B-meson decay constants, B^0-$\overline{B^0}$-mixing and the $B^*B\pi$ coupling with domain-wall light quarks and relativistic heavy quarks

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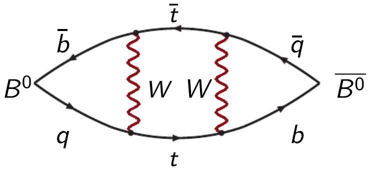
Phenomenological Importance

- $B - \bar{B}$-mixing allows us to determine CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

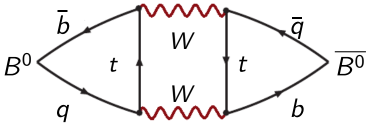
$$\left.\begin{array}{r} |V_{td}^* V_{tb}| \text{ for } B_d-\text{mixing} \\ |V_{ts}^* V_{tb}| \text{ for } B_s-\text{mixing} \end{array}\right\} \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

- Non-perturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$



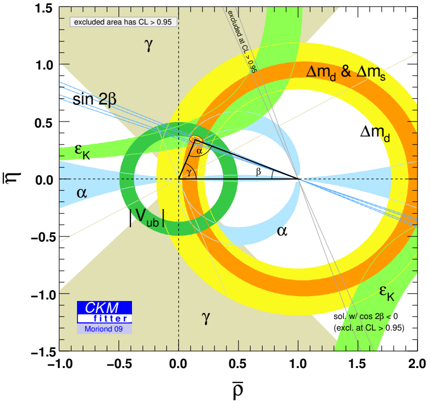
- CKM matrix elements are extracted by

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



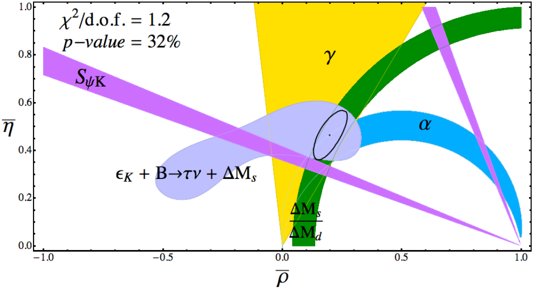
B-meson mixing and the Unitarity Triangle

- The apex of the unitarity triangle is constrained by the ratio of B_s to B_d oscillation frequencies (Δm_q)
- Δm_q is experimentally measured to better than a percent [BABAR, Belle, CDF]
- Dominant error comes from the uncertainty on the lattice QCD calculation of the ratio ξ ($\sim 3\%$)
- A precise determination is needed to help constrain physics beyond the Standard Model

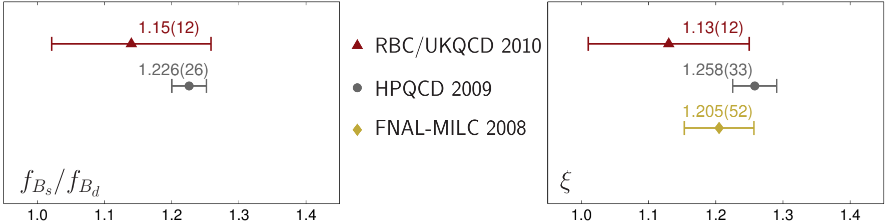


Unitarity Fit without Semileptonic Decays

- A unitarity fit without V_{ub} or V_{cb} is possible [Lunghi and Soni]
- Avoids 1-2 σ tension between inclusive and exclusive determinations of both V_{ub} and V_{cb}
- Requires precise determination of f_B (and also of $B \to \tau\nu$ and ΔM_s)



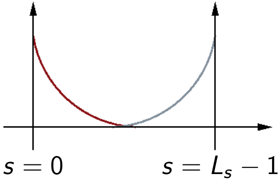
Lattice Calculations of *B*-meson mixing Parameters



- HPQCD and FNAL-MILC result both based on the asqtad-improved staggered ensembles generated by MILC
- RBC/UKQCD result only exploratory study computed on 16^3 lattices and using static approximation for the *b*-quarks
- This project aims for an independent cross-check at high precision using domain-wall light-quarks and relativistic heavy quarks performing also precision computations of decay constants and $B^*B\pi$ coupling
- Project started 2009/10 and we ask for time to continue in 2010/11

Light Quark and Gluon Action

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992, Shamir 1993]
  - Five dimensional formulation with an approximate chiral symmetry
  - Left-handed modes are bound to 4-d brane at $s = 0$, right-handed modes to a 4-d brane at $s = L_s - 1$
  - Overlap exponentially suppressed
  - Renormalization simplified due to reduced operator mixing

