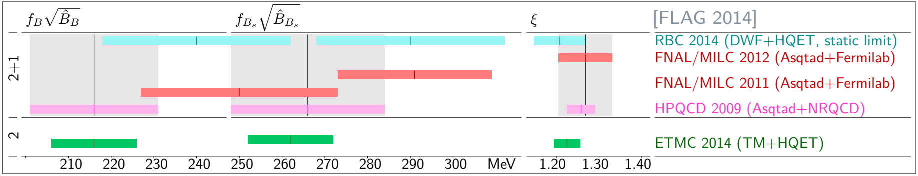
Error budget [Y. Aoki et al. 2014]



- Reduce statistical errors using AMA [Blum et al. 2012]
- Reduce chiral extrapolation error with new ensembles with $M_\pi = 139$ MeV
- Reduce renormalization error with a nonperturbative method (Piotr Korcyl)
- Account for $1/m_b$ corrections

Comparison with other results



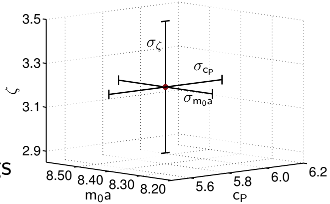


Relativistic Heavy Quarks

Relativistic Heavy Quark (RHQ) action

[Christ, Li, and Lin 2007]
[Lin and Christ 2007]

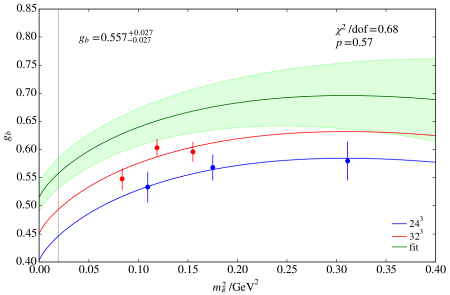
- Builds upon Fermilab approach [El-Khadra et al. 1997]
- Close relation to Tsukuba formulation [S. Aoki et al. 2003]
- Treat heavy quark mass to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$ with resulting errors of $O(\vec{p}^2 a^2)$
- Similar size discretization errors in heavy-light quantities as in light-light
- Well defined continuum limit
- Requires (perturbative) mixed-action renormalization factors
  - 1-loop results (Christoph Lehner)
- Tune 3 parameters of the anisotropic clover action nonperturbatively using the B_s system [PRD86 (2012) 116003]
- Validate by computing $b\bar{b}$ masses and splittings

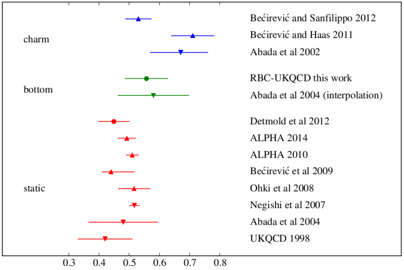


Coupling constant $g_{B^* B\pi}$ [PoS(Lattice2013)408]

(Ben Samways, Jonathan Flynn)

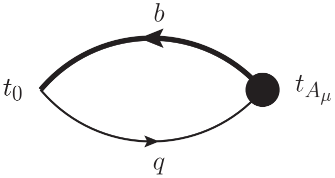
- Strong coupling $g_{B^* B\pi}$ parameterizes $\langle B\pi | B^* \rangle$
- Related to leading order LEC $g_b = g_{B^* B\pi} \cdot f_\pi / (2M_B)$ of HMχPT
- g_b important for chiral extrapolations of f_B, B_B, ξ, $f_+^{B\pi}$, $f_0^{B\pi}$, ...
- First determination at physical b-quark mass
- Not accessible experimentally





B-meson decay constant [arXiv:1404.4670]

(Ruth Van de Water, OW)



- Use point-source light quark and generate Gaussian smeared-source heavy quark
- Compute on the lattice Φ_{B_q}

