# $B$-meson decay constants and $B^0-\overline{B^0}$-mixing with domain-wall light and relativistic heavy quarks

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http://rbc.phys.columbia.edu/USQCD/B-physics/

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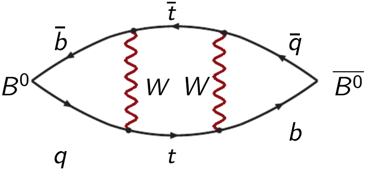
# Phenomenological Importance

- $B-\bar{B}$-mixing allows us to determine CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

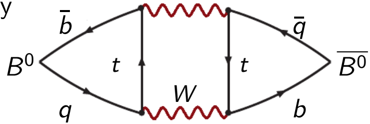
$$\left.\begin{array}{r} |V^*_{td}V_{tb}|\,\text{for}\,B_d-\text{mixing} \\ |V^*_{ts}V_{tb}|\,\text{for}\,B_s-\text{mixing} \end{array}\right\} \Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2}\eta_B S_0 m_{B_q} f_{B_q}^2 B_{B_q} |V^*_{tq}V_{tb}|^2$$



- Non-perturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$

- CKM matrix elements are extracted by

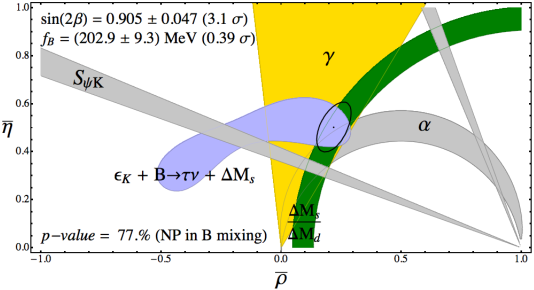
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



- Experimental error of $\Delta m_q$ is better than a percent; lattice uncertainty for $\xi$ is about 3%

# Unitarity Fit without Semileptonic Decays [Lunghi and Soni 2009]

- Avoids 1-2 $\sigma$ tension between inclusive and exclusive determinations of both $V_{ub}$ and $V_{cb}$
- Requires precise determination of $f_B$ (and also of $B \to \tau\nu$ and $\Delta M_s$)

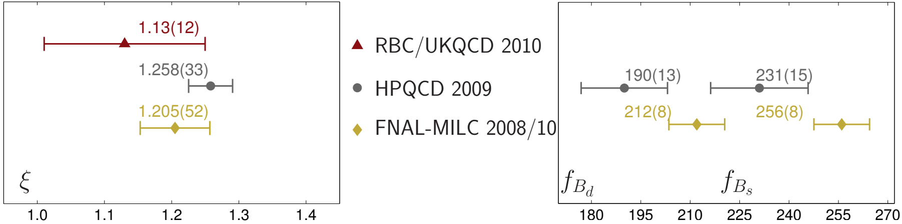


# Possible Deviations from the Standard Model

[Lunghi and Soni 2010]

- Experimental value for $\sin(2\beta)$ is $3.3\sigma$ lower than SM expectation
- Measured value for $BR(B \to \pi l \nu)$ is $2.8\sigma$ lower than predicted
- Most likely source of deviation is in $B_{d(s)}$ mixing and $\sin(2\beta)$; less likely in $B \to \tau\nu$

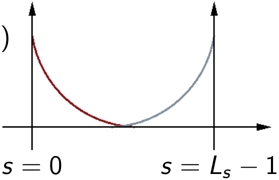
# Lattice Calculations of *B*-meson mixing Parameters



- HPQCD and FNAL-MILC result both based on the asqtad-improved staggered ensembles generated by MILC (FNAL-MILC uses new $r_1$)
- RBC/UKQCD result only exploratory study computed on $16^3$ lattices and using static approximation for the *b*-quarks
- This project aims for an independent cross-check at high precision using domain-wall light-quarks and relativistic heavy quarks performing
- Project started 2009/10 and we ask for time to continue in 2011/12