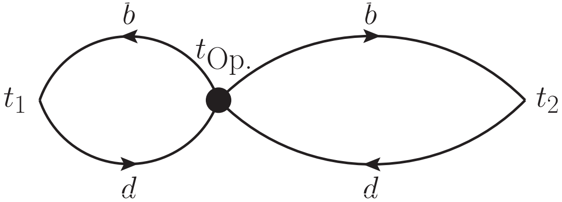


- Iwasaki gauge action [Iwasaki 1983]
  - Improves chiral symmetry and reduces residual quark mass when combined with domain-wall sea quarks [Y. Aoki et al. 2004]

Relativistic Heavy Quark Action for the *b*-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the *b*-quarks in 2-point and 3-point correlation functions
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie] by tuning all parameters of the clover action non-perturbatively
- Matching of lattice action to continuum through $\mathcal{O}(pa)$
  - Errors will be of $\mathcal{O}(a^2 p^2)$
  - Heavy quark is mass treated to all orders in $m_b a$
    $\Rightarrow$ coefficient of the $\mathcal{O}(a^2 p^2)$ error is a function of $m_b a$
  - This function is bounded to be $\leq \mathcal{O}(1)$ [El Khadra, Kronfeld, Mackenzie]
  - Heavy-light spectrum quantities can be computed with discretization errors of the same order as in light-light quantities

$B^0 - \overline{B^0}$ mixing matrix element calculation



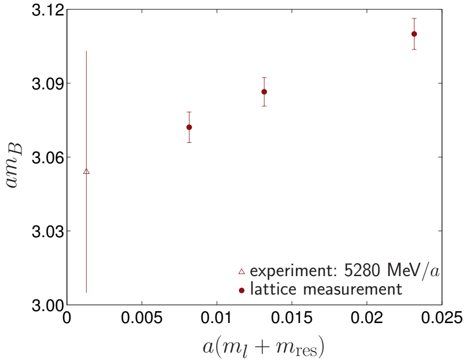
- Location of four-quark operator is fixed
- Location of B-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating form operator location
- Propagators can be used for B- and $\overline{B}$-meson
- Project out zero-momentum component using a Gaussian sink

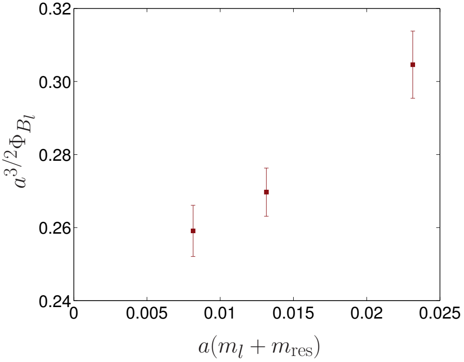
Operator Improvement and Matching

- Rotate b-quark at the source to reduce discretization errors in the heavy-light current and the four-fermion operator
- Compute rotation parameter d_1 at tree-level in tadpole-improved lattice PT (improving operator to $\mathcal{O}(\alpha_s ap)$)
- Renormalization factors for matching of lattice operators to continuum operator are computed using 1-loop tadpole-improved lattice PT (truncation errors $\mathcal{O}(\alpha_s ap)$)
- Only one other operator at $\mathcal{O}(1/m_b)$ mixes with desired operator (at this order)
- For ratio ξ much of the perturbative truncation error should cancel
  Phenomenologically most important quantity should be most reliable

Status of Computation and Preliminary Results

- First focused at generating domain-wall light quark propagator: 24^3 ($a \approx 0.11$fm) and one ensemble on 32^3 ($a \approx 0.08$ fm) completed
- Now working on computation of 2-point and 3-point functions
- Point-Point correlators on 24^3 ensembles
- Significantly improved plateau quality over exploratory study on 16^3 (static heavy quarks)

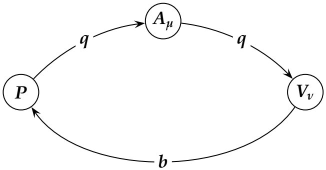




Importance of $B^*B\pi$ Coupling

- Largest systematic uncertainty in many lattice calculations of B-meson quantities is chiral and continuum extrapolation
- Reduce systematic error with improved determination of the $B^*B\pi$ coupling
- $B^*B\pi$ coupling is defined from the matrix element of the light quark axial vector current between B and B^* states
- Determines the leading interaction in the chiral Lagrangian for heavy-light B-mesons
- Enters χPT expressions as coefficient of chiral logarithms for the ratio of $B^0-\overline{B^0}$ mixing matrix elements or form factors of semileptonic decays $B\to\pi$ and $B\to D^{(*)}$
- But, phenomenologically only poorly known; uncertainty of $\mathcal{O}(40\%)$