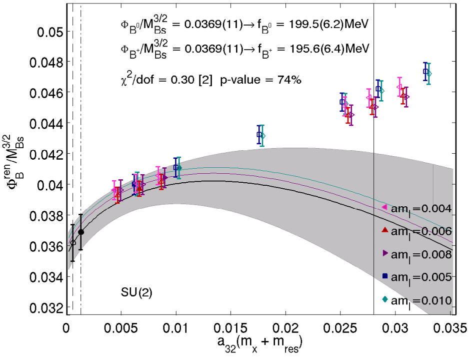
$$f_B = \Phi^{\text{ren}}_{B_q} \cdot a_{32}^{-3/2} / \sqrt{M_{B_q}}$$

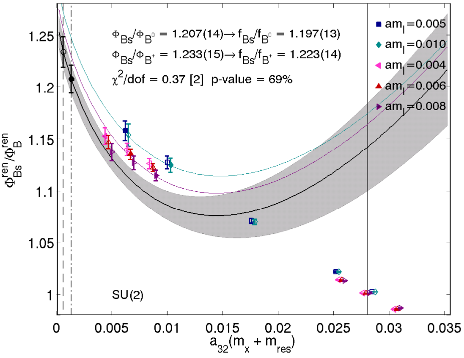
- Improve axial current at 1-loop ($O(\alpha_S a)$, perturbatively computed coefficient)
- Use mostly-nonperturbative renormalization [El-Khadra et al. 2001]

$$Z_V^{bl} = \varrho^{bl} \cdot \sqrt{Z_V^{bb} Z_V^{ll}}$$

- Coefficients computed at 1-loop results (Christoph Lehner)
- Compute dimensionless ratios over $M_{B_s}^{3/2}$ to avoid explicit a-dependence

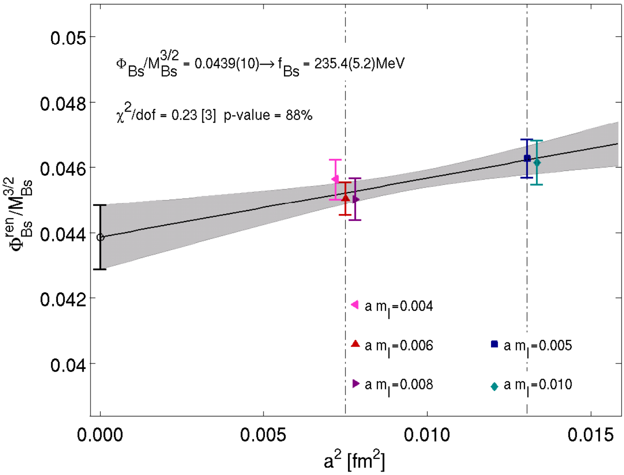
Chiral-continuum extrapolation of f_B and f_{B_s}/f_B [arXiv:1404.4670]





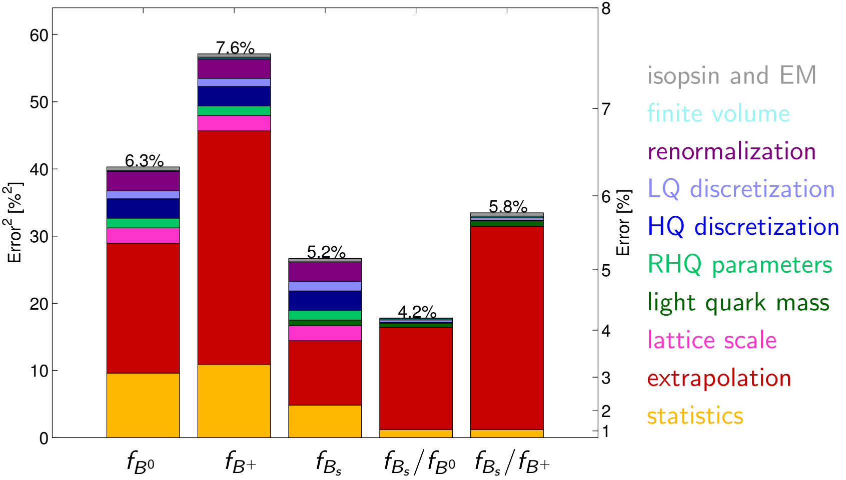
- NLO SU(2) HMχPT to data with unitary M_π
- Only data points with filled symbols included in the fit ($M_\pi \lesssim 425$ MeV)
- $g_{B^*B\pi} = 0.57(8)$ [PoS(Lattice2013)408] ▸ $f_\pi = 130.4$ MeV PDG ▸ $\Lambda_\chi = 1$ GeV
- Statistical errors only