# 2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992, Shamir 1993]
- Iwasaki gauge action [Iwasaki 1983]



| L | $a$(fm) | $m_l$ | $m_s$ | $m_\pi$(MeV) | approx. # configs. | # time sources |
|---|---|---|---|---|---|---|
| 24 | $\approx$ 0.11 | 0.005 | 0.040 | 331 | 1636 | 1 |
| 24 | $\approx$ 0.11 | 0.010 | 0.040 | 419 | 1419 | 1 |
| 24 | $\approx$ 0.11 | 0.020 | 0.040 | 558 | 345 | 8 |
| 32 | $\approx$ 0.08 | 0.004 | 0.030 | 307 | 628 | 2 |
| 32 | $\approx$ 0.08 | 0.006 | 0.030 | 366 | 889 | 2 |
| 32 | $\approx$ 0.08 | 0.008 | 0.030 | 418 | 544 | 2 |

[C. Allton et al. 2008, Y. Aoki et al. 2010]

# Relativistic Heavy Quark Action for the *b*-Quarks

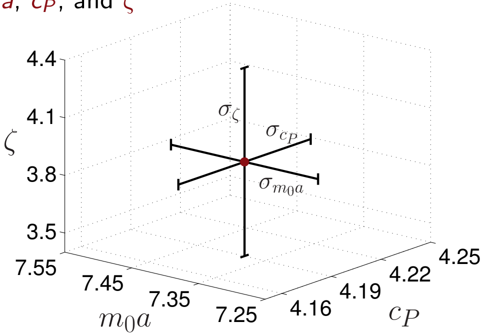
- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the *b*-quarks in 2-point and 3-point correlation functions
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie] by tuning all parameters of the clover action non-perturbatively
- Matching of lattice action to continuum through $\mathcal{O}(pa)$
  - Errors will be of $\mathcal{O}(a^2p^2)$
  - Heavy quark mass is treated to all orders in $m_ba$ $\Rightarrow$ coefficient of the $\mathcal{O}(a^2p^2)$ error is a function of $m_ba$
  - This function is bounded to be $\leq \mathcal{O}(1)$ [El Khadra, Kronfeld, Mackenzie]
  - Heavy-light spectrum quantities can be computed with discretization errors of the same order as in light-light quantities

# Tuning the Parameters for the RHQ Action

$$S = \sum_{n,n'} \bar{\Psi}_n \left\{ m_0 + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[ \vec{\gamma} \cdot \vec{D} - \frac{a\left(\vec{D}\right)^2}{2} \right] - a \sum_{\mu\nu} \frac{i c_P}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \Psi_{n'}$$

- Start from an educated guess for $m_0 a$, $c_P$, and $\zeta$

$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix} \pm \begin{bmatrix} \sigma_{m_0 a} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \sigma_{c_P} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \sigma_\zeta \end{bmatrix}$$



- Compute for all seven parameter sets

| | | | |
|---|---|---|---|
| spin-averaged mass | $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$ | → | 5403.1(1.1) MeV |
| hyperfine-splitting | $\Delta_M = (M_{B_s^*} - M_{B_s})$ | → | 49.0(1.5) MeV |
| ratio | $\frac{M_1}{M_2} = M_{\text{rest}}/M_{\text{kinetic}}$ | → | 1 |

- Assuming linearity

$$Y_r = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_r = J^{(3\times3)} \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_r + A^{(3\times1)} \qquad (r = 1, \dots, 7)$$

and defining

$$J = \left[ \frac{Y_3 - Y_2}{2\sigma_{m_0 a}}, \frac{Y_5 - Y_4}{2\sigma_{c_P}}, \frac{Y_7 - Y_6}{2\sigma_\zeta} \right] \qquad A = \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}_1 - J \times \begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}_1$$