Computing $g_{B^* B\pi}$



- Reuse the computationally expensive domain-wall light quark propagators for contraction with newly generated sequential propagators
- Result can be non-perturbatively renormalized;
  the nonperturbative renormalization factor for axial current (Z_A) is known for domain-wall fermions

Proposed Generation of DWF Light Quark Propagators

a(fm)	L	m_l	m_s	m_{val}^{dwf}	# configs 2009/10	# configs 2010/11
≈ 0.08	32	0.004	0.030	0.004, 0.006, 0.008, 0.025, 0.030	300	300
≈ 0.08	32	0.006	0.030	0.004, 0.006, 0.008, 0.025, 0.030	450	450
≈ 0.08	32	0.008	0.030	0.004, 0.006, 0.008, 0.025, 0.030	300	300
≈ 0.11	24	0.005	0.040	0.005, 0.01, 0.02, 0.03, 0.04	800	800
≈ 0.11	24	0.010	0.040	0.005, 0.01, 0.02, 0.03, 0.04	850	850
≈ 0.11	24	0.020	0.040	0.005, 0.01, 0.02, 0.03, 0.04	350	350

Proposed Generation of Sequential DWF Propagators

≈ 0.08	32	0.004	0.030	0.004	600
≈ 0.08	32	0.006	0.030	0.006	900
≈ 0.08	32	0.008	0.030	0.008	600
≈ 0.11	24	0.005	0.040	0.005	1600
≈ 0.11	24	0.010	0.040	0.01	1700
≈ 0.11	24	0.020	0.040	0.02	700

32^3 $a = 0.08$ fm domain-wall propagators	3.063 $\times 10^6$ jpsi core-hours
24^3 $a = 0.12$ fm domain-wall propagators	1.322 $\times 10^6$ jpsi core-hours
32^3 $a = 0.08$ fm clover propagators	0.076 $\times 10^6$ jpsi core-hours
24^3 $a = 0.12$ fm clover propagators	0.092 $\times 10^6$ jpsi core-hours
32^3 $a = 0.08$ fm sequential domain wall propagators	1.709 $\times 10^6$ jpsi core-hours
24^3 $a = 0.12$ fm sequential domain wall propagators	1.162 $\times 10^6$ jpsi core-hours
2-point and 3-point correlators, code testing and analysis	0.742 $\times 10^6$ jpsi core-hours
Total	8.166 $\times 10^6$ jpsi core-hours

- Majority of time devoted to domain-wall propagator generation
- All domain-wall propagators are saved on tape
- Preference to continue running on Fermilab clusters
- Would like to retain rights to use these propagators for D-meson decay constants and beyond the Standard Model contributions to $B^0 - \overline{B^0}$ mixing

Conclusion

- This project aims for a precise determination of neutral B-meson mixing parameters, decay constants and the $B^* B\pi$ coupling
  - Using $2 + 1$ flavor dynamical domain-wall light quarks
  - Nonperturbatively tuned relativistic heavy quarks
  - Computation uses two lattice spacings, multiple quark masses, and heavy-meson chiral perturbation theory
- Fulfills one of the key goals in flavor physics of USQCD [2007 white paper]
- Result will place an important constraint in the quark flavor sector when used in unitarity triangle analysis

Webpage

http://quark.phy.bnl.gov/~witzel/ (temporary)
http://rbc.phys.columbia.edu/USQCD/ (future)

Question from the SPC:

> [T]his project will lead to determinations of ξ, decay constants and bag parameters using different quark actions than in the past, thus providing valuable cross checks of lattice results. That being said, it would be useful to the SPC to see an estimate of what the final error budget would look like and how that compares with what is already available in the literature.

Tentative Error Budget for ξ

	FNAL-MILC	HPQCD	DWF+RHQ
statistics	3.1%	} 2.0% (statistics + chiral extrapolation)	$\lesssim$ 3%
chiral extrapolation	2.1%		$\sim$2%
uncertainty in $g_{B^* B\pi}$	0.3%	1.0%	$\lesssim$ 1%
renormalization factors	$<$ 0.5%	0.7%	$\lesssim$ 2%
scale and quark mass uncertainties	0.7%	1.0%	$\lesssim$ 1%
finite volume error	$<$ 0.1%		$\lesssim$ 0.5%
relativistic correction		0.4%	
(heavy-quark) discretization	0.2%	0.3%	$\lesssim$ 1%
total	4.3%	2.6%	$\lesssim$ 4%

- Conservative estimate based on comparison with static result and the work of other collaborations