Continuum extrapolation of f_{B_s} [arXiv:1404.4670]



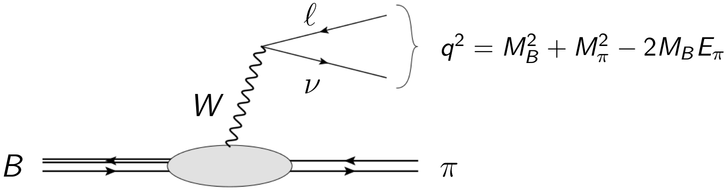
- Data for Φ_{B_s} show no sea-quark mass dependence
- Average data at same lattice spacing
- Assume a^2 scaling to remove LQ and gluon discretization errors
- Statistical errors only

Graphical error budget [arXiv:1404.4670]



Exclusive semileptonic decays: example $B \to \pi\ell\nu$ [arXiv:1501.05373]

(Taichi Kawanai)



- Conventionally parameterized by

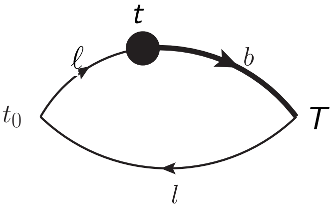
$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_B^3}\left[\left(M_B^2+M_\pi^2-q^2\right)^2 - 4M_B^2M_\pi^2\right]^{3/2}\times\left|f_+(q^2)\right|^2\times\left|V_{ub}\right|^2$$

experiment known nonperturbative input CKM

- Allows determination of CKM matrix element $|V_{ub}|$

$B \to \pi\ell\nu$: computation of lattice form factors $f_\parallel$ and $f_\perp$

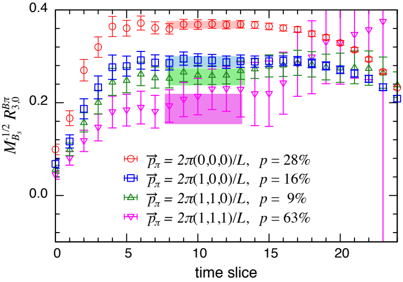
(which are linearly related to f_+ and f_0) [arXiv:1501.05373]

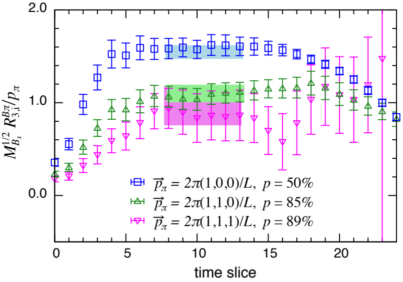


$$f_\parallel = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_\perp = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

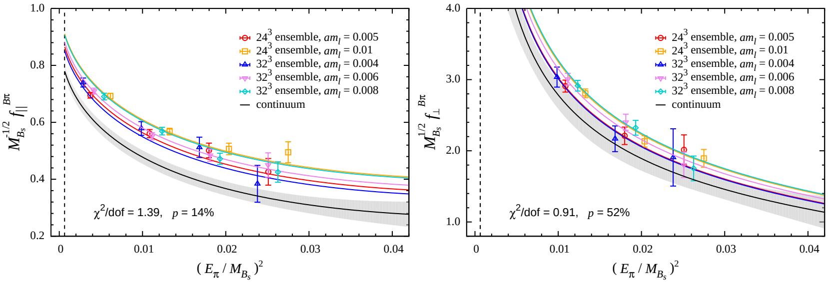
$$R_\mu^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{C_2^\pi(t)C_2^B(T-t)}\sqrt{\frac{2E_\pi}{e^{-E_\pi t}e^{-M_B(T-t)}}}$$