- We extract the RHQ parameters and iterate until result is inside uncertainties

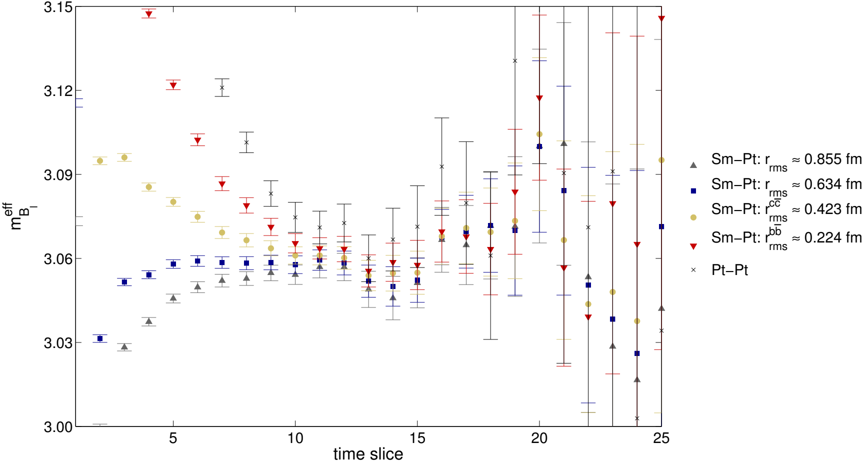
$$\begin{bmatrix} m_0 a \\ c_P \\ \zeta \end{bmatrix}^{\text{RHQ}} = J^{-1} \times \left( \begin{bmatrix} \overline{M} \\ \Delta_M \\ \frac{M_1}{M_2} \end{bmatrix}^{\text{PDG}} - A \right)$$

# Improvement of Tuning

- Tuning method pioneered on $24^3$ (a $\approx$ 0.11fm) by Min Li [M. Li 2009]
  Further studies by Hao Peng on $32^3$ (a $\approx$ 0.08fm) [H. Peng 2010]
  Exploratory studies; results not suitable for production

- Improvements and new setup
  - Use of point-source strange quark operators and Gaussian-smeared heavy quarks
  - Performed optimization study of smearing parameters
  - Significantly increased statistics
  - Only use of heavy-light quantities
  - Check on linearity assumption

# Improving the Signal by Smearing of Source



- Reduction of excited state contamination
- 818 measurements, $m^l_{\mathrm{sea}} = m^l_{\mathrm{val}} = 0.005$, $m_0 a = 7.38$, $c_P = 3.89$, $\zeta = 4.19$

# Tuned Parameters $24^3$

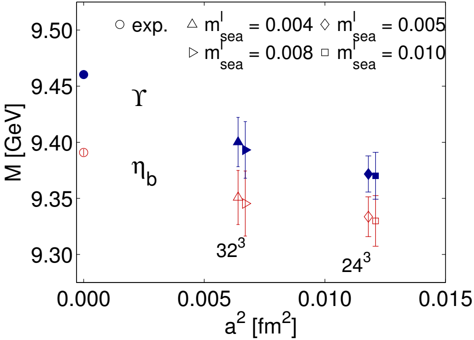
| $m^l_{sea}$ | $m_0a$ | $c_P$ | $\zeta$ |
|---|---|---|---|
| 0.005 | 8.4(1) | 5.7(2) | 3.1(1) |
| 0.010 | 8.5(1) | 5.8(3) | 3.1(2) |

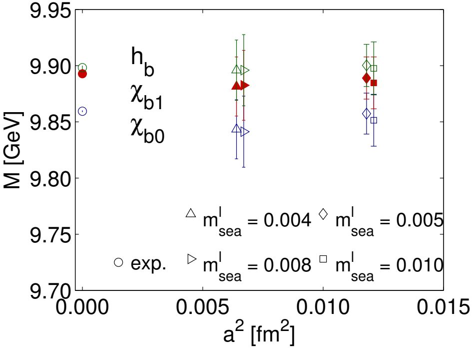
# Tuned Parameters $32^3$

| $m^l_{sea}$ | $m_0a$ | $c_P$ | $\zeta$ |
|---|---|---|---|
| 0.004 | 4.00(8) | 3.6(2) | 2.0(1) |
| 0.006 | in progress | | |
| 0.008 | 3.97(9) | 3.6(2) | 2.0(1) |

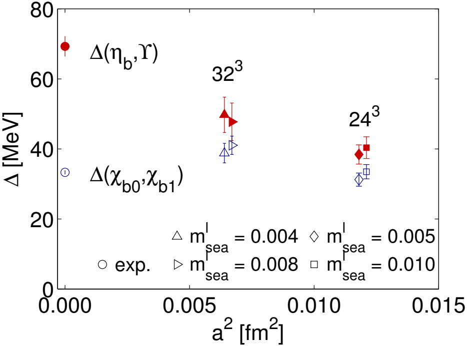
# Predictions for the Heavy-Heavy Masses

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the $B_s$ system we can predict bottomonium states and mass splittings