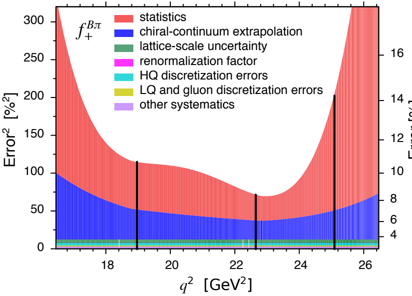
Chiral-continuum extrapolation using SU(2) hard-pion χPT

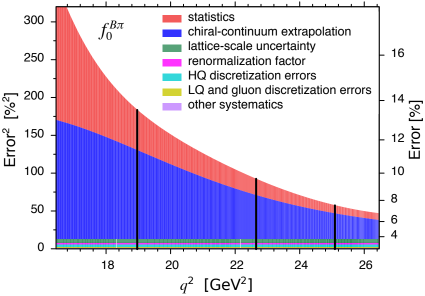
[arXiv:1501.05373]



- Extract $f_{||}$ and $f_{\perp}$ for three different q^2 values
- Convert results to f_+ and f_0
- Estimate all systematic errors and add in quadrature

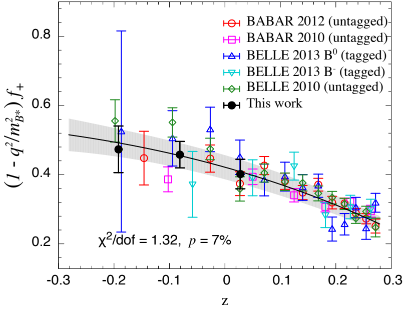
Graphical error budgets [arXiv:1501.05373]

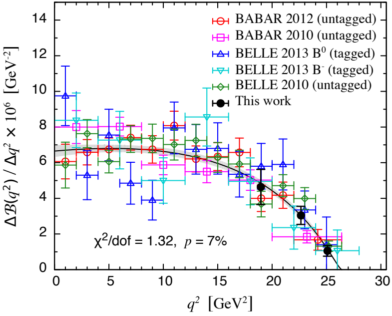




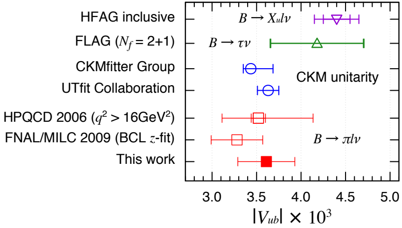
Total:	10.6%	8.4%	14.3%	13.6%	9.6%	7.6%

Determination of $|V_{ub}|$ [arXiv:1501.05373]





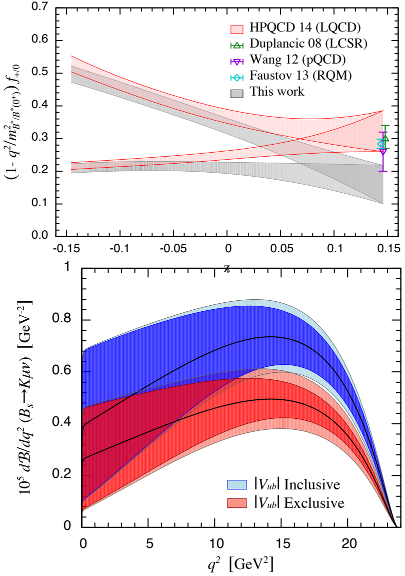
- $|V_{ub}| = 3.61(32) \cdot 10^{-3}$



$B_s \to K\ell\nu$ [arXiv:1501.05373]

- Calculation analogous to $B \to \pi\ell\nu$, but smaller uncertainties
- Lattice simulations agree for simulated data, differ by less than 2σ for extrapolated $q^2 = 0$

- Given an experimental measurement of branching fractions at $q^2 \gtrsim 13$ GeV one may distinguish between curves corresponding to $|V_{ub}|_{\text{excl.}}$ and $|V_{ub}|_{\text{incl.}}$



Work in progress

- Improve chiral-continuum extrapolation by adding new ensembles with physical pions / finer lattice spacing
- Reduce statistical errors using AMA [Blum eta al. 2012]

- B^0–$\overline{B^0}$ mixing

- Rare B-decays (Edwin Lizarazo)

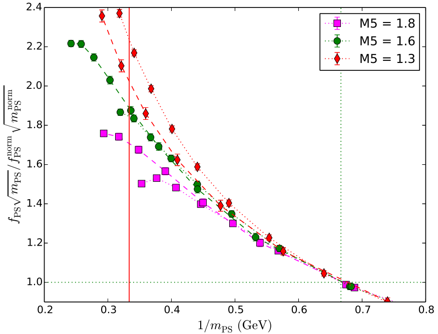
Domain-Wall Heavy Quarks

Heavy domain-wall fermions

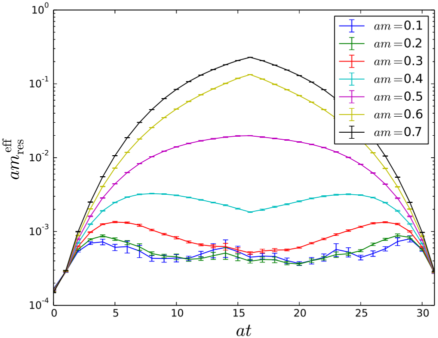
[Jüttner et al. 2015]

(Tobias Tsang, Andreas Jüttner)

- Exploration of Möbius DWF parameters: am_q, M_5, and L_s
  - Quenched study on small lattices with $a^{-1} = 2 \ldots 5.6$ GeV and $L \approx 1.6$ fm
  - Monitoring "η_s," "D_s," and "η_c," quantities



- Optimal DW height $M_5 = 1.6$

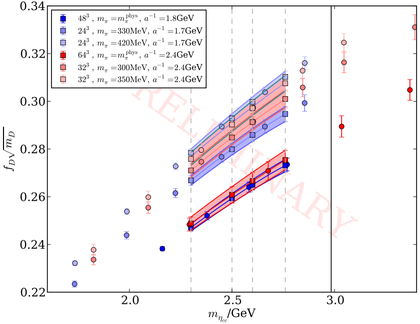


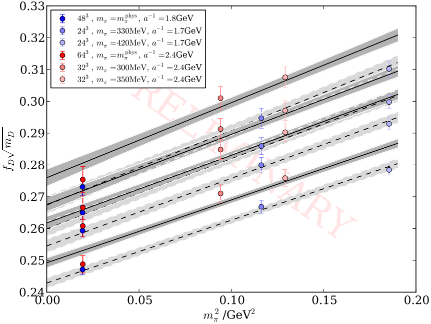
- Unphysical behavior for $am_q > 0.4$

Charm physics on dynamical 2+1 flavor DWF ensembles

(Andreas Jüttner) [Jüttner et al. 2015]

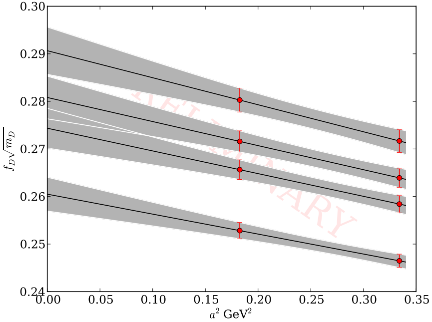
- Simulate several "c"-quarks per ensemble and then interpolate
- Möbius DWF: $am_q \leq 0.45$, $M_5 = 1.6$, $L_s = 12$
- Use Z_2 noise sources on every 2nd time plane and one-end-trick