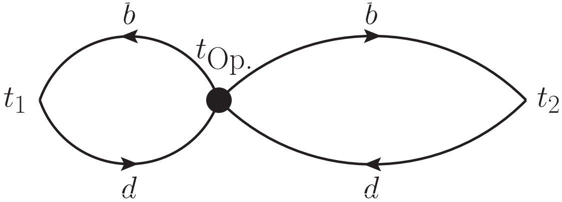


# Predictions for the Heavy-Heavy Mass-Splittings



- Publication on tuning and bottomonium spectroscopy is in preparation

# $B^0 - \overline{B^0}$ mixing matrix element calculation

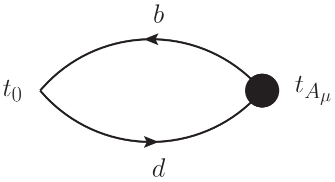


- Location of four-quark operator is fixed
- Location of $B$-mesons is varied over all possible time slices
- Need: one point-source light quark and one point-source heavy quark originating from operator location
- Propagators can be used for $B$- and $\overline{B}$-meson
- Project out zero-momentum component using a Gaussian sink

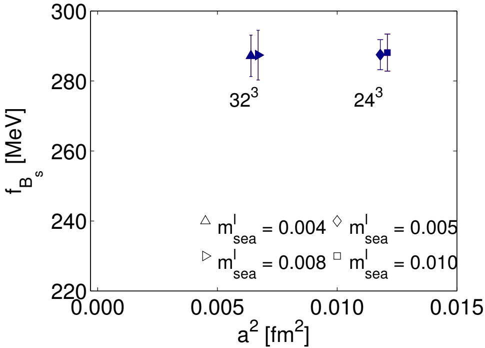
# Operator Improvement and Matching

- Rotate $b$-quark at the source to reduce discretization errors in the heavy-light current and the four-fermion operator
- Compute rotation parameter $d_1$ at tree-level in tadpole-improved lattice PT (improving operator to $\mathcal{O}(\alpha_s ap)$)
- Renormalization factors for matching of lattice operators to continuum operator are computed using 1-loop tadpole-improved lattice PT (truncation errors $\mathcal{O}(\alpha_s ap)$)
- Only one other operator at $\mathcal{O}(1/m_b)$ mixes with desired operator (at this order)
- For ratio $\xi$ much of the perturbative truncation error should cancel
  Phenomenologically most important quantity should be most reliable

# *B* meson decay constant calculation



- Re-use: point-source light quark and generate Gaussian smeared-source heavy quark
- Best signal found for using point sinks

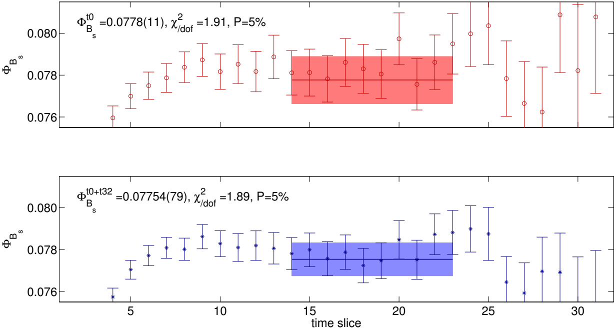


- Preliminary result for $f_{B_s}$
- Renormalization factor and coefficient for $\mathcal{O}(a)$ improvement only computed at tree-level
- Expect 1-loop correction to be 10-20%

# Proposed Generation of DWF Light Quark Propagators

| L | $m^l_{sea}$ | $m_{val}$ | time source per config | # propagators 2009-2011 | # propagators 2011/2012 |
|---|---|---|---|---|---|
| 32 | 0.004 | 0.004, 0.006, 0.008, 0.025, 0.030 | 2 | 628 | 628 |
| 32 | 0.004 | 0.0272 | 2 | 1256 | — |
| 32 | 0.006 | 0.004, 0.006, 0.008, 0.025, 0.030 | 2 | 445 | 1333 |
| 32 | 0.006 | 0.0272 | 2 | 1778 | — |
| 32 | 0.008 | 0.004, 0.006, 0.008, 0.025, 0.030 | 2 | 544 | 544 |
| 32 | 0.008 | 0.0272 | 2 | 1088 | — |
| 24 | 0.005 | 0.005, 0.010, 0.020, 0.030, 0.040 | 1 | 1636 | — |
| 24 | 0.005 | 0.0343 | 1 | 1636 | — |
| 24 | 0.010 | 0.005, 0.010, 0.020, 0.030, 0.040 | 1 | 1419 | — |
| 24 | 0.010 | 0.0343 | 1 | 1419 | — |
| 24 | 0.020 | 0.005, 0.010, 0.020, 0.030, 0.040 | 1 | 345 | — |
| 24 | 0.020 | 0.0343 | 8 | 2760 | — |

# Second source per configuration for $32^3$ ensembles



- Leading order contribution for decay amplitude on $m^l_{\text{sea}} = 0.004$, $m_{\text{val}} = 0.0272$, 628 configurations
- Adding second source reduces statistical error by expected factor of $\sqrt{2}$

# Requested Computing Time

| | |
|---|---|
| $32^3$ $a \approx 0.08$ fm domain-wall propagators | 7.313 $\times 10^6$ jpsi core-hours |
| $32^3$ $a \approx 0.08$ fm clover propagators | 1.755 $\times 10^6$ jpsi core-hours |
| $24^3$ $a \approx 0.11$ fm clover propagators | 0.428 $\times 10^6$ jpsi core-hours |
| 2-point and 3-point correlators and analysis | 0.915 $\times 10^6$ jpsi core-hours |
| Total | 10.411 $\times 10^6$ jpsi core-hours |

- Majority of time devoted to domain-wall propagator generation
- All domain-wall propagators are saved on tape
- Preference to continue running on Fermilab clusters
- Would like to retain rights to use these propagators for $D$-meson decay constants and beyond the Standard Model contributions to $B^0 - \overline{B^0}$ mixing

# Projected Error Budget

| | $f_B$ | $\xi$ |
|---|---|---|
| statistics | 3% | 3% |
| chiral extrapolation | 3% | 2% |
| uncertainty in $g_{B^*B\pi}$ | 1% | 1% |
| renormalization factors | 5% | 2% |
| scale and quark mass uncertainties | 2% | 1% |
| finite volume error | 1% | 0.5% |
| (heavy-quark) discretization | 2% | 1% |
| total | 7% | 4% |

- Conservative estimate based on comparison with static result and the work of other collaborations — hopefully we do even better
- Expect competitive results to [FNAL-MILC 2008/10] and [HPQCD 2009]

# Conclusion

- This project aims for a precise determination of $B$-meson decay constants and neutral $B$-meson mixing parameters
  - Using $2+1$ flavor dynamical domain-wall light quarks
  - Nonperturbatively tuned relativistic heavy quarks
  - Computation uses two lattice spacings, multiple quark masses, and heavy-meson chiral perturbation theory
- Fulfills one of the key goals in flavor physics of USQCD [2007 white paper]
- Result will place an important constraint in the quark flavor sector when used in unitarity triangle analysis
- We expect (preliminary) results for $f_B$ and $B^0 - \overline{B^0}$ mixing next year

# Question from the SPC

This proposal addresses phenomenologically very important quantities such as the $B$ meson decay constant and neutral $B$ meson mixing parameters. The SPC would like to learn more about your long term plans for $B$ Physics using domain-wall light and relativistic heavy quarks. What kind of errors do you want to achieve in the long term, and will they be small enough to have phenomenological impact?

- The authors and the RBC collaboration are committed to continue the heavy-light physics program in the future. The internal discussion for future ensemble generation on QCDCQ aka BG/Q is in progress.
- Within a year we hopefully know for sure what our biggest uncertainties are and we intend to address those first.
- Results from different methods with a few percent errors are important for a strong phenomenological impact.

# Possibilities for future activities

- Adding a third, finer lattice spacing to the set of DWF-Iwasaki ensembles for improving the continuum extrapolation.
  Unsolved problem of frozen topology
- Enhance the chiral extrapolation by generating a DWF-Iwasaki ensemble with $a \approx 0.08$fm in a larger volume with lighter pions
- Reduce uncertainties from renormalization by using (mostly) non-perturbative renormalization
- Extend computation to other quantities:
  $B \to \pi l \nu$, BSM operators, charm physics