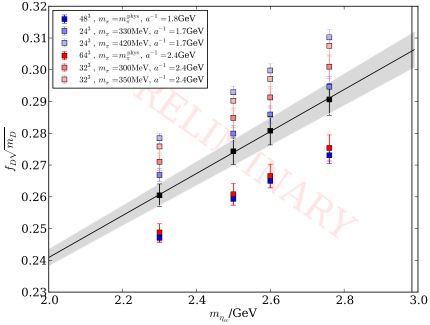




- First interpolate each ensemble to common reference masses "η_c" ($\eta_c = 2.3$, 2.5, 2.6, 2.76 GeV)
- Next interpolate each "η_c" reference mass to the physical pion mass at fixed lattice spacing

Charm physics on dynamical 2+1 flavor DWF ensembles





- Perform a continuum extrapolation for each "η_c" reference mass
- Finally interpolate results to physical c-quark mass
- Preliminary data – incomplete statistics

- Ensemble with third, finer lattice spacing will significantly improve control of continuum extrapolation

From charm to bottom

(Ava Khamseh, Peter Boyle, Andreas Jüttner, OW)

- Charm and heavier than charm-quarks can be simulated using DWF
- Simulations of domain-wall bottom-quarks are not (yet) feasible

- Options
  - Extrapolate from the charm region to the b-quark mass à la ETMC's ratio method [Blossier et al. 2010]
  - Interpolate from charm region to the static limit
  - Interpolate ratios between charm region and static limit

- Work in progress
  - Improve on statistical uncertainties by using HDCG [Boyle 2014]
  - Carry out nonperturbative renormalization

Outlook

- Short range: RHQ action provides fastest access to important B-physics quantities like semi-leptonic form factors
  - First results are posted to the arXiv
  - Improvements on existing and study of other quantities is under way
- Long range: heavy DWF look promising to achieve very precise results
  - Parameters for "good" DWF c-quarks explored; runs in progress
  - Extrapolation to b-quarks is studied
  - Possible comeback of the static action: turn extrapolation into an interpolation!
- Production of next generation 2+1+1 DWF ensembles has started!
  - Dynamical charm quarks; finer lattice spacing; huge volumes

Appendix

Error budget [arXiv:1404.4670]

	f_{B^0}(%)	f_{B^+}(%)	f_{B_s}(%)	f_{B_s}/f_{B^0}(%)	f_{B_s}/f_{B^+}(%)
statistics	3.1	3.3	2.2	1.1	1.1
chiral-continuum extrapolation	4.4	5.9	3.1	3.9	5.5
lattice-scale uncertainty	1.5	1.5	1.5	0.1	0.1
l- and s-quark mass uncert.	0.1	0.1	0.9	0.8	0.9
RHQ parameter tuning	1.2	1.2	1.2	0.1	0.1
HQ discretization errors	1.7	1.7	1.7	0.3	0.3
LQ and gluon discr. errors	1.1	1.1	1.2	0.6	0.6
renormalization factor	1.7	1.7	1.7	0.0	0.0
finite volume	0.4	0.5	0.0	0.5	0.5
isospin-breaking and EM	0.7	0.7	0.7	0.1	0.7
total	6.3	7.6	5.2	4.2	5.8

$B \to \pi \ell \nu$: relating form factors f_+ and f_0 to $f_\parallel$ and $f_\perp$

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_0 \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

- Using the B-meson rest frame we compute on the lattice

$$f_\parallel(E_\pi) = \langle \pi | V^0 | B \rangle / \sqrt{2 M_B} \quad \text{and} \quad f_\perp(E_\pi) p_\pi^i = \langle \pi | V^i | B \rangle / \sqrt{2 M_B}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 - M_\pi^2} \left[(M_B - E_\pi) f_\parallel(E_\pi) + (E_\pi^2 - M_\pi^2) f_\perp(E_\pi) \right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_B}} \left[f_\parallel(E_\pi) + (M_B - E_\pi) f_\perp(E_\pi) \right]